Applications of the KKM property to coincidence theorems, equilibrium problems, minimax inequalities and variational relation problems

Monica Patriche

Faculty of Mathematics and Computer Science, University of Bucharest, 14 Academiei Street, Bucharest, Romania
e-mail: `monica.patriche@yahoo.com`

The aim of this talk is twofold: firstly, to establish a Fan type geometric result and to apply it in order to obtain some coincidence-like theorems for the case when the images of the correspondences are not convex. Further, new theorems concerning the existence of solutions for equilibrium problems are provided. For coincidence theorems, the reader is referred, for instance, to [2], [3], [10], [11], [13], [19]. The equilibrium problems have been studied, for example, in [1], [2], [3], [6], [7], [9], [12], [18]. Our goal is also to investigate whether the class of minimax inequalities can be extended. In fact, we obtain a new general minimax inequality of the following type: $\inf_{x \in X} \sup_{y \in Y} t(x,y) \leq \frac{\sup_{y \in Y} \inf_{z \in Z} q(y,z)}{\inf_{z \in Z} \sup_{x \in X} p(x,z)}$. Its study is motivated and inspired by the results obtained, for instance, in [1], [2], [3], [19], which concern the three-function inequality: $inf_{x \in X} t(x,x) \leq \sup_{y \in Y} \inf_{z \in Z} q(y,z) + \sup_{z \in Z} \inf_{y \in Y} p(z,y)$. We intend to connect, in forthcoming papers, the present results with the new ones, which consider the equilibrium in games, and are established in [15] or [16]. Another recent result, a contribution of the author, regarding minimax inequalities for discontinuous correspondences, is [14]. In this first part, the originality consists of introducing a new type of properly quasi-convex-like correspondences, which proved to play an important role in our results. The method of proof is based on the well known KKM property.

There exists a large literature containing applications of the KKM property to coincidence theorems, equilibrium theorems, maximal element theorems and minimax inequalities. We refer the reader, for instance, to M. Balaj [2], Lin, Ansari and Wu [10], Lin and Wan [11] or Park [13]. Secondly, we explore how the KKM principle can promote new more theorems which show the existence of solutions for some classes of variational relation problems. We emphasize that, here, the method of application of the KKM property is new and provides new hypotheses for our research. The presented results are mainly published in [17].

Bibliography

[1] M. Balaj, An intersection theorem with applications in minimax theory and equilibrium problem, *J. Math. Anal. Appl.* 336 (2007) 363–371

[2] M. Balaj, Coincidence and maximal element theorems and their applications to generalized equilibrium problems and minimax inequalities, Nonlinear Anal. 68 (2008), 3962-3971.

[3] M. Balaj and L. J. Lin, Fixed points, coincidence points and maximal elements with applications to generalized equilibrium problems and minimax theory, *Nonlinear Anal.* 70 (2009), 393–403.

[4] M. Balaj, Solution existence for two types of mixed variational relation problems, Applicable Analysis, DOI: 10.1080/00036811.2015.1006204

[5] A. Dhara and D.T. Luc, A solution method for linear variational relation problems, J. Global Optimization 59 (2014) 729-756

[6] X. P. Ding, T. M. Ding, KKM type theorems and generalized vector equilibrium problems in noncompact FC-spaces, J. Math. Anal. Appl. 331 (2007) 1230–1245

[7] B. V. Dinh, P. G. Hung and L. D. Muu, Bilevel Optimization as a Regularization Approach to Pseudomonotone Equilibrium Problems, Numerical Functional Analysis and Optimization 35 (2014) 539-563

[8] M. Fakhar, M. Lotfipour, J. Zafarani, On the Br´ezis Nirenberg Stampacchia-type theorems and their applications, Journal of Global Optimization 55 (2013), 751–770

[9] S.-M. Guu and J. Li, Vector quasi-equilibrium problems: separation, saddle points and error bounds for the solution set, Journal of Global Optimization, 58, (2014), 751-767

[10] L.J. Lin, Q. H. Ansari and J. Y. Wu, Geometric properties and coincidence theorems with applications to generalized vector equilibrium problems, J. Optim. Theory Appl. 117(2003) 121-137

[11] L. J. Lin, W.P. Wan, KKM type theorems and coincidence theorems with applications to the existence of equilibrium, J. Optim. Theory Appl. 123 (2004) 105-122.

[12] L. J. Lin, H.L. Chen, The study of KKM theorems with applications to vector equilibrium problems and implict vector variational inequalities problems. J. Glob. Optim. 32 (2005), 135-157

[13] S. Park, Coincidence points and maximal elements of multifunctions on convex spaces, Comment. Math. Univ. Carolina 36 (1995) 57-67.

[14] M. Patriche, Minimax theorems for set-valued maps without continuity assumptions, Optimization, 65 (2016), 957-976

[15] M. Patriche, Existence of equilibrium for generalized games in choice form and applications, Optimization, 65 (2016), 2135–2151

[16] M. Patriche, Existence of the equilibrium in choice, J. Optim. Theory Appl., 175 (2017), 58–171

[17] M. Patriche, Applications of the KKM property to coincidence theorems, equilibrium problems, minimax inequalities and variational relation problems, Numerical Functional Analysis and Optimization, 38 (2017), 4, 523-548

[18] P. H. Sach and N. B. Minh, New Results on Henig Proper Generalized Vector Quasiequilibrium Problems, Numerical Functional Analysis and Optimization, 36 (2015), 387-403

[19] M. Yang and N. Huang, Coicidence theorems for noncompact RC-maps in abstract convex spaces with applications, Bull. Korean Math. Soc. 49 (2012), 1147–1161