The quantum dynamics of a nanomechanical resonator coupled to two quantum dots

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***Abstract* — We investigate the quantum dynamics of a system composed of two initially excited closely-spaced quantum dots fixed on a quantum mechanical resonator. The system is submitted to environmental effects and obeys the conditions for Dicke-type collective interactions. We analytically solve the system dynamics and compare it to a single-quantum-dot case, in order to identify the inside mechanics of the influence of the quantum dot collective interaction to the dynamics of the mechanical resonator's phonon field.**

***Index Terms* — optomechanics, phonon superradiance, quantum dots, two-atom systems, collective effects**

I. INTRODUCTION

Two-atom quantum systems are one of the most trivial models for the study of collective effects and have an important role in the theoretical and experimental research of quantum optical effects such as quantum interferences, atomic entanglement and superradiance. Quantum interferences had been observed for two photodissocciated atoms of a Ca_2 molecule where a single photon is shared among the atoms [1]. A two-atom-cavity interferometer was first built in analogy with the Young's interferometer, where each trapped atom acts as slit and the interference occurs due to indistinguishable pathways of the atoms-field interaction [2]. Depending on the superposition of the pathways, this type of interferences shows cavity-induced saturation of the resonance fluorescence spectra or photon bunching effect [3]. One of the basic model of quantum atomic entanglement effects is the two-atom system either placed in a cavity [4] or interacting with the surrounding vacuum field [5]. The observation of the Rydberg blockade regime between two neutral atoms was reported in [6, 7], which is a crucial prerequisite towards the creation and control of atomic entanglement. The dipole-dipole interaction may also be manipulated when the environmental vacuum field is considered in the case of two three-level Λ-type atoms, where the surrounding reservoir induces the coupling of orthogonal transitions. Therefore, the spatial orientation of these atoms determines the steady-state dynamics of the system [8]. The coherent trapping effect is influenced as well by the dipole-dipole interaction of the two two-level atoms trapped in a cavity [9].

Two-atom systems may be considered as the edge for collective effects. Dicke collective behaviour [10] of superradiance and subradiance was first observed in the spontaneous emission variation for two trapped Ba_{138}^{+} ions [11]. Non-decaying atomic collective states were predicted for two-atom systems prepared in a subradiant state [12]. Meanwhile, significant results have been achieved for collective effects in condensed matter physics [13]. Particularly, superradiance was observed in a collection of artificial atoms as quantum dots [14]. Moreover, phonon superradiance effects were predicted for molecular nanomagnets [15] and nanomechanical resonators as vibrating membranes [16], while subradiance with phonons was reported for a system using coupled quantum dots, in analogy with the subradiant photon effect for a two-ion system [17].

In this paper, one investigates the model of two identical two-level quantum dots embedded on a quantum mechanical resonator, such as a vibrating membrane [18] or a nanobeam [19]. The artificial atoms interact with the surrounding electromagnetic field, as well as with the nanoresonator's single-mode phonon field. The aim of this study is to explain the mechanics of the phonon field behaviour within the superradiant regime. This regime is reached when the emitters are spaced closer to each other comparing to their transition wavelength. Therefore, this two-emitter system allows an analytical treatment for the inside dynamics of the collective interaction within the superradiant regime and shows a significant enhancement of the phonon signal of the quantum mechanical resonator.

This article is structured as follows. In section II, the analytic model is described and the equation of motion of interest are solved. In section III, one discusses the behaviour of the mechanical resonator mean phonon number and its relation to the collective population dynamics. One compares the investigated model with the single quantum dot case. The summary is given in section IV.

