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A Second Order-Cone Programming Relaxation for Facility Location Problem

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Abstract: **In this paper we discuss the numerical solving of one of the well known problems related to the production systems. Namely, it is about facility location problem, which is reformulated in terms of non-convex minimization problems with quadratic constraints. After relaxation of the above problem, a model based on second-order cone programming is obtained.**

Keywords: Facility Location Problem; Second Order-Cone Programming; Relaxation

I. INTRODUCTION

In the field of production systems organization, there is a problem known as the facility location problem. This problem occupies an important place in the modern economic world, currently being researched intensively and widely by specialists.

The facility location task involves grouping of equipment, machinery, available space, etc. to determine the best solution to an objective function, while the industrial information system requirements are satisfied [1-3].

Most of mathematical models of the considered problem are formulated in terms of linear programming with binary variables. They belong to the class of NP-hard problems [4, 5]. Applied methods encounter difficulties in the case of solving large dimension problems. Therefore, various approximate methods are used. With these methods, one can provide a "good" solution in a reasonable time. Approximate methods are divided into two major classes: heuristic methods and local search methods based on relaxation of the considered problems, on condition that the variables are integers [5].

In this paper, we propose reformulation and relaxation of the investigated problem, in terms of second order cone programming (SOCP), which is an important branch of convex programming [6].

II. NOTATION

We use the following notation and symbols throughout the paper: $\Re^n$ is the Euclidean space of *n*-dimensional column vectors with the inner product $x^T y = \sum_{i=1}^{n} x_i y_i$, and $\|x\|_2 = \sqrt{x^T x}$. The superscript "*T*" indicates transposition. $\Re_+$ is the set of all non-negative real numbers. e to denote a vector with each component equal to one, and e_j to denote a vector with all components equal to zero, except for j^{th} component, which is equal to one (the dimension is clear from the context). The matrix I_m represents the identity matrix of the $m \times m$ dimension; o and O_m stand for zero vectors, and zero matrixes, respectively.

III. MATHEMATICAL PROGRAMMING MODEL

The mathematical model of the facility location problem can be written as [1, 3, 5]:

$$\sum_{i\in F} f_i y_i + \sum_{i\in F}\sum_{j\in D} c_{ij} x_{ij} \to \min \tag{1}$$

subject to

$$x_{ij} \le y_i, i \in F, j \in D, \tag{2}$$

$$\sum_{i\in F} x_{ij} = 1, j \in D, \tag{3}$$

$$x_{ij}, y_i \in \{0,1\}, i \in F, j \in D. \tag{4}$$

The following notations are used above: $D = \{1,2,\ldots,n\}$ - a finite set of "clients"; $F = \{1,2,\ldots m\}$ - a finite set of possible "facilities"; f_i - fixed costs for opening facilities, $f_i \in \Re_+$ for every facility $i \in F$; $c_{ij} \in \Re_+$ - the costs of service for $\forall i \in F, \forall j \in D$; $x_{ij} = 1$, if facility *i* serves the client *j*, otherwise