SCATTERING OF A TWO-DIMENSIONAL SELF-HEALING BEAM AT THE INTERFACE OF A NANOSTRUCTURED MEDIUM WITH THE VACUUM

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Abstract. In this paper the Fresnel's formula are generalized to the case of a two-dimensional (2D) self-healing beam incident on the interface of the vacuum with a nonmagnetic, homogeneous on average and isotropic nanostructured medium with spatial dispersion. This approach is used for simplifying the problem of finding effective parameters of the nanostructured medium. It is found that, although the 2D beam can have only a finite length and finite energy in contrast to the 3D one, conditions are provided for the conservation of beam configuration and for ensuring his self-healing properties.

Key words: nanostructured medium, self-healing beam, reflection/refraction, scattering indicatrix, dielectric function, additional boundary conditions, longitudinal/transverse mode.

1. INTRODUCTION

Fabrication and investigation of various nanostructured media has been of increasing interest in the last decade. Particularly, development of new methods for investigating nanostructured materials is of a great importance for today`s technologies. Usually, a nanostructured medium represents a complex heterogeneous system composed of different components. The scattering of light at the boundaries of inhomogeneities and the related complex physical processes hinders finding of averaged parameters of such a medium [1–3]. Using self-healing beams (for instance Bessel beams) allows one to avoid such difficulties [4].

We propose in this paper to make use of 2D self-healing, non-diffractive beams, with dimensionality like the Fresnel equations, to simplify the issue of finding the effective parameters when probing a nanostructured medium.

2. MATHEMATICAL MODEL OF BEAM REFLECTION FORM THE INTERFACE OF THE NANOSTRUCTURED MEDIUM

We will use in the section a generalization [5] of the mathematical model of nanostructured medium [3] for the case of a non-magnetic and, on average, a homogeneous and isotropic porous medium [3]. A phenomenological approach will be applied, based on using the dielectric function of inhomogeneous medium ε(r,t). For a medium with spatial dispersion one can write [6]:

$$D_i(\vec{r},\omega) = \varepsilon_0 \int d^3r' \, \varepsilon_{ik}(\vec{r}-\vec{r}',\omega) E_k(\vec{r}',\omega), \tag{1}$$

where $\vec{D}(\vec{r})$ is the induction, $\vec{E}(\vec{r})$ is the electric field strength.

The $\varepsilon_{ik}(\vec{r},\omega)$ function is expressed as:

$$\varepsilon_{ik}(\vec{r},\omega) = \frac{1}{(2\pi)^{3/2}} \int d^3k \tilde{\varepsilon}_{ik}(\vec{k},\omega) \exp(i\vec{k}\vec{r}), \tag{2}$$

where

$$\tilde{\varepsilon}_{ik}(\vec{k},\omega) = \tilde{\varepsilon}_l(k,\omega)\left(\delta_{ik} - \frac{k_i k_k}{k^2}\right) + \tilde{\varepsilon}_t(k,\omega)\frac{k_i k_k}{k^2}, \tag{3}$$

$$\tilde{\varepsilon}_t(k,\omega)\big|_{k,\omega\to 0} = \varepsilon_{eff}, \tag{4}$$

$$\tilde{\varepsilon}_l(k,\omega)\big|_{\omega\to 0} = (1/2\pi)^{3/2}\left[\varepsilon_{eff}{}^{-1} + \eta^{-1}k^2/(k^2-h^2)\right]^{-1}. \tag{5}$$

With such a choice of $\tilde{\varepsilon}_{ik}(\vec{k},\omega)$, different modes of electromagnetic oscillations will exist in the system: the usual bright modes (transversal oscillations) with two types of polarizations, which interact with matter by means of the effective parameter ε_{eff} [6], and the dark (longitudinal) mode described by the wave vector *h*, which interacts with matter by means of the effective parameter

$$\eta = [\varepsilon_{Coul}{}^{-1} - \varepsilon_{eff}{}^{-1}]^{-1}, \tag{6}$$

where

$$\varepsilon_{Coul} = [1 + f(\varepsilon^{-1} - 1)], \tag{7}$$

f is the ration of the material content in the porous medium as compared to the vacuum content, ε is the dielectric constant of the material [5].