II. THE MODEL

The model consists of two identical initially excited two-level quantum dots (QDs) embedded on a nanomechanical resonator as a nanobeam, a membrane or a multilayered acoustical cavity. The QDs are described by their transition frequency ω_{qd} among the excited state $|e\rangle_i$ and the ground state $|g\rangle_i$ where $i = \{1,2\}$ is the QD index. The QDs may spontaneously decay at a rate γ and interact equally with the quantum mechanical resonator with a coupling constant η. The nanomechanical resonator is described in the good cavity limit by a single-mode phonon field of frequency ω and the bosonic operators of annihilation and creation, respectively, b and $b^\dagger$. The thermal environment damps the mechanical resonator with a rate κ and is described as a thermal reservoir with a mean phonon number $\bar{n} = (\exp(\hbar\omega/k_B T) - 1)^{-1}$, where k_B is the Boltzmann constant and T is the ambient temperature. The system Hamiltonian is defined as follows:

$$H = \hbar\omega_{qd} S_z + \hbar\omega b^\dagger b + \hbar\eta S_{22}(b^\dagger + b)\,, \qquad (1)$$

where the first two terms describe the free QDs and the free nanoresonator Hamiltonians and the last term defines the phonon-QDs interaction. The collective atomic operators are given as: $S_z = \sum_{i=1}^{2}(|e\rangle_{ii}\langle e| - |g\rangle_{ii}\langle g|)/2 = S_{22} - 1$, $S^+ = \sum_{i=1}^{2}|e\rangle_{ii}\langle g|$ and $S^- = \sum_{i=1}^{2}|g\rangle_{ii}\langle e|$, and obey the standard SU(2) commutation relations: $[S^+, S^-] = 2S_z$ and $[S_z, S^\pm] = \pm S^\pm$.

The system dynamics is defined via the master equation of the density matrix operator, expressed as:

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \kappa\bar{n}\mathcal{L}(b^\dagger) + \kappa(1 + \bar{n})\mathcal{L}(b) + \gamma\mathcal{L}(S^-)\,, \qquad (2)$$

where the Liouville superoperator $\mathcal{L}$ for a given operator o is defined as $\mathcal{L}(o) = 2o\rho o^\dagger - o^\dagger o\rho - \rho o^\dagger o$. The first term of the master equation is the coherent term defined by the system Hamiltonian, the next two terms represent the pumping and damping of the mechanical resonator by the thermal reservoir, while the last term describes the spontaneous emission of the separate QDs as well as their interaction through the surrounding vacuum field within the superradiant condition. This condition is satisfied when the QDs are spaced closer to each other than their transition wavelength. The master equation is solved via the system of equations of motion for the parameters of interest, i.e., the resonator mean phonon number $\langle b^\dagger b\rangle$ and the QDs collective population $\langle S_z\rangle$. The corresponding system of linear coupled differential equations of motion is defined as:

$$\begin{aligned}
\frac{\partial\langle S_z\rangle}{\partial t} &= -2\gamma\langle S^+S^-\rangle,\\
\frac{\partial\langle S^+S^-\rangle}{\partial t} &= 8\,\gamma\{1 + \langle S_z\rangle - \langle S^+S^-\rangle\},\\
\frac{\partial\langle b^\dagger b\rangle}{\partial t} &= i\eta\{\langle S_z b\rangle - \langle S_z b^\dagger\rangle + \langle b\rangle - \langle b^\dagger\rangle\}\\
&\quad -2\kappa\langle b^\dagger b\rangle + 2\kappa\bar{n}\,,\\
\frac{\partial\langle S_z b\rangle}{\partial t} &= -(\kappa + i\omega)\langle S_z b\rangle - 2\gamma\langle S^+S^-b\rangle\\
&\quad -i\eta\{2 + 2\langle S_z\rangle - \langle S^+S^-\rangle\}\,,\\
\frac{\partial\langle S^+S^-b\rangle}{\partial t} &= -(\kappa + 8\gamma + i\omega)\langle S^+S^-b\rangle - 2i\eta\{1 + \langle S_z\rangle\}\\
&\quad + 8\gamma\{\langle S_z b\rangle + \langle b\rangle\}\,,\\
\frac{\partial\langle b\rangle}{\partial t} &= -(\kappa + i\omega)\langle b\rangle - i\eta\{1 + \langle S_z\rangle\}
\end{aligned} \qquad (3)$$

together with the Hermitian conjugates of these equation. The dynamics of the equations of motion may be solved analytically considering the initial conditions of its variables: $\langle b^\dagger b\rangle_{t=0} = \bar{n}$, $\langle S_z\rangle_{t=0} = 1$ and all the other variables value zero at $t = 0$. The system may be prepared in such an initial state by exciting the QDs with a short pulse laser of duration $\tau \ll 1/\eta$ in order to not affect the phonon-QDs dynamics. One finds the following expression of the nanomechanical resonator mean phonon number:

