The Iterative Algorithm of Tuning Controllers to the Models Object with Inertia and Time Delay

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Abstract—**In this paper is proposed the iterative algorithm of tuning the typical controllers *P, PI, PID* to the model objects with inertia and time delay. In the proposed algorithm it is using the maximal stability degree method for tuning controllers. In the result of this studying it is proposed the algorithm of tuning controllers and the procedure of determining the system's performance in dependence of maximal stability value.**

Index Terms— **the iterative algorithm, the maximal stability degree method, tuning of controllers**.

I. INTRODUCTION

At the automation of many slow technological processes the mathematical objects' models of control process are represented as the models with respectively order inertia and time delay [1, 2].

The procedure of tuning controllers to the model object with inertia and time delay becomes difficult. In this paper is analyzing the model object (fixed part) with inertia (second order) and time delay with transfer function which is presented in the follow form

$$H_{PF}(s) = \frac{ke^{-ds}}{(T_1 s + 1)(T_2 s + 1)} = \frac{ke^{-ds}}{a_0 s^2 + a_1 s + a_2}, \qquad (1)$$

where the parameters of object $k, T_1, T_2, d, a_0, a_1, a_2$ are known.

To the model object (1) with known parameters *k, T_1, T_2, d* it is proposed to tune the standard controllers *PI* and *PID* using the maximal stability degree (MSD) method [3] and to analyze the dynamic of control system for the case when it is varying the object's parameters from the nominal values *k, T_1, T_2, d*, keeping the tuning parameters of controllers *PI* and *PID*.

II. THE TUNING ALGORITHM OF CONTROLLERS

Assume that the control system is formed of object with transfer function *$H_{PF}(s)$,* which is presented in relation (1), and transfer function of controller *$H_R(s)$* with typical control laws *PI, PID*, Fig. 1.

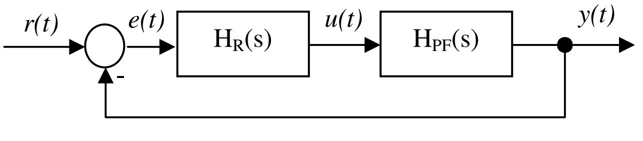

Fig. 1 Structure scheme of control system

It will be tune the typical algorithms of tuning *PI* and *PID* for the model object with known parameters, using the M.S.D. method.

For the tuning the PID controller using the MSD method it was applying the algebraic expressions [4,5], which are the analytical expressions

$$-c_0 J^3 + c_1 J^2 - c_2 J + c_3 = 0, \qquad (2)$$

$$k_R = (\exp(-dJ)/k)(a_0 dJ^3 - (da_1 + 3a_0)J^2 + (da_2 + 2a_1)J - a_2); \qquad (3)$$

$$k_i = 1/T_i = (\exp(-dJ)/k)(a_0 J^3 - a_1 J^2 + a_2 J) - k_R J. \qquad (4)$$

where $c_0 = d^2 a_0; c_1 = a_1 d^2 + 6da_0;$

$$c_2 = a_2 d^2 + 4da_1 + 6a_0; c_3 = 2da_2 + 2a_1.$$

The expression (2) is a function of object's parameters and unknown value of stability degree *J*. Solved the expression (2) it is determined the optimal value of stability degree *J_{opt},* which presented the smallest positive and real root or the real positive part of complex root.

From expressions (3) and (4) were determined the optimal values of parameters k_p and k_i of *PI* controller.

In the case of tuning parameters of *PID* controller using the MSD method it was applying the algebraic expressions [4,5], which are the analytical expressions

$$c_0 J^3 - c_1 J^2 + c_2 J - c_3 = 0, \qquad (5)$$

$$k_R = (\exp(-dJ)/k)(a_0 dJ^3 - (da_1 + 3a_0)J^2 + (da_2 + 2a_1)J - a_2) + 2k_d J; \qquad (6)$$

$$k_i = 1/T_i = (\exp(-dJ)/k)(a_0 J^3 - a_1 J^2 + a_2 J) - k_d J^2 + k_R J; \qquad (7)$$

$$k_d = (\exp(-dJ)/2k)(a_0 d^2 J^3 - (d^2 a_1 + 6da_0)J^2 + (d^2 + 4da_1 + 6a_0)J - 2d - 2a_1), \qquad (8)$$

where $c_0 = d^3 a_0; c_1 = a_1 d^3 + 9d^2 a_0;$

$$c_2 = a_2 d^3 + 6d^2 a_1 + 18da_0;$$

$$c_3 = 3a_2 d^2 + 6da_1 + 6a_0.$$

The expression (5) is a function of object's parameters and unknown value of stability degree *J*. Solved the expression (5) it is determined the optimal value of stability degree *J_{opt},* which presented the smallest positive and real root or the real positive part of complex root.