The Green's function for the potential $G(\vec{r}-\vec{r}')$ is actually the modified Coulomb's law for the given medium

$$G(\vec{r}-\vec{r}\,')=\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{eff}\,|\vec{r}-\vec{r}\,'|}+\frac{\cos(h\,|\vec{r}-\vec{r}\,'|)}{\eta\,|\vec{r}-\vec{r}\,'|}\right]. \tag{8}$$

Note that the field near any charge, including the one bound at the interface of media, splits into two terms:

$$G(\vec{r}-\vec{r}\,')\Big|_{|\vec{r}-\vec{r}\,'|\to 0}\to\frac{1}{4\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{eff}\,|\vec{r}-\vec{r}\,'|}+\frac{1}{\eta\,|\vec{r}-\vec{r}\,'|}\right], \tag{9}$$

so that each of modes give an independent contribution to the formation of the total charge field, according to the effective dielectric constants of each mode ε_{eff}, η.

At the same time, the above mentioned phenomenological approach does not allow one to estimate the value of dark mode wave vector *h*. Dark modes of the type used in this paper can exist in 3D systems with elements of 1D structure (cylinders, channels, pores, etc.) [7]. We will consider for simplicity that the porous system can generally be described as a set of 2D lattices filling the 3D space and being oriented at various angles with respect to each other. A formula from ref. [3] is used for estimations.

The phenomena of reflection/refraction on the surface of porous material should be considered in details for a quantitative description of the beam reflection, when al list two types of modes can exist independently of each other in a medium.

Recall that in a classical Fresnel problem only one type of modes, namely the transversal ones, are taken into consideration, and two boundary conditions for the electric *E* and magnetic *H* fields are enough to be used for the problem of reflection/refraction. The conservation of tangential components of E_y and H_x for the s-polarization, and E_x and H_y for the p-polarization should be considered when using the notations from ref. [6].

In our case, if two different types of modes do exist in the porous material, it is enough to introduce only one *additional boundary condition* (ABC). Further, the problem will be solved by finding the reflection coefficient R for the plane transversal wave incident from the vacuum on the surface of porous material for different polarizations and different types of the porous material. It was shown in a previous study [3] that there is only one reflected (r) wave in the vacuum for a transversal wave incident from the vacuum on the vacuum/porous material boundary for the p-polarization, and this reflected wave is also transversal. At the same time there are both the transversal (T) and longitudinal (L) components for the transmitted (t) wave in the porous medium (Fig. 1).

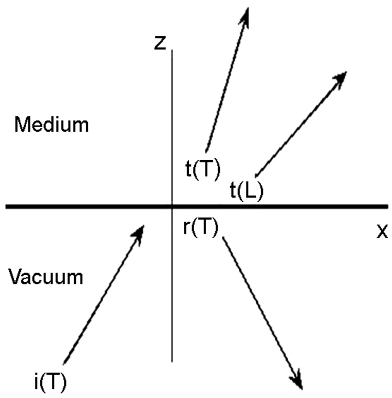

Fig. 1 – The diagram of the reflection/refraction of the incident (i) transversal wave from the vacuum on the vacuum/porous material boundary for the p-polarization.

The basic relations used for electric and magnetic fields for the “bright” and “dark” modes in the porous medium for the p-polarization can be presented as follows.

