About Cartesian Product of Two Subcategories

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Summary. We examine a categorial construction which permit to obtained a new reflective subcategories with a specal properties.
Key words: Reflective subcategories, the pairs of conjugated subcategories, the right product of the two subcategories.
Results. Let $\mathcal{K}$ be a coreflective subcategory, and $\mathcal{R}$ a reflective subcategory of the category of locally convex topological vector Hausdorff spaces $\mathcal{C}_2\mathcal{V}$ with respective functors $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$ and $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$.

Concerning of the terminology and notation see [1]. Note by $\mu\mathcal{K} = \{m \in \mathcal{M}ono \mid k(m) \in \mathcal{I}so\}$, $\varepsilon\mathcal{R} = \{e \in \mathcal{E}pi \mid r(e) \in \mathcal{I}so\}$. Further for an arbitrary object X of the category $\mathcal{C}_2\mathcal{V}$ we examine the follows construction: let $k^X : kX \longrightarrow X$ is $\mathcal{K}$-coreplique, and $r^{kX} : kX \longrightarrow rkX$-replique of the respective objects. On the morphism k^X and r^{kX} we construct the cocartesian square

$$\overline{v}^X \cdot k^X = u^X \cdot r^{kX}. \tag{1}$$

Definition 1. *1. The full subcategory of all isomorphic objects with the type of objects is called* $\overline{v}X$ *cartesian product of the subcategories* $\mathcal{K}$ *and* $\mathcal{R}$*, noted by* $\overline{v} = \mathcal{K} *_{dc} \mathcal{R}$.

2.The diagram of cartesian product is called the diagram of cartesian product of the pair of conjugate subcategories $(\mathcal{K}, \mathcal{R})$ *(Diagram (RCP)).*

Definition 2. *The full subcategory of all isomorphic objects with the objects of type* $\overline{v}X$ *is called cartesian product of the subcategories* $\mathcal{K}$ *and* $\mathcal{R}$*, note by* $\overline{\mathcal{W}} = \mathcal{K} *_{sc} \mathcal{R}$.

Lemma 1. $\mathcal{R} \subset \mathcal{K} *_{dc} \mathcal{R}$.

Theorem 1. *The application* $X \mapsto \overline{v}X$ *defined a functor*

$$\overline{v} : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K} *_{dc} \mathcal{R}.$$

We examine the following condition:

(RCP) For any object X of the category $\mathcal{C}_2\mathcal{V}$ in the diagram (RCP) the morphism u^X belongs to the class $\mu\mathcal{K}$.

Theorem 2. *Let it be a pairs of the subcategories* $(\mathcal{K}, \mathcal{R})$ *verify the condition (RCP). Then* $\overline{v}$ *it is a reflector functor.*

Theorem 3. *Let* $\mathcal{K}$ *be a coreflective subcategory, but* $\mathcal{R}$ *is a reflective subcategory of the category* $\mathcal{C}_2\mathcal{V}$*,* $\widetilde{\mathcal{M}}$ *- the subcategory of the spaces with Mackey topology,* $\mathcal{S}$ *is the subcategory of the spaces with weak topology. If* $\mathcal{K} \subset \widetilde{\mathcal{M}}$*, but* $\mathcal{S} \subset \mathcal{R}$*, then the pair of subcategories* $(\mathcal{K}, \mathcal{R})$ *verify condition (RCP) the cartesian product is a reflective subcategory.*

Examples. 1. *For any coreflective subcategory* $\mathcal{K}$ *we have* $\mathcal{K} *_{dc} \Pi = \Pi$*,* Π*-reflective subcategory of the complete space with weak topology.*

2. *For any coreflective subcategory* $\mathcal{K}$ *we have* $\mathcal{K} *_{dc} \mathcal{S} = \mathcal{S}$*,* $\mathcal{S}$*-reflective subcategory of the space with weak topology.*

Theorem 4. *Let* $(\mathcal{K}, \mathcal{L})$ *a pair of conjugate subcategories, and* $\mathcal{R}$ *a reflective subcategory of the category* $\mathcal{C}_2\mathcal{V}$*. Then:*

1. $\mathcal{K} *_{dc} \mathcal{R} = \mathcal{Q}_{\varepsilon\mathcal{L}}(\mathcal{R})$*, where* $\mathcal{Q}_{\varepsilon\mathcal{L}}(\mathcal{R})$ *is the full subcategory of all* $\varepsilon\mathcal{L}$*-factorobjects of objects of the subcategory* $\mathcal{R}$*.*
2. $\mathcal{K} *_{dc} \mathcal{R}$ *is a reflective subcategory of the category* $\mathcal{C}_2\mathcal{V}$*.*
3. *The subcategory* $\mathcal{K} *_{dc} \mathcal{R}$ *is closed in relation to* $\varepsilon\mathcal{L}$*-factorobjects.*
4. $\overline{v} \cdot k = r \cdot k$*.*
5. *If* $r(\mathcal{K}) \subset \mathcal{K}$*, then the coreflector functor* $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$ *and the reflector* $\overline{v} : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K} *_{dc} \mathcal{R}$ *commute:* $k \cdot \overline{v} = \overline{v} \cdot k$*.*

Theorem 5. *Let* $\mathcal{K}$ *(respective* $\mathcal{R}$*) a coreflective subcategory (respective: reflective)of the category* $\mathcal{C}_2\mathcal{V}$*, those functors* $k : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{K}$ *and* $r : \mathcal{C}_2\mathcal{V} \longrightarrow \mathcal{R}$ *commute:* $k \cdot r = r \cdot k$*. Then*

$$\mathcal{K} *_{dc} \mathcal{R} = \mathcal{K} *_{d} \mathcal{R}.$$

Bibliography

[1] Botnaru D.,*Structures bicatégorielles complémentaires*, ROMAI Journal, v.5, N.2, 2009, p. 5-27.

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