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The effects of Ethernet LANs' fragmentation

Ion Bolun, Victor Andronatiev

Abstract

Three configurations of Ethernet networks that operate in duplex mode at relatively low load of transmission media are investigated. Configurations differ by number of collision domains and switches used in the network. To investigate the dependences among basic parameters, analytic models of the functioning of such networks are elaborated. Calculations made for a relatively large set of values of initial data confirm the necessity of case dependent special investigations on the physical configuration of concrete Ethernet networks. It has been determined, for example, that the fragmentation doesn't necessarily permit to increase the number of stations in the network; it may be even vice versa.

1 Introduction

Currently the Ethernet, Fast Ethernet and Gigabit Ethernet local networks are largely used. Moreover, the efficient capitalization of their resources is important. In many cases one of the reserves consists in the fragmentation of networks - their construction from many interconnected segments [3, 4]. Such a fragmentation usually permits:

- grouping in a fragment of stations with higher traffic interchange;
- to increase the total data traffic in the network;
- to decrease the mean time of messages delay in the network;
- to increase the reliability of services offered etc.

At the same time, fragmentation implies additional costs with equipment to interconnect the network's segments. Therefore, for a rational networks' fragmentation, it is necessary to make a quantitative appreciation of the respective effects. Some aspects of such appreciation are discussed in this paper.

2 Preliminary considerations

Investigations are based on networks or network fragments with only one collision domain. Some results of the research of networks with only one collision domain and homogenous stations are published in [1-3]. In these papers the boundaries of networks performance parameters, such as the mean delay time of packets and the mean utilization of transmission media are estimated.

At the same time, data packets are considered of fixed length, which, as mentioned in [3], sometimes causes significant deviations from the real situation, improving the real network performance. Moreover, in paper [2] the nature of flow of messages generated by network stations is not taken into account; in calculations only the mean value of respective parameters is used, such as: the mean time that a station doesn't have packets to transmit and the mean waiting time until the beginning of packet transmission. In [3] the flows of packets generated by stations are considered of Poisson type, but at the same time it is supposed that each station transmits each packet before the next one arrives; this corresponds to the reality only in the case of messages which do not exceed the maximum length of one packet; in addition it is considered that the network contains an infinite number of stations.

Thus, investigations in [1-3] are oriented to marginal appreciations of the performance parameters of local networks with only one collision domain. In the present paper an attempt is made to use models that would allow the evaluation of performance parameters for the cases of relatively low transmission media load ρ, aiming to appreciate the effects of fragmentation of local networks in many collision domains. It is well known that the Ethernet technology is working efficiently at a load that doesn't exceed 30-40 % [3]. At such a load, there may be signals' collisions in the transmission media, but the probability that there will be more than one collision per packet is very small.

There are investigated Ethernet local networks containing N homogeneous stations, each of which generates requests to other stations in the network with the transmission of a Poisson flow of messages of rate β; the repartition of messages' size is exponential and its average is L. To be mentioned that the exponential repartition of messages' size and data packets' size are accepted in many investigations [3-5, etc.]. Stations of the same collision domain are distributed uniformly between marginal stations. The data transmission speed in the media is V. It is required to investigate the network's basic operation features when the following constraints are observed:

- the load ρ of the network transmission media doesn't exceed a given value ρ_0
$$\rho \leq \rho_0; \tag{1}$$
- the mean delay time θ of a message in the network doesn't exceed a given value θ_0
$$\theta \leq \theta_0. \tag{2}$$

Messages are transmitted through the network by packets, the size of which is considered of exponential repartition with the average value ν and the heading of length s. Thus, the number of packets per message is equal to $L/(\nu - s)$. The processing time for disassembling messages in packets and for the inverse operation of assembling packets in messages at stations can be considered proportional with the number of packets per message, that is equal to $aL/(\nu - s)$, where a is a proportionality coefficient.

Let T be the mean delay time of a packet in the network (the mean waiting time + the mean transmission time), then

$$\theta = \frac{L}{\nu - s}\left(T + a\right). \tag{3}$$

Taking into consideration relation (3), the constraint (2) can be represented, at packets' level, as

$$T \leq T_0, \tag{4}$$

where T_0 is the upper limit of the mean time of a packet delay in the network; the following relation takes place between constants θ_0 and T_0:

$$\theta_0 = \frac{L}{\nu - s}\left(T_0 + a\right). \qquad (5)$$

Switches (bridges) are used to interconnect different network fragments. It is considered that the network operates in the duplex mode with the memorization of frames by switches and each collision domain of it forms a separate fragment. Thus, the network's physical configuration is determined by the content of its fragments and the topology of their interconnection through switches.