1. For the usual transverse wave (“bright” mode):

$$\begin{aligned}
&\vec{E} = C\,\frac{Z_0}{k_0} e^{i(\sqrt{\varepsilon_{eff} k_0^2 - k_x^2}\, z + k_x x - \omega t)} \{\sqrt{\varepsilon_{eff} k_0^2 - k_x^2} / \varepsilon_{eff},\quad 0,\quad -k_x / \varepsilon_{eff}\} \\
&\vec{H} = C\, e^{i(\sqrt{\varepsilon_{eff} k_0^2 - k_x^2}\, z + k_x x - \omega t)} \{0,\quad 1,\quad 0\} \\
&\vec{P}_w = C \frac{\varepsilon_0 Z_0}{k_0} e^{i(\sqrt{\varepsilon_{eff} k_0^2 - k_x^2}\, z + k_x x - \omega t)} \{(1 - 1/\varepsilon_{eff})\sqrt{\varepsilon_{eff} k_0^2 - k_x^2},\quad 0,\quad -(1 - 1/\varepsilon_{eff}) k_x\}
\end{aligned} \tag{10}$$

where {x,y,z} is a vector, $k_0 = \omega/c$ is the numeric value of the wave vector in vacuum, $Z_o = \sqrt{\mu_o/\varepsilon_o} = 376.6\ \Omega$ is the vacuum impedance, $\vec{k} = \{k_x, 0, \sqrt{\varepsilon_{eff} k_0^2 - k_x^2}\}$, C is a coefficient determined from the solution of the following linear equation:

$$A = \begin{bmatrix}
-k_x / h, & -\sqrt{\varepsilon_{eff} k_0^2 - k_x^2} / k_0 \sqrt{\varepsilon_{eff}}, & -\sqrt{k_0^2 - k_x^2} / k_0, & \sqrt{k_0^2 - k_x^2} / k_0 & (Ex) \\
0, & -\sqrt{\varepsilon_{eff}}, & 1, & 1 & (Hy) \\
-(\eta - 1)\sqrt{h^2 - k_x^2} / h, & (\varepsilon_{eff} - 1) k_x / k_0 \sqrt{\varepsilon_{eff}}, & 0, & 0 & (Pz) \\
t(\mathrm{L}), & t(\mathrm{T}), & r(\mathrm{T}), & i(\mathrm{T}) &
\end{bmatrix}, \tag{11}$$

where, for instance, the first column describes the transmitted longitudinal wave, the last column describes the incident transverse wave, the first row is for the components of the electric field (E_x) etc.

2. For the longitudinal wave (“dark” mode):

$$\vec{E} = C\frac{Z_0}{k_0}e^{i(k_x x + z\sqrt{h^2 - k_x{}^2} - \omega t)}\{k_x, \quad 0, \quad \sqrt{h^2 - k_x{}^2}\}$$
$$\vec{H} = Ce^{i(k_x x + z\sqrt{h^2 - k_x{}^2} - \omega t)}\{0, \quad 0, \quad 0\} \quad . \tag{12}$$
$$\vec{P}_d = C\frac{\varepsilon_0 Z_0}{k_0}e^{i(k_x x + z\sqrt{h^2 - k_x{}^2} - \omega t)}\{(\eta - 1)k_x, \quad 0, \quad (\eta - 1)\sqrt{h^2 - k_x{}^2}\}$$

According to ref. [3], the value of the dark mode wave vector h is defined as follows:

$$h \approx \frac{1}{a}\sqrt{\varepsilon a^2 k_0^2 + \frac{f_h}{G}}\,, \tag{13}$$

where

$$f_h = f + 1/\varepsilon\,. \tag{14}$$

3. ADDITIONAL BOUNDARY CONDITIONS AND THE REFLECTION COEFFICIENT

The component of the polarization perpendicular to the surface of the medium at the boundary of the porous medium with the vacuum has to be equal to zero [8]. We will use additional boundary condition in the form of

$$\left[\vec{P}_\Sigma\right]_\perp \equiv 0\,, \tag{15}$$

where

$$\vec{P}_\Sigma = (\vec{P}_w + \vec{P}_d) \tag{16}$$

is the electrical polarization of the medium at the boundary with the vacuum, summarized for all the modes.

The Eq. (15) transforms to a trivial identity 0≡0 in the case of s-polarization, which means that the longitudinal wave is not excited. In the case of p-polarization, the longitudinal wave is excited, but the polarization vector is differently expressed for the bright and dark modes. Therefore, we have

$$\vec{P}_w = (\varepsilon_{eff} - 1)\vec{E}\,, \tag{17}$$

for the transversal bright mode, and

$$\vec{P}_d = (\eta - 1)\vec{E}\,. \tag{18}$$

for the longitudinal dark mode.

