About algebraic characterization of quasi-varieties of loops

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We will present a characterization of the quasi-varieties of loops in the language of the filtered product. From the definition of quasi-variety it results that, if M is a quasi-variety, then $SM = M$, $PM = M$.

We will note: by $P_f M$ the class of isomorphic loops of the filtered product of loops from M; by $P_u M$ the class of isomorphic loops of the ultrafiltered product of loops from M; by $P_{fin} M$ the class of isomorphic loops to the direct product of loops in M (with finite support).

For any class M of loops the intersection of all quasivarieties containing M is a quasi-variety. This is the smallest quasi-variety containing M, which is denoted by qM and is called the quasi-variety generated by M. If $M = \{L\}$ consists of a single loop L, then instead of qM we know simply qL. The following statement takes place.

Theorem 1. *Any quasi-variety of loops is generated by the set of all its finite-generated loops.*

The class of loops M is called: filtered-closed if $P_f M = M$; ultra-dark if $P_u M = M$; multilicative-closed if $PM = M$; hereditary if $SM = M$.

The following statements are true.

Theorem 2. *Any quasi-variety of loops is filtered-closed.*

Theorem 3. *Suppose that the loop $L \in qM$, where M is some class of loops. Then $L \in SP_u P_{fin} M$.*

Theorem 4. *Let M be the set of all finite loops, then $qM = SP_u M$*

From Theorems 2, 3 we obtain the following theorems.

Theorem 5 (Mal'tsev)**.** *A class M of loops is quasi-variety if and only if:* `i)` *M is filtered-closed;* `ii)` *M is hereditary.*

Theorem 6 (Mal'tsev)**.** $QM = SP_f M$.

Theorem 7. $qM = SPP_uM$.

In particular, when M is a finite totality of finite loops, from Theorem 7 it follows:

Theorem 8. *If M is a total of finite loops, then $qM = SPM$.*

Theorem 9 (sign of belonging)**.** *Let M be a class of loops. If the finite generated loop L belongs to the quasiarity qM, then $L \in SPM$.*

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