A POSTERIORI ERROR ESTIMATION AND ADAPTIVE SOLUTION OF ELLIPTIC VARIATIONAL INEQUALITIES OF THE SECOND KIND

Viorel Bostan[1], Weimin Han[2], and B.D. Reddy[3]

Abstract. In this paper, we perform an a posteriori error analysis for adaptive finite element solutions of elliptic variational inequalities of the second kind. A general framework for a posteriori error estimates is established by using duality theory in convex analysis. We then turn to an analysis of some particular a posteriori error estimates of residual type. Efficiency of the error estimators are investigated. Numerous numerical examples are included to illustrate the effectiveness of the a posteriori error estimates in adaptive solutions of the variational inequalities.

1 Introduction

The finite element method today is the dominant numerical method for solving most problems in structural and fluid mechanics. It is widely applied to both linear and nonlinear problems. For practical use of the method, one of the most important problems is the assessment of the reliability of a finite element solution. The reliability of the numerical solution hinges on our ability to estimate errors after the solution is computed; such an error analysis is called a posteriori error analysis. A posteriori error estimates provide quantitative information on the accuracy of the solution and are the basis for the development of automatic, adaptive procedures for engineering applications of the finite element method.

The interest in a posteriori error estimation for the finite element method began in the late 1970's. The pioneering work on the topic was done in [4, 5]. Since then, a posteriori error analysis and adaptive computation in the finite element method have attracted many researchers, and a variety of different a posteriori error estimates have been proposed and analyzed. Some comprehensive summary accounts can be found in [1], [6] and [42].

Most of the work so far on a posteriori error analysis has been devoted to ordinary boundary value problems of partial differential equations. In applications, an important family of nonlinear boundary value and initial-boundary value problems is that associated with variational inequalities, that is, problems involving either differential inequalities or inequality boundary conditions. Mechanics is a rich source of variational inequalities (cf. e.g. [37]), and some examples of problems that give rise to variational inequalities are obstacle and contact problems, plasticity and visco-plasticity problems, Stefan problems, unilateral problems of plates and shells, and non-Newtonian flows involving Bingham fluids. An early comprehensive reference on the topic is [14], where many nonlinear boundary value problems in mechanics and physics are formulated and studied in the framework of variational

[1]Department of Mathematics, University of Iowa, Iowa City, IA 52242, U.S.A. E-mail: vbostan@math.uiowa.edu
[2]Department of Mathematics, University of Iowa, Iowa City, IA 52242, U.S.A. E-mail: whan@math.uiowa.edu
[3]Department of Mathematics and Applied Mathematics, University of Cape Town, 7700 Rondebosch, South Africa. E-mail: bdr@maths.uct.ac.za