From multiple possible variants, the investigations in this paper will confine to networks of three configurations:

1) networks with only one collision domain;

2) networks with a switch to which m homogeneous fragments-collision domains, with n stations each, are connected

$$n = \frac{N}{m}; \qquad (6)$$

3) networks with two switches, to each of which m homogeneous fragments-collision domains, with n stations each, are connected

$$n = \frac{N}{2m}. \qquad (7)$$

Obviously, the rate μ of packets transmission through network media is calculated as

$$\mu = \frac{V}{\nu}. \qquad (8)$$

There are of interest such aspects of the problem as:

- determination of parameters θ and ρ for a given network;
- finding the maximum number N_{max} of stations for a given network;

- comparison of the performance of networks of different configurations.

These aspects will be investigated in sections 3–6. The analytic models of networks of each of these three configurations are made up in sections 3–5, and some results of calculations for a set of real values of the respective parameters are described in section 6.

3 Networks of configuration 1

Let there be a network with a single collision domain in the frame of which all stations are interconnected. Two analytic models of such networks will be investigated: (1) model 1a, which doesn't take into account the time of signal propagation σ through the transmission media, nor the mean delay time ω caused by a possible collision and (2) model 1b, which takes into account parameters σ and ω. The quantitative comparison of these two models is made in subsection 6.2.

3.1 Model 1a – an elementary analytic model.

When constraint (4) is observed and ρ_0 has reasonable values, it is acceptable, in practical calculations, to not take into account the influence of collisions on packets' delay time in the network. The time of signal propagation through the transmission media will be not taken into account, as well. The opportunity of such suppositions is confirmed for a large set of real values of initial data in subsection 6.2.

So, the network can be represented as a queue system with one server – the transmission media. The mean time of a packet delay T_1 in the network, taking into account the Poisson nature of packets flow (see [1]) and the exponential repartition of packets' size, are determined as [5]

$$T_1 = \frac{1}{\mu - \lambda_1}, \tag{9}$$

where the rate λ_1 of packets flow in the transmission media is calculated like

$$\lambda_1 = N\gamma. \tag{10}$$

Taking into account relations (8) and (10), formula (9) takes the following form

$$T_1 = \frac{\nu}{V - N\nu\gamma}. \tag{11}$$

At the same time, load ρ_1 of the transmission media, taking into account relations (8) and (10), is

$$\rho_1 = \frac{\lambda_1}{\mu} = \frac{N\nu\gamma}{V}. \tag{12}$$

The analytic model of the network, from the viewpoint of aspects suggested in section 2, is determined by relations (11) and (12) for parameters T_1 and ρ_1.

Determination of the maximum number N_{1max} of stations in the network.

The observance of constraints (1) and (3) or (4) imposes the limitation of number N of stations in the network. Proceeding from the condition not to exceed the mean delay time T_0 of packets in the network (see constraint (4)) and taking into account formula (11), one has

$$\frac{\nu}{V - N\nu\gamma} \leq T_0 \tag{13}$$

or

$$N_{1\,\max}(T_0) = \frac{VT_0 - \nu}{\nu\gamma T_0}. \tag{14}$$

As well, proceeding from the condition that the load mustn't exceed ρ_0 (see constraint (1)), taking into account the formula (12), one has

$$\frac{N\nu\gamma}{V} \leq \rho_0 \tag{15}$$

or

$$N_{1\,\max}(\rho_0) = \frac{\rho_0 V}{\nu\gamma}. \tag{16}$$

Combining constraints (14) and (16) with regard to the maximum number of stations in the network, one has

$$N_{1\max} = \min\left\{\frac{\rho_0 V}{\nu\gamma}; \qquad \frac{VT_0-\nu}{\nu\gamma T_0}\right\}. \tag{17}$$

3.2 Model 1b – a more exact analytic model.

In order to consider the mean time σ of signal propagation through the media between the source station and the destination station and also the mean delay time ω caused by a possible collision, one adds values σ and ω to expression (9) for T_1

$$T_1 = \frac{1}{\mu-\lambda_1} + \sigma + \omega. \tag{18}$$

Let's determine the values of σ and ω. For a station placed at a distance l (in time) from one of the two marginal stations (Fig. 1) and a uniform distribution of stations between the two marginal stations, the mean distance r (in time) to one of the other $n-1$ stations of the collision domain is

$$r = \frac{l}{\tau}\int_0^l t\frac{1}{\tau}dt + \frac{\tau-l}{\tau}\int_0^{\tau-l} t\frac{1}{\tau}dt = \frac{l}{2} + \frac{(\tau-l)^3}{2\tau^2}. \tag{19}$$

