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On weak-functionally complete systems of formulas containing paraconsistent negation in a paraconsistent logic

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Abstract

We consider a paraconsistent logic and establish the conditions for a system of its formulas containing paraconsistent negation to be functionally complete in it.

Keywords: paraconsistent negation, paraconsistent logic, functionally complete systems of formulas.

1 Introduction

A theory is called inconsistent if it has as its theorems formulas A and $\neg A$ [1]. If a theory derives all the formulas as its theorems, it is called trivial. These two notions are different, but they coincide in familiar to us systems. A theory is paraconsistent if it is inconsistent, but it is not trivial [2].

Paraconsistent logic is motivated not only by philosophical considerations, but also by its applications and implications. One of the applications is automated reasoning (information processing). Consider a computer which stores a large amount of information. While the computer stores the information, it is also used to operate on it, and, crucially, to infer from it. Now it is quite common for the computer to contain inconsistent information, because of mistakes by the data entry operators or because of multiple sourcing. This is certainly a problem for database operations with theorem-provers, and so has drawn much attention from computer scientists. Techniques for removing inconsistent information have been investigated. Yet all have limited applicability, and, in any case, are not guaranteed to produce consistency. (There is no algorithm for logical falsehood.) Hence, even if steps are taken to get rid of contradictions when they are found, an underlying paraconsistent logic is desirable if hidden contradictions are not to generate spurious answers to queries [3].

Other motivation for investigations of paraconsistent logics is the part of artificial intelligence research called belief revision, which is one of the areas that have been studied widely. Belief revision is the study of rationally revising bodies of belief in the light of new evidence. Notoriously, people have inconsistent beliefs. They may even be rational in doing so. For example, there may be apparently overwhelming evidence for both something and its negation. There may even be cases where it is in principle impossible to eliminate such inconsistency. For example, consider the 'paradox of the preface'. A rational person, after thorough research, writes a book in which they claim $(A_1\& \ldots \& A_n)$. But they are also aware that no book of any complexity contains only truths. So they rationally believe $\sim (A_1\& \ldots \& A_n)$ too. Hence, principles of rational belief revision must work on inconsistent sets of beliefs. So, a more adequate account can be based on paraconsistent logic [4]. Other applications of paraconsistent logics are known in robot control [5], in air traffic control [6], in defeasible deontic reasoning [7], in information systems [8] and medicine. Connections between paraconsistent logics, adaptive logics and diagnosis are investigated in [9] and [10].

2 The problem

It is a well known class of problems in logic, algebra, discrete mathematics and cybernetics dealing with the possibility of obtaining some functions (operations, formulas) from other ones by means of a fixed set of tools. The notion of expressibility of Boolean functions through other ones by means of superpositions goes back to the works of E. Post [11], [12]. He described all closed (with respect to superpositions) classes of 2 valued Boolean functions. The problem of completeness (with respect to expressibility), which requires to determine the necessary and sufficient conditions for all formulas of the logic under investigation to be expressible via the given system of formulas, is also investigated. In 1956 ([13, p. 54], [14]) A. V. Kuznetsov established the theorem of completeness according to which we can build a finite set of closed with respect to expressibility classes of functions in the k-valued logics such that any system of functions of this logic is complete if and only if it is not included in any of these classes. In 1965 [15] Rosenberg I. established the criterion of completeness in the k-valued logics formulated in terms of a finite set of pre-complete classes of functions, i.e. in terms of maximal, incomplete and closed classes of functions.

In the present paper we investigate the conditions of completeness with respect to expressibility of the systems of formulas of the modal logic $S5$ containing the paraconsistent negation of the logic $S5$.

The standard language of $S5$ is based on propositional variables and logical connectives: &, $\vee$, $\rightarrow$, $\neg$, $\Box$, and $\Diamond$. We consider the paraconsistent negation $\sim$ of $S5$ [16] as follows:

$$\sim a =_{Def} \Diamond\neg a.$$

The logic $S5$ can be considered, according to [16], as a paraconsistent logic since it contains a paraconsistent negation. The logic $S5$ is characterized by the axioms and rules of inference of the classical propositional logic, the following axioms:

$$\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B),$$
$$\Box A \rightarrow A,$$
$$\Diamond A \rightarrow \Box\Diamond A,$$

and the necessity rule of inference: from A infer $\Box A$.

The formula F is said to be *weak-expressible in the logic L via a system of formulas* Σ, if F can be obtained from propositional variables, constants and formulas of Σ applying a finite number of times: a) the rule of substitution of equivalent formulas in the logic L, and b) the rule of weak substitution, which permits, being given formulas A and B, to substitute one of them in another instead of a given corresponding propositional variable [17], [18], [19].

The system of formulas Σ is said to be *(weak-functionally) complete (with respect to the weak-expressibility) in the logic* L, if all formulas of the calculus of L are weak-expressible in the logic L via formulas of Σ [17],[18], [19].

3 Main result

The main result of the paper is the following.

Theorem. *There is an algorithm that decides weather a system of formulas containing the paraconsistent negation* $\sim$ *is weak-functionally complete in the modal logic* $S5$.

4 Conclusion

We can consider in the same manner the problems of (parametric, positive, implicit) expressibility of the systems of formulas containing the paraconsistent negation in the modal logic $S5$, too. Also we can investigate the problem of weak-functional systems containing paraconsistent negation in other logical systems.

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