From expressions (6), (7) and (8) were determined the optimal values of parameters k_p, k_i and k_d of *PID* controller.

The tuning parameters of *PI* and *PID* controller - k_p , k_i and k_d are the function of known parametrs of control object and of the unknown value *J* stability degree of control system: *k_p=f(J), k_i=f(J), k_d=f(J)* (view relations (3), (4), (6), (7) and (8)). Based on these relations in the case of known object's parameters and in the case of variation stability degree $J \geq 0$ in the strict limits it was made the respectivily calculations and obtained the dependences $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for *PI* and *PID* controllers.

This curves for the respectively object and respective *PI* or *PID* controllers present some parabolic with branches facing down and situated in the first (top) and forth (branches) cadrans. The optimal tuning values of the controller are the maximal value of the respectivily curves and the maximal stability degree value *J* is optimal.

To verify the performance of control system it was made the simulation. If the performance don't sutisfy the imposed performance, it will be in the iterative process choose the other sets of values of controllers parameters from curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$, the procedure will repeat until the performance of system will be satisfied.

III. APPLICATION AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning the typical controllers *PI, PID* to the model of object (1) which has the following parameters $T_1=2$, $T_2=5$, $d=1$, $a_0=10$, $a_1=7$, $a_2=1$ and it will be using the procedure exposes above for tuning typical controllers.

Doing the respectively calculations in conformity with the elaborated algorithm for the given object are obtained the following results:

- for the *PI* controller:
  $J_{opt}=0.2127$, $k_p=0.9906$, $k_i=0.198$ or $T_i=5.05$;
- for the *PID* controller:
  $J_{opt}=0.635$, $k_p=6.745$, $k_i=1.245$ or $T_i=0.8$, $k_d=8.511$.

To check the obtained results in case of the tuning controllers *PI* and *PID* using the proposed method, it was using the Ziegler-Nichols (ZN) method.

In conformity of this method it was obtained the follow critical parameters of the control system: $k_{cr}=15.7$, $T_{cr}=8$ s. Using this values it was determinate the optimal values of *PI* controller: $k_{popt}=7.065$ and $k_i=0.156$ $(T_{iopt}=6.4\ s)$ and for *PID* controller: $k_{popt}=11.775$; $k_i=0.208$ $(T_{iopt}=4.8\ s)$; $k_{dopt}=0.8\ s$; $T_{fopt}=0.08\ s$.

For verify the obtained results in case of tuning controllers *PI, PID* to the model object (1) using the MSD and ZN methods, it was made the computer simulation of the control system in MATLAB.

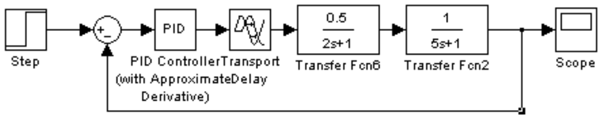

Fig. 2 Simulation diagrams of the control system

The obtained results of computer simulation are presented in Fig. 3*a, b*. Fig. 3*a* is presented the transition process of control system with *PI* controller tuning after MSD (curve 1) and ZN (curve 2) methods, but in Fig. 3*b* are presented the transition process of control system with *PID* controller tuning after MSD (curve 1) and ZN (curve 2) methods. Analyzing the obtained transition process of the control system with *PI* and *PID* controllers tuning, the obtained performances of the system are presented in the table 1.

