

Diagrammatic analysis of the Hubbard model II: Superconducting state

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Diagrammatic analysis for normal state of Hubbard model proposed in our previous paper^[1] is generalized and used to investigate superconducting state of this model. We use the notion of charge quantum number to describe the irreducible Green's function of the superconducting state. As in the previous paper we introduce the notion of tunneling Green's function and of its mass operator. This last quantity turns out to be equal to correlation function of the system. We proved the existence of exact relation between renormalized one-particle propagator and thermodynamic potential which includes integration over auxiliary interaction constant. The notion of skeleton diagrams of propagator and vacuum kinds were introduced. These diagrams are constructed from irreducible Green's functions and tunneling lines. Identity of this functional to the thermodynamic potential has been proved and the stationarity with respect to variation of the mass operator has been demonstrated.

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I. INTRODUCTION

The present paper generalizes our previous work^[1] on diagrammatic analysis of the normal state of the Hubbard model^[2-4] to the superconducting state.

Now we shall assume the existence of pairing of charge carriers and non-zero Bogolyubov quasi-averages^[5] and, consequently, of the Gor'kov anomalous Green's functions^[6].

The main property of the Hubbard model consists in the existence of strong electron correlations and, as a result, of the new diagrammatic elements with the structure of Kubo cumulants and named by us as irreducible Green's functions. These functions describe the main charge, spin and pairing fluctuations of the system.

The new diagram technique for such strongly correlated systems has been developed in our earlier papers^[7-17]. This diagram technique uses the algebra of Fermi operators and relies on the generalized Wick theorem which contains, apart from usual Feynman contributions, additional irreducible structures. These structures are the main elements of the diagrams.

In superconducting state, unlike the normal one, the irreducible Green's functions can contain any even number of fermion creation and annihilation operators, whereas in normal state the number of both kinds is equal. Therefore we need an automatic mathematical mechanism which takes into account all the possibilities to consider the interference of the particles and holes in the superconducting state.

With this purpose we use the notion of charge quantum number, introduced by us in ^[7] and called α -number, which has two values $\alpha = \pm 1$ according to the definition

$$C^\alpha = \begin{cases} C & , \quad \alpha = 1; \\ C^+ & , \quad \alpha = -1. \end{cases} \quad (1)$$

Were C is a Fermion annihilation operator. In this new representation the tunneling part of the Hubbard Hamiltonian can be rewritten in the form

$$\begin{aligned} H' &= \sum_{\sigma} \sum_{\vec{x} \vec{x}'} t(\vec{x}' - \vec{x}) C_{\vec{x}', \sigma}^+ C_{\vec{x} \sigma} \\ &= \frac{1}{2} \sum_{\alpha=-1,1} \sum_{\sigma} \sum_{\vec{x} \vec{x}'} \alpha t_{\alpha}(\vec{x}' - \vec{x}) C_{\vec{x}', \sigma}^{-\alpha} C_{\vec{x} \sigma}^{\alpha}, \end{aligned} \quad (2)$$

with the definition of the tunneling matrix elements

$$\begin{aligned} t_1(\vec{x}' - \vec{x}) &= t(\vec{x}' - \vec{x}) \\ t_{-1}(\vec{x}' - \vec{x}) &= t(\vec{x} - \vec{x}') \\ t(\vec{x} = 0) &= 0. \end{aligned} \quad (3)$$

In this charge quantum number representation the operator H' has an additional multiple α for every vertex of the diagrams and additional summation over α . All the Green's functions depend of this number.

In interaction representation operator H' has a form

$$\begin{aligned} H'(\tau) &= \frac{1}{2} \sum_{\alpha\sigma} \sum_{\vec{x} \vec{x}'} \alpha t_{\alpha}(\vec{x}' - \vec{x}) \\ &\quad \times C_{\vec{x}', \sigma}^{-\alpha} (\tau + \alpha 0^+) C_{\vec{x} \sigma}^{\alpha}(\tau), \end{aligned} \quad (4)$$

The main part of the Hubbard Hamiltonian

$$\begin{aligned} H &= H^0 + H', \\ H^0 &= \sum_i H_i^0, \\ H_i^0 &= -\mu \sum_{\sigma} C_{i\sigma}^+ C_{i\sigma} + U n_{i\uparrow} n_{i\downarrow}, \end{aligned} \quad (5)$$

contains the local part H^0 , where μ is the chemical potential and U is the Coulomb repulsion of the electrons.