

## FRactal GEOMETRY IN SUPERCONDUCTIVITY

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The influence of fractal geometry on superconductivity has been analyzed for superconductors with different Euclidean dimensions. The complexity of fractal structures gives rise to a „multi-crossover“ behavior in one-dimensional Nb/Cu fractal multilayers, „quasi-2D Parks-Little effect“ for two-dimensional fractal networks, and temporal fractality for three-dimensional high- $T_c$  superconducting ceramic samples. Artificial fractal structures serve as a suitable model object for simulations and for experimental studies of disordered superconductors and superconducting devices with complicated topology.

### INTRODUCTION

The last two decades witnessed an explosive growth in theoretical and experimental contributions to fractal geometry. It is of general interest to investigate how familiar physical properties are modified in fractals, first because of numerous examples of fractal structures in nature, and second because of their similarity to inhomogeneous materials. At the same time novel superconducting materials and devices with complicated topology raised new problems and questions. How does the complexity of billion interconnections of superconducting computer with Josephson junctions influence its parameters? How strong is the influence of the fractal geometry in a superconducting cable consisting of thousands of thin superconducting filaments in a non-superconducting matrix? What is the origin of long-time relaxation processes in magnetic field observed in high- $T_c$  superconductors? Some of these problems of superconducting systems with fractal structure are discussed.

### 1. ONE-DIMENSIONAL FRACTAL MULTILAYERS

The Nb/Cu multilayers (ML) were prepared in an ultrahigh vacuum system (base pressure  $10^{-10}$  mbar) onto sapphire substrates using computer-controlled electron beam evaporation, details of the system have been described elsewhere [1]. The geometry of prepared fractal ML followed the triadic Cantor set

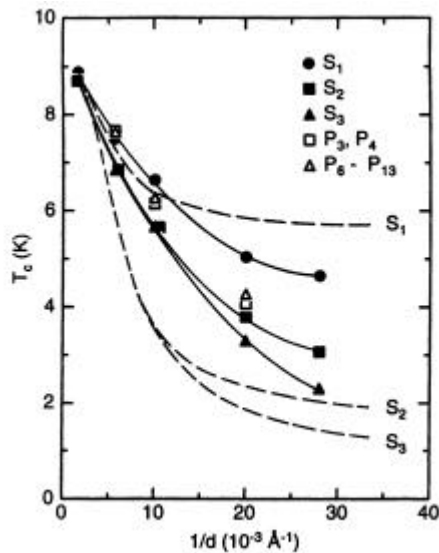


Fig.1.  $T_c$  of fractal and periodic Nb/Cu ML vs. inverse Nb-thickness  $1/d$ . Solid lines are guides to the eye. Dashed curves show the theoretical calculation [3] for fractals  $S_1, S_2, S_3$  with  $D_f = 0.63$ .

[2]. By varying the dividing factor  $r$  ( $0 < r < 0.5$ ) we could change a fractal dimension of the multilayer,  $D_f = \ln 2 / \ln(1/r)$ , and obtained structures with a fractal dimension between two limits  $D_f = 0$  (for a single film  $r = 0$ ) and  $D_f = 1$  (for a periodic ML  $r = 0.5$ ). The total thickness  $d_{tot}$  of a fractal multilayer  $S_n$  with a number of repeat scales  $n$  is given by  $d_{tot}(S_n) = (1/r)^{n-1} (d/r)$ . The type of layering of ML strongly determines their  $T_c$  and  $B_{c2}(T)$  behavior, as one can see in Fig.1 and Fig.2. The critical temperature for fractal ML decreases with increasing of the fractal scales number  $n$ , according to a scaling model calculated by Yuan and Whitehead [3], while for simple periodic ML it remains constant with increasing number of periods  $p$ . The influence of fractality on a critical magnetic field parallel to the layers is demonstrated in Fig.2, where data for samples with different geometries (single Nb film, periodic ML, and fractal ML) are presented. At low temperatures all samples show a two-dimensional behavior, i.e. square-root dependence  $B_{c2H}(T) \sim (1 - T/T_c)^{1/2}$ . The single film clearly exhibits the 2D behavior of  $B_{c2H}(T)$  at all temperatures  $T < T_c$ , whereas the dependence  $B_{c2H}(T)$  for fractal ML and periodic ML distinctly changes above the crossover temperature  $T_{cr}$