$$\begin{aligned}
\langle b^\dagger b\rangle &= \bar{n} + \bar{a}e^{-2\kappa t} - \bar{b}(t)e^{-4\gamma t}\\
&\quad -\bar{c}e^{-(4\gamma+\kappa)t}\left[\bar{d}(t)\cos(\omega t) + \bar{e}(t)\sin(\omega t)\right],
\end{aligned} \qquad (4)$$

where the coefficients of the expression (4) are defined as:

$$\bar{a} = 2\eta^2(448\gamma^4 - 496\gamma^3\kappa + 2\kappa^2(\kappa^2 + \omega^2) + 4\gamma^2(49\kappa^2 + 3\,\omega^2) - 3\gamma(11\kappa^3 + 3\kappa\omega^2))/(\alpha\,\zeta),$$

$$\bar{b}(t) = 2\eta^2(-2\kappa^2(\kappa^2 + \omega^2) + \gamma\kappa(3\kappa^2 + 7\omega^2 - 2\kappa^3 t - 2\kappa\omega^2 t) + 4\gamma^2(\kappa^2 - \omega^2 + \kappa^3 t + \kappa\,\omega^2 t))/(\alpha\beta),$$

$$\bar{c} = 8\eta^2/(\beta\zeta),$$

$$\bar{d}(t) = (-16\gamma^2(4\gamma^2\kappa^2 + 3\gamma\kappa^3 - 4\gamma^2\omega^2 + 7\gamma\kappa\omega^2 + \kappa^2\omega^2 + \omega^4 + 4\gamma(2\gamma\kappa + \omega^2)(\kappa^2 + \omega^2)t) + \beta(52\gamma^2 - 13\gamma\kappa + \kappa^2 + \omega^2 + 2\gamma(48\gamma^2 - 12\gamma\kappa + \kappa^2 + \omega^2)t)),$$

$$\bar{e}(t) = \gamma\omega(\,\beta + 16\gamma(8\gamma^2\kappa - \kappa(\kappa^2 + \omega^2) + \gamma(\kappa^2 + 5\omega^2)) + 8\gamma(\beta + 8\gamma(2\gamma - \kappa)(\kappa^2 + \omega^2))t)$$

while $\zeta = ((\kappa - 4\gamma)^2 + \omega^2)^2$, $\alpha = (\kappa - 2\gamma)^2$ and $\beta = (\kappa^2 + \omega^2)^2$.

The expression of the collective population of the two-QD system is deduced as:

$$\langle S_z\rangle = e^{-4\gamma t}(4\gamma t + 2) - 1\,. \qquad (5)$$

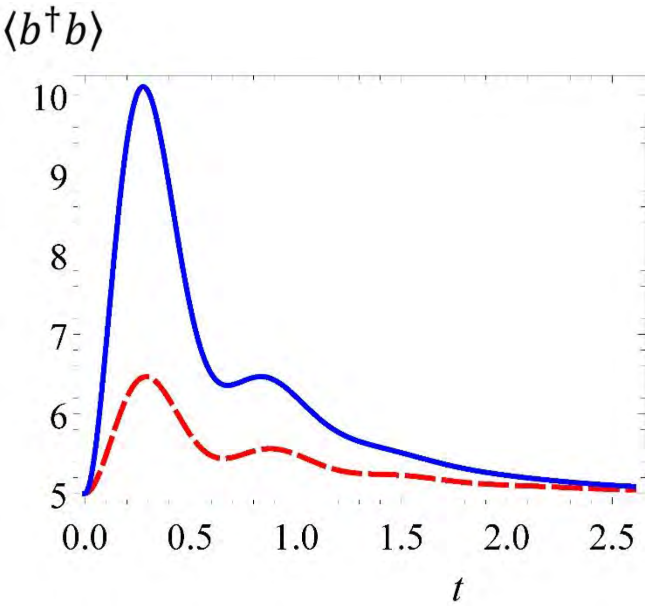

Figure 1: The mean phonon number of the nanomechanical resonator $\langle b^\dagger b\rangle$ for a two-QD system (continuous line) and for a single-QD system (dashed line) as functions of time *t*. The other system parameters are $\kappa/\gamma = 0.8$, $\eta/\gamma = 8$,

$\omega/\gamma = 10$ and $\bar{n} = 5$.