Here τ is the network diameter in time units – the signal propagation time through the media between the marginal stations. Then, in average on the set of n stations, one has

$$\sigma = \int_0^\tau r\frac{1}{\tau}dl = \int_0^\tau \left[\frac{l}{2} + \frac{(\tau-l)^3}{2\tau^2}\right]\frac{1}{\tau}dl = \frac{3}{8}\tau. \tag{20}$$

Obviously,

$$\omega = p(\sigma+\eta), \tag{21}$$

where p is the probability of one collision during the transmission of the current packet, the mean transmission time before the collision is numerically equal to σ, and η is the mean time before the beginning of packet retransmission after the collision. The following relation takes place:

$$p = \rho \frac{\sigma}{\frac{\nu}{V}} = \frac{3}{8}\rho\tau\mu = \frac{3}{8}\lambda_1\tau. \qquad (22)$$

Here $\rho = \lambda_1/\mu$ is the media load (without taking into consideration the collisions) and, at the same time, the probability that when a new packet arrives the transmission media is busy. Taking into consideration that the ratio ν/V is the mean packet transmission time, the ratio $\sigma/(\nu/V)$ is the probability that the source station of the new packet doesn't identify the busy state of the media and so it will start the transmission, which will, in the ultimate respect, lead to a collision.

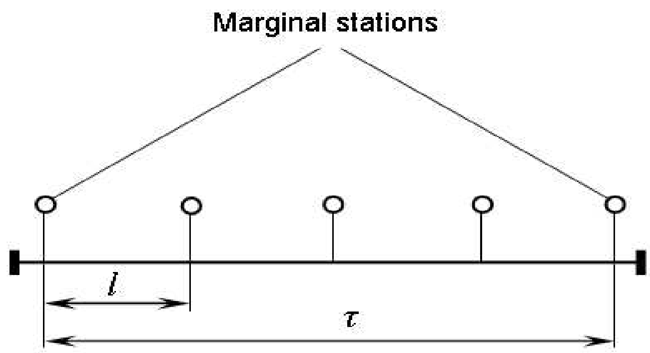

Figure 1. A collision domain.

After the collision, each of the two stations will begin the transmission after a random period of time which is uniformly distributed in the interval $[0;b]$. So, the probability that at least one of the two stations will begin the transmission after the period of time t is

$$P(x_1 \leq t \text{ or } x_2 \leq t) = 1 - \left(\frac{b-t}{b}\right)^2 = \frac{t(2b-t)}{b^2}, \qquad (23)$$

from which the repartition $f(t)$ is

$$f(t) = \frac{dP(x_1 \leq t \text{ or } x_2 \leq t)}{dt} == \frac{2}{b^2}(b-t). \qquad (24)$$

So,

$$\eta = \int_0^b t f(t) dt = \int_0^b \frac{2t(b-t)}{b^2} dt = \frac{b}{3}. \tag{25}$$

By substituting (22) and (25) in (21), one can obtain

$$\omega = \frac{3}{8}\rho\tau\mu\left(\frac{3}{8}\tau + \frac{b}{3}\right) = \frac{3}{8}\lambda_1\tau\left(\frac{3}{8}\tau + \frac{b}{3}\right) = h\lambda_1, \tag{26}$$

where

$$h = \sigma(\sigma + \eta) = \frac{3}{8}\tau\left(\frac{3}{8}\tau + \frac{b}{3}\right). \tag{27}$$

Taking into account relations (20) and (26), formula (18) takes the form

$$T_1 = \frac{\nu}{V - N\nu\gamma} + \frac{3}{8}\tau\left[1 + N\gamma\left(\frac{3}{8}\tau + \frac{b}{3}\right)\right]. \tag{28}$$

The load ρ_1 of the transmission media, taking into account relation (12), is

$$\rho_1 = \frac{\lambda_1}{\mu} + \lambda_1\omega = \frac{\lambda_1}{\mu} + \lambda_1 h\lambda_1 = h\lambda_1^2 + \frac{\lambda_1}{\mu} = N\gamma\left(N\gamma h + \frac{\nu}{V}\right). \tag{29}$$

The network analytic model 1b, from the point of view of the aspects proposed in section 2, is determined by relations (28) and (29) for parameters T_1 and ρ_1.

Determining the maximum number N_{1max} of stations in the network.

From the condition that the mean delay time of packets in the network should not exceed T_0 (see constraint (4)), taking into account formula (28), one has

$$\frac{\nu}{V - N\nu\gamma} + \frac{3}{8}\tau\left[1 + N\gamma\left(\frac{3}{8}\tau + \frac{b}{3}\right)\right] \le T_0. \tag{30}$$

Then from (30) one has

$$N_{1\max}(T_0) = \frac{hV + T_0'\nu + \sqrt{(hV + T_0'\nu)^2 + 4h\nu(\nu - T_0'V