The nanostructured medium constituted from a material with dielectric constant ε and the concentration f, and from vacuum with concentration (1-f), can be considered as a metamaterial with some effective parameters. In the case of absence of dark modes, the nanostructured medium is described by usual effective parameters [9]. In the presence of a dark mode with parameters h, ε_{eff}, η, additional boundary conditions are used at the boundary of the medium, as discussed above Eq. (15). In such a case, the reflection coefficient from the nanostructured medium is calculated from the matrix presenting the linear Eq. (11), and it is given by the following expression:

$$R = \frac{\varepsilon_{eff} k_z - \sqrt{\varepsilon_{eff} k^2 - k_x^2} - \left[\left(\varepsilon_{eff} - 1\right) / \left(\eta - 1\right)\right]\left(k_x / h\right)^2 \sqrt{h^2 - k_x^2}}{\varepsilon_{eff} k_z + \sqrt{\varepsilon_{eff} k^2 - k_x^2} + \left[\left(\varepsilon_{eff} - 1\right) / \left(\eta - 1\right)\right]\left(k_x / h\right)^2 \sqrt{h^2 - k_x^2}} . \quad (19)$$

The reflected ray shifts to different directions depending on the sign of the refraction index (Fig. 3). The observed virtual image of the light source also shifts in different directions relative to the mirror image at the interface medium/vacuum.

4. DISTRIBUTION OF THE ELECTRIC FIELD INTENSITY FOR A TWO-DIMENSION SELF-HEALING BEAM

Our further calculations will be performed for a coherent light source emitting in the z direction [10]. The 2D wavefield of the beam is described by the following formula

$$A(\rho,\theta) = \sum_{l=-M}^{M} S(l) \exp(il\theta - i\alpha(l)) H_l^{(1)}(k_0\rho) , \quad (20)$$

where

$$S(l) = \exp\left[\left(0.008i\left|l\right|^2 / M\right) - \left(0.5\left|l\right|^2 / M\right)\right], \quad (21)$$

ρ and θ are the polar coordinates, z = ρ·cosθ, A = E(H) is the electrical(magnetic) field perpendicular to the ZOX plane, $\alpha(l) = \arg(H_l^{(1)}(\mathrm{M}))$, $H_l^{(1)}(.)$ is the Hankel function, and M >> 1. Using of the 2D self-healing non-diffractive beam is convenient for generalization of the Fressnel formula, which is also essentially two-dimensional. One can see that the self-healing beam of two types of polarization can obtain his properties at $\rho \approx M/k_0$ in a limited interval of 10^{-2} Mλ (Fig. 2). His width is of the order of λ and the width practically does not change when M→∞.

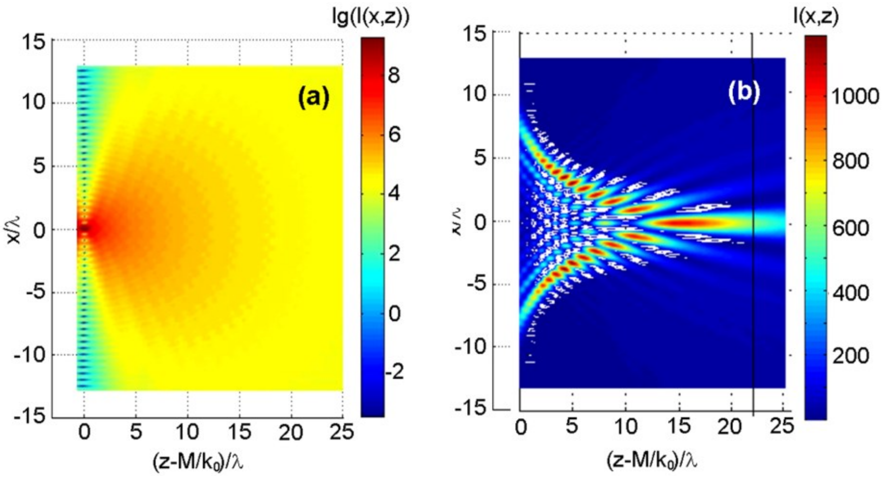

Fig. 2 – (a) Distribution of intensity for a 2D Gauss beam with S(*l*) ≡ 1) for M = 2000.
(b) Distribution of intensity for a 2D self-healing beam with S(*l*) defined from equation (21) for M = 2000.