III. RESULTS AND DISCUSSIONS

The dynamics of the quantum mechanical resonator mean phonon number and the collective QD population are represented in figures 1 and 2, respectively. The behaviour of the system is compared to the single-QD case that we had treated in [16]. One observes that the phonon signal is significaly enhanced and experiences a stronger damping effect comparing to single-QD case, while the collective population decays faster.

In order to highlight the exponential behaviour of the collective population decay of figure 2, it was given in logarithmic scale as an expression of the normalized excited state population that satisfies the relation $\langle S_z \rangle/N = \langle S_{22} \rangle/N - 1/2$. The asymptotical behaviour for longer time durations is exponential and the two-atom case has a twice faster decay. This is a characteristic feature of superradiant dynamics that becomes N times faster, where N is the number of collectively interacting emitters.

The superradiant feature of faster dynamics of the QD collective decay is transferred to the behaviour of the quantum mechanical resonator. This effect was reported for larger QD samples in [16], however the analytic expression of the two-QD case allows a better understanding for the mechanics of this effect. Namely, the general form of the expression of $\langle b^\dagger b \rangle$ of equation (4) is similar to the single-QD case, with a single main difference – a twice faster spontaneous emission. Therefore, the two-QD analytic case resembles to the case of a single QD with a twice bigger spontaneous emission rate that couples twice stronger to the nanomechanical resonator. Although this is a usual description of superradiant mechanics, the coefficients of the equation (4) are different than the ones of the single-QD case. Moreover, some of them became time dependent for the two-QD case, so that no complete analogy can be made with the single-QD case.

Hence, superradiant features are found within the general behaviour of the two-QD system and are expressed through a twice faster dynamics of the QD decay. This features are completely transferred to the mechanical resonator and its coupling to the two QDs resembles to a stronger coupling to a faster single QD. However, the system admits some particularities that are specific only to the dynamics of the two-level atom system and cannot equate with the single-QD case.

IV. SUMMARY

The quantum dynamics of a nanomechanical resonator coupled to a two-quantum dot collectively decaying system had been investigated. This system allows an analytic treatment of the dynamics of the quantum mechanical resonator and shows superradiant features due to the QD collective behaviour. A strong resemblance with the dynamics of a single-QD system was identified, leading to a deeper understanding of the mechanics of the phonon superradiance effect. An enhanced phonon emission have been observed.

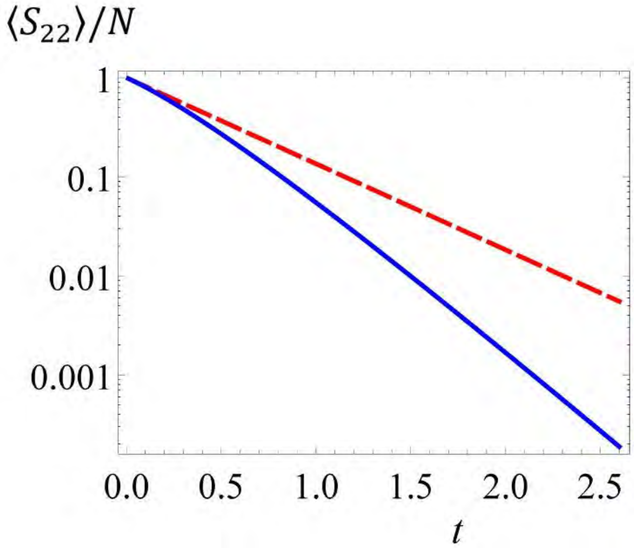

Figure 2: The normalized atomic excited populations $\langle S_{22} \rangle/N$ of the two-QD system (continuous line) and of the single-QD system (dashed line), in logarithmic scale, as functions of time *t*. The other system parameters are $\kappa/\gamma = 0.8$, $\eta/\gamma = 8$, $\omega/\gamma = 10$ and $\bar{n} = 5$.

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