(marked by arrows). In the region  $T_{cr} < T < T_c$ ,  $B_{c2II}(T)$  can be described by

$$B_{c2II}(T) \sim (1 - T/T_c)^f \tag{1}$$

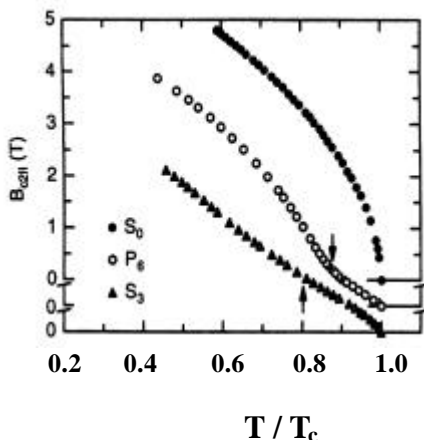


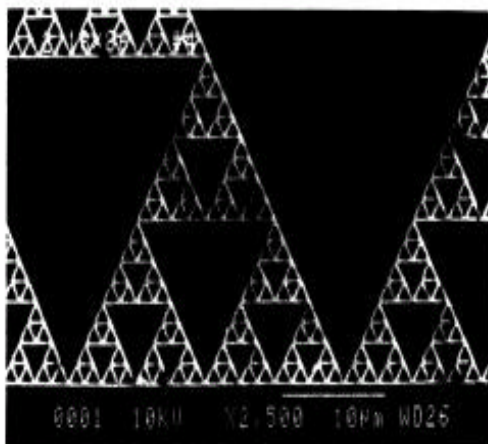
Fig.2. Parallel critical magnetic field  $B_{c2II}(T)$  vs. reduced temperature  $T/T_c$  for a single Nb film  $S_0$ , a periodic ML  $P_6$ , and fractal  $S_3$  with  $D_f = 0.63$ ,  $d_{Nb} = 175 \text{ \AA}$ .

where the exponent  $f$  strongly depends on the type of layering: for periodic ML three-dimensional behavior with  $f = 1$  is observed, whereas fractal ML shows  $f = 0.75$ . According to the scaling theory [4] near  $T_c$  a temperature dependent superconducting coherence length  $\xi_\lambda(T)$  becomes successively comparable to the different fractal scales ( $n = 1,2,3,\dots$ ). This leads to the „multi-crossover“ or fractal behavior of the parallel critical magnetic field  $B_{c2II}(T)$  with non-integer exponent  $f$ .

## 2. TWO-DIMENSIONAL FRACTALS – SIERPINSKI GASKET

A family of regular fractal networks – Sierpinski carpet (SC) and Sierpinski gasket (SG) have an Euclidean dimension  $d = 2$ , and fractal dimension  $D_f = \ln 3 / \ln 2 = 1.585$  for SG and  $D_f = 1.8928$  for SC [2]. Because of their dilatational symmetry, statistical, mechanical, transport and superconducting properties of SG are exactly solvable, making these fractals an attractive model system. Experimental measurements of the superconducting-to-normal phase boundary,  $T_c(B)$ , for SG-fractal network, prepared from superconducting Al thin film, demonstrate a very unusual oscillating behavior of the transition temperature (Fig.3b) with a fine structure as a consequence of Parks-Little effect in a two-dimensional network of holes. Self-similarity of the fine structure of the  $T_c(B)$  oscillations is described by the Goldman equation [6]

a)



b)

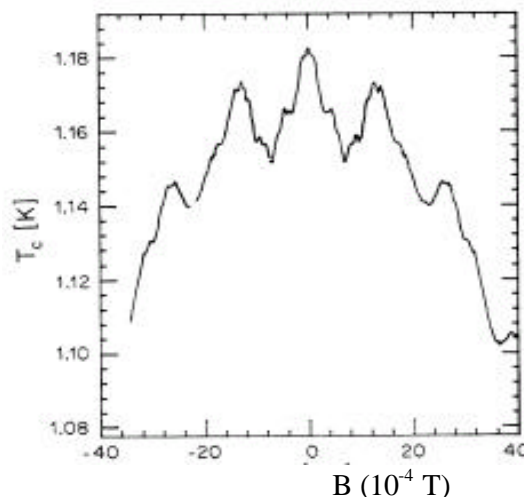


Fig.3. a) An electron micrograph of a fourth-order Sierpinski gasket, prepared from 100 nm thick Al film; b) The superconducting transition temperature as a function of external magnetic field,  $T_c(B)$ , for the Sierpinski gasket [6].

$$\Delta T_c/T_{c0}(\phi/\phi_0 = 0.5) = [\xi(0)/L_0]^2 \arccos^2[\varepsilon(\phi/\phi_0 = 0.5)/z] = 0.024 \quad (2)$$