The 2D self-healing beams feature their properties only on a finite length, for instance of 10λ÷100λ for M = $3{\cdot}10^3$. However, the 3D beams actually have, in practice, also only a finite length. On the other hand, in contrast to 3D beams, the overall emission power of a 2D beam is finite even theoretically.

The 2D beam upon scattering on the interface splits into separate plane waves, which can be detected at such distances from the interface, at which the Fraunhofer diffraction is usually observed. The fact that the beam is self-healing allows one to not take into consideration the scattering inside the nanostructured medium [4, 11], to neglect the phenomenon of back scattering, and to simplify the problem of finding the effective parameters of the nanostructured medium.

5. SCATTERING OF THE 2D SELF-HEALING BEAM AT SMALL ANGLES OF INCIDENCE AT THE INTERFACE

Each of the plane waves changes the component of the wave vector normal to the surface $k_\perp$ to a similar value but with opposite direction upon interaction with the interface. The amplitude of the wavefield changes proportionally to the R value from the Eq. (19) upon the reflection from the surface.

The radiation beam represents by itself a waveguide for its energy. We assume in calculations that the components of the wave vectors along the beam axis are directed towards the direction of energy flow (Fig. 3). However, their values are different for different waves, since the geometrical sizes of beams are finite, and the value of k_0 from Eq. (20) is just the module of the wave vector for any of plane waves constituting the beam. Therefore, first we find the components of the wave vectors $k_\perp$ perpendicular to the beam axis, and then the components of the wave vectors $k_{||}$ along the beam axis. The components of the wave vectors $k_\perp$ perpendicular to the beam axis are found from the Fourier decomposition of the field amplitude distribution along the line perpendicular to the beam axis.

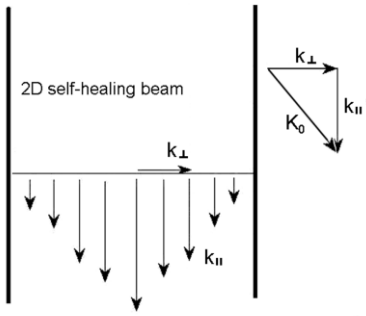

Fig. 3 – Structure of the 2D self-healing beam.

Figure 4c illustrates scattering indicatrix of a 2D self-healing beam for small angles of incidence at the interface with a nanostructured medium, calculated for different types of polarizations, such as *s*-polarization (black curve); *p*-polarization without taking into consideration the existence of black mode (blue curve); and *p*-polarization with taking into consideration the existence of black mode (red curve). The scattering indicatrix for the case of reflection from an ideally reflecting surface with R = –1 is shown in Fig. 4a for the purpose of comparison, and the indicatrix of scattering a plane wave by a strip with the width d = λ is illustrated in Fig. 4b. One should notice that calculations shown that the indicatrix is practically not affected by introducing absorption in the medium at the level of $\varepsilon = 16 - 2i$.