TABEL I. PERFORMANCE OF CONTROL SYSTEM

Perform. of control system		ε , %	t_r	σ ,%	λ	t_c
M.S.D. method	PI	5	23			
	PID	5	19	40	2	4
Z.N. method	PI	5	80	19	2	6
	PID	5	80	57	>10	4

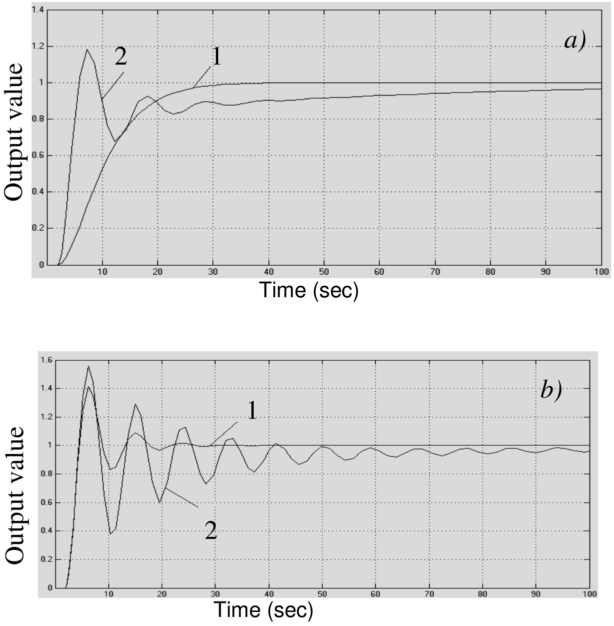

Fig. 3 The transient processes of control systems with *PI, PID* controllers

The control system with *PID* controller tuning by MSD method has the higher performance then performance of control system with *PID* controller tuning by ZN method.

For the realtions $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ of *PI* and *PID* controllers (see realtions (3), (4), (6), (7) and (8)) with the known parameters values of object and at the variation of stability degree *J* in the respectively limits were made the iterative calculations and the obtained results are presented in Fig. 4, 5.

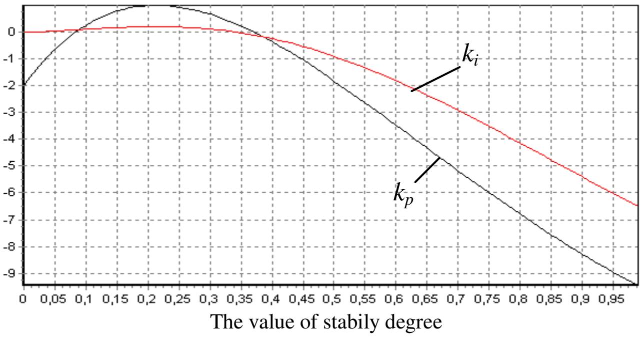

Fig. 4 The dependence of *PI* controller parameters of the stability degree value

It can be observed from Fig. 4 and 5 that for the optimal stability degree J_{opt} value the sets of values of tuning parameters of *PI* and *PID* controller are the maximal for the object's parameters (1).

For analyse the set of performance of control system with *PI* and *PID* controllers from fig. 4 and 5 it was chosen the sets of values *J* - k_p, k_i for the *PI* controller (table 2) and *J* -

k_p , k_i, k_d for the *PID* controller (Table III) and it was made the computer simulation of the control system with *PI, PID* controllers, and the transition processes are presented in Fig.6, 7, where the number of the curves it is coincided with the number of the values of the set's variant from Tables II and III (the marked position 1 from Tables II, III present the optimal values).

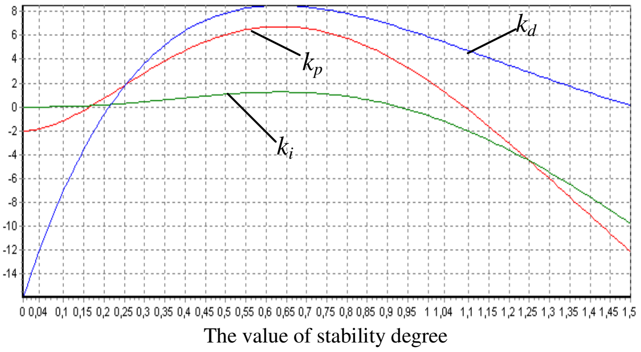

Fig. 5 The dependence of PID controller parameters of the stability degree value

TABEL II. THE VALUES OF THE *PI* CONTROLLER'S PARAMETERS.

