Intermediate representation of hyperbolic manifolds by equidistant polyhedra

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The n-dimensional hyperbolic manifold is usually considered as a homogeneous complex. One compact polytope is sufficient to describe the manifold by pairwise identifying its faces. We discuss an "intermediate" way of representing the manifold by an equidistant (generalized) polyhedron over compact basis as a submanifold of codimension one.

First such representation was proposed for a symmetric 3-submanifold of the Davis hyperbolic

4-manifold on regular 120-cells. A totally geodesic 2-submanifold (or border), which is a surface of genus 4 with a Platonic map $\{5, 5\}$, serves as a compact basis for equidistant polyhedra. We call them also lens hyperbolic polytopes. Remark that from the combinatorial point of view, the above Platonic surface $\{5, 5\}$ of genus 4 is exactly the large star dodecahedron $\{5, 5/2\}$.
For some equidistant polyhedra with elliptic, parabolic and hyperbolic incidences of hyperfaces at vertices, respectively over Platonic maps $\{4, 5\}$, $\{5, 4\}$, $\{5, 5\}$ on surface of genus 4, we construct examples that lead to compact or non-compact hyperbolic 3-manifolds. The geometry of such manifolds is described. In dimension 4 the star regular polytope $\{5, 3, 5/2\}$, or 3-submanifold locally geodesic immersed in Davis 4-manifold, can be considered as a compact basis for an equidistant 4-dimensional polyhedron over Platonic map $\{5, 3, 5\}$.
In a general case, we start with cells complexes over regular (semiregular or k-regular) maps on totally geodesic hyperbolic submanifolds and indicate pairs of faces of the lens polytope that lead to hyperbolic manifolds. Thus, using the proposed method, we construct manifolds starting from their submanifolds not, as usually, from fundamental polytopes. Algebraic aspects of this approach (embedding and extensions of the fundamental group) are discussed.