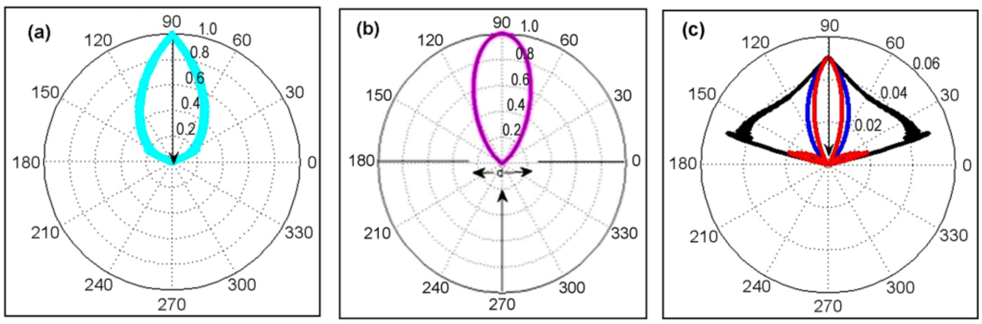

Fig. 4 – (a) Scattering indicatrix for an ideally reflecting surface with R = –1. (b) Indicatrix of scattering a plane wave by a strip with the width d = λ. (c) Scattering indicatrix from an interface with nanostructured medium without absorption with $\varepsilon = 16$ and $f = 0.1$: the black curve is for the case of *s*-polarization, the blue curve is for *p*-polarization without taking into consideration the existence of black mode, and the red curve is for *p*-polarization with taking into consideration the existence of black mode.

One can see from Fig. 4 that scattering of a 2D self-healing beam for small angles of incidence at the interface with an ideally reflecting interface occurs mainly in the direction of backscattering similarly to scattering a plane wave by a strip. The same is true for scattering a 2D beam with *p*-polarization by an interface with a nanostructured medium without taking into consideration the existence of the black mode. However, the scattering indicatrix is significantly modified if one takes into consideration the existence of the black mode. One can see that two scattering wings emerge in the direction nearly perpendicular to the incident beam direction, *i.e.* parallel to the interface. The wings are greatly enlarged in the case of s-polarization.

6. CONCLUSIONS

The results of this study based on generalization of the Fresnel`s formula to the case of a two-dimensional beam incident on the interface of the vacuum with a nonmagnetic, homogeneous on average and isotropic nanostructured medium with spatial dispersion show that the beam features properties of self-healing on a finite length, for instance of the order of 10 to 100 wavelengths for $M = 3\cdot 10^3$.

Using of a 2D self-healing beam allows one to avoid taking into consideration scattering inside the nanostructured medium and to neglect the phenomenon of backscattering, and therefore to substantially simplifies the problem of finding effective parameters of the nanostructured medium. Since a large number of various plane waves are summarily contained in the radiation upon scattering of the incident 2D beam, one can determine the effective parameters of the nanostructured medium from the measured scattering features.

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REFERENCES

1. S. Ya. Prislopski, E.K. Naumenko, I.M. Tiginyanu, L. Ghimpu, E. Monaico, L. Sirbu, S.V. Gaponenko, Optics Lett. **36**, 3227–3229 (2011).
2. Yu. A. Kravtsov and A.I. Saichev, Sov. Phys. Usp. **25**, 494–508 (1982).
3. V.V. Sergentu, S. Ya. Prislopski, E.V. Monaico, V.V. Ursaki, S.V. Gaponenko, I.M. Tiginyanu, J. Opt. **18**, 125008 (2016).
4. F.O. Fahrbach and A. Rohrbach, Nature Commun. **3**, 632 (2012).
5. V.V. Sergentu, V. Prilepov, V. Zalamai, P. Gashin, Mold. J. Phys. Sci. **15**, 150–157 (2016).

6. L.D. Landau, E.M. Lifshitz, L.P. Pitaevskii, *Electrodynamics of Continuous Media*, Butterworth-Heinemann, Oxford, 1984.
7. V. Sergentu and V. Ursaki, Proceedings IFMBE **55**, 234–238 (2016).
8. S.I. Pekar, *Crystal optics and additional light waves*, Benjamin Cummings Pub. Co, Menlo Park, 1983.
9. L.A. Golovan, V. Yu. Timoshenko, P. K. Kashkarov, Phys.-Usp. **50**, 595–612 (2007).
10. V. Sergentu and V. Ursaki, Rom. Rep. Phys. **69**, 404 (2017).
11. Q. Zhan, Adv. Opt. Photonics **1**, 1–57 (2009).