Nr.	*J*	k_p	k_i
1	**0.2127**	**0.9906**	**0.198**
2	0.1	0.253	0.097
3	0.15	0.776	0.161
4	0.25	0.924	0.182
5	0.3	0.651	0.106

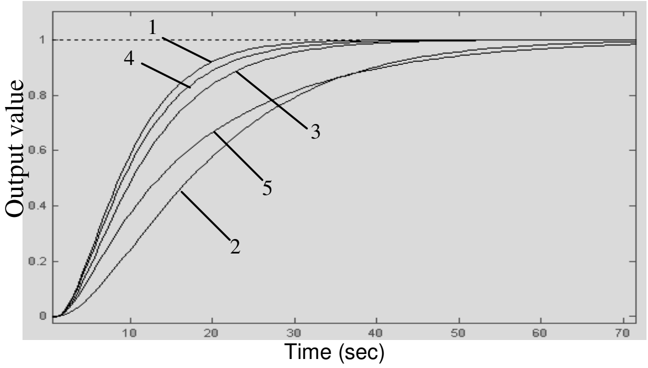

Fig. 6 The transient process of control systems with *PI* controller

TABEL III. THE VALUES OF THE PI CONTROLLER'S PARAMETERS

Nr.	*J*	k_p	k_i	K_d
1	**0.635**	**6.745**	**1.245**	**8.511**
2	0.35	3.65	0.564	4.916
3	0.5	5.261	1.035	7.78
4	0.75	6.332	1.101	8.217
5	0.9	4.427	0.315	7.05

Analyzing the performance of the control system from Fig. 6 it was observed that **the optimal performances** were obtained for curves with $J_{opt}=0.2127, \varepsilon = -5$ % and t_r=23 s, which correspond with stability degree calculated from relation (2), but for the other transitional processes the tuning time are: for curve 2 – 57 s, curve 3 – 31 s, curve 4 – 28 s, curve 5 – 61 s.

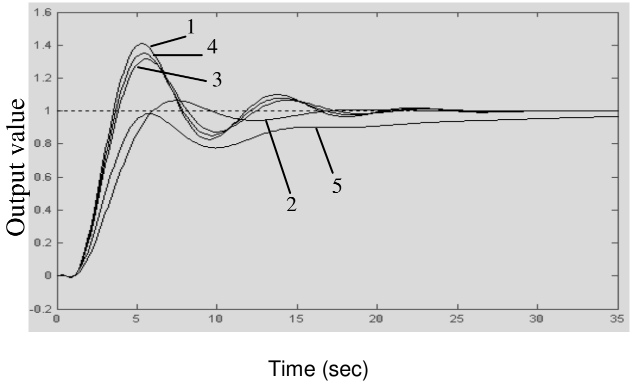

Fig. 7 The transient process of control systems with *PID* controller

For the control system with *PID* controller the transition processes are presented in Fig. 7 and the performance of the system are presented in the Table IV. The optimal performances are obtained for the 2 curve. The 1 curve from Fig. 7 presented the transition process of control system with values sets of the *PID* controller for the optimal stability degree value $J_{opt}=0.635$.

TABEL IV. THE VALUES OF THE *PID* CONTROLLER'S PARAMETERS

Nr.	t_r, s	σ ,%	λ	t_c, s
1	19	40	2	3
2	**16**	**9**	**1**	**6**
3	17	30	2	3,2
4	18	35	2	3,1
5	37			5,5

To compare the performance of control system with *PID* controller with tuning parameters by MSD and ZN methods, in the fig. 8 are presented the transition processes of the system: the curve *1* is presented the transition process of control system with *PID* controller tuning by MSD method for $J_{opt}=0.635$; the curve *2* is presented the transition process of control system with *PID* controller tuning by ZN method; the curves *3* is presented **the optimal transition process** of the control system with *PID* controller tuning by M.S.D. method, which was obtained for the variation of value *J* for the sets of values which are presented in the table 3 (curve 2 from Fig. 7).

For the control system with *PID* controller tuning by MSD and ZN methods, which the transition process are presented in Fig. 8, were analyzed the distribution of poles of characteristic equation of the control system in the complex plan which were calculated in the MATLAB and presented in Fig. 9.

In Fig. 9 are presented the domination poles: *1* – for the control system with *PID* controller tuning by MSD method, the minimal pole has value - 0.182; *2* - for the control system with *PID* controller tuning by ZN method, the minimal pole has value - 0.0154; 3 - for the control system with *PID* controller (optimal) tuning by MSD method, the minimal pole has value - 0.163;