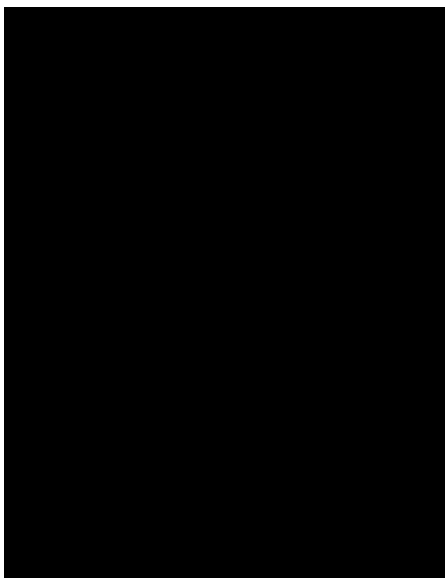


Fig.4. Cross-section of the Nb<sub>3</sub>Sn multifilament commercial cable, produced by Airco Company [5].

here  $\phi_0 = hc/2e$  is elementary flux quantum,  $\xi(0) = 0.26$  mkm is the GL-coherence length of the Al film,  $z$  is the node coordination number of the Sierpinski gasket ( $z = 4$  for the planar SG),  $L_0 = 1.73$  mkm is the length of the Al elementary triangle in the gasket (Fig.3a). Experimentally, the critical temperature decreasing in magnetic field at the points  $\phi/\phi_0 = 0.5$  is  $\Delta T_c/T_{c0}(\phi/\phi_0 = 0.5) = 0.025$  is in perfect agreement with theoretical equation (2). The last result reflects a direct influence of fractal complexity on superconductivity. One can compare the artificially prepared Sierpinski gasket in Fig.3a with the real picture of fractal-like structure of a multifilament superconducting cable (Fig. 4) and make conclusion about necessity to take into account the influence of fractal geometry on superconducting properties of such materials.

### 3. THREE-DIMENSIONAL SUPERCONDUCTING FRACTALS

3D fractals demonstrate a more complex behavior, than the 1D multilayers or 2D networks. Properties of the 3D fractals are characterized by either spatial or temporal fractality. One of the „natural“ 3D fractal system is the high- $T_c$  ceramic oxide, where the long-time relaxation effects are observed.

The relaxation of the thermoremanent magnetization  $M$  (the quantity of magnetization, remained in superconductor after switching off the external magnetic field) was investigated as a function of time for various high- $T_c$  ceramics, using rf-SQUID magnetometer [7]. As one can see in Fig.5, where results for some investigated samples are shown, all the experimental time-dependences  $M(t)$  are well described by the logarithmic law

$$M(t) = M_0 - A \ln t, \quad (3)$$

where the remanet magnetization at the moment  $t = 0$ ,  $M_0$ , and decay rate,  $A$ , are constants. This relaxation behavior (3) is caused by existence of the hierarchy of superconducting loops in the fractal network of ceramics („Sierpinski pyramid“, a three-dimensional variant of the planar Sierpinski gasket, serves as a mathematical model of such porous media). This leads to a wide range of relaxation times for magnetic flux motion, and, as a result the temporal fractal behavior of the superconducting porous media in magnetic field is observed.

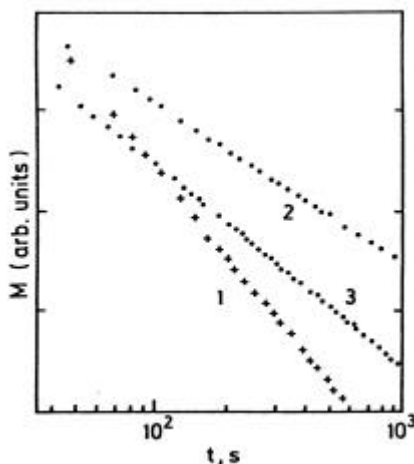


Fig. 5. Time dependences of the thermoremanent magnetization  $M(t)$  for YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-x</sub> (1), Bi<sub>2</sub>Sr<sub>2</sub>CaCu<sub>2</sub>O<sub>10-x</sub> (2), and BaPb<sub>0.75</sub>Bi<sub>0.25</sub>O<sub>3</sub> (3) polycrystalline samples.  $T = 4.2$ K, external magnetic field  $B = 2$  mT.

### 4. CONCLUSION

One can see that the complexity of superconducting systems with fractal geometry directly influences their main properties. This correlation between geometry and superconducting properties should be taken into account when one designs superconducting devices and materials with complex topology.

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