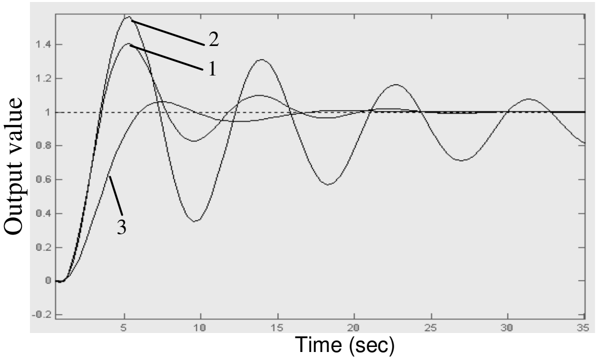

Fig. 8 The transient process of control system with *PID* controller

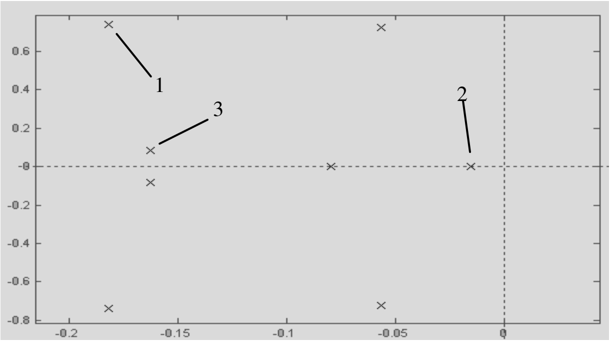

Fig. 9 The distribution of the characteristic equation's poles

Analyzing the distribution of poles of characteristic equations of control system with *PID* controller tuning by MSD and ZN methods it can be observed that the relative stability of the control system with PID controllers tuning by MSD method has the reserve of stability much higher (≥ 10 times) than the reserve of stability of the control system with *PID* controller tuning by ZN method.

It follows that the robustness of control system with *PID* controller tuning by MSD method in the case of the variation of parameters of the control object (1) is higher than the robustness of control system with *PID* controller tuning by ZN method.

IV. CONCLUSION

As a result of the study, the following conclusion can be made:

- It is proposed the grafo-analitical method of tuning *PI*, *PID* controllers to the model objects with inertia of the third order and time delay, which permitted to obtain the settled performance.
- For the control system with *PI* controller tuning by MSD method, the transition process of system is aperiodic and optimal for the given values of object (see the curve 1, Fig. 6). The transition process of control system with *PI* controller tuning by ZN method is oscillating. The performances of control system with *PI* controller tuning by MSD method are better then the performances of control system with *PI* controller tuning by ZN method.
- For the control system with *PID* controller tuning by MSD method the transition process of system are oscillating for the given values of the object (see the curve 1 Fig. 3 *b*), but with better performance then the performance of control system with controller tuning by ZN method.
- Analyzing the transition processes of the control system with *PID* controller from Fig. 7 it can be seen that making the variation of the degree stability of the control system were obtained the sated performance of the control system (curve 3, Fig. 8). In this case it is necessary to make the iterative calculations for the difference value sets of the *J*- k_p, k_i, k_d for the *PID* controller.
- Control system with *PID* controller tuning by MSD method has t_r the reserve of stability much higher (≥ 10 times) than the reserve of stability of the control system with *PID* controller tuning by ZN method.
- The robustness of control system with *PID* controller tuning by MSD method in the case of the variation of parameters of the control object (1) is higher than the robustness of control system with *PID* controller tuning by ZN method.

REFERENCES

[1] R. K. Dorf, R. X. Bishop. Sovremennîe sistemî upravlenia (Modern Control Systems). Moskva: Laboratoria Bazovîh Znanii, 2004. - 832s.

[2] Ş. Preitl, R. E. Precup. Introducere în ingineria reglării automate, Timişoara: Editura Politehnica, 2001. - 334 p.

[3] G. I. Zagarii, A. M. Shubladze. Sintez system upravlenia na osnove criteria maximalnoi stepeni ustoichivosti. (The Synthesis of the Control System According to the Maximal Stability Degree), Moskva: Energoatomizdat, 1998. - 198 s.

[4] B. Izvoreanu, I. Fiodorov, "The Synthesis of Linear Regulators for periodic Objects with Time Delay According to the Maximal Stability Degree Method", Preprints the Fourth IFAC Conference on System Structure and Control, Ed. Tehnică Bucuresti, pp. 449 – 454, 1997.

[5] I. Fiodorov, B. Izvoreanu, M. Pisarenco, "Comparative Analysis of Regulators Tuning Methods to Models of Objects with Inertia", Buletinul Institutului Politehnic din Iaşi, Tomul L(LIV), Fasc.5A, Electrotehnică, Energetică, Electronică, pp. 63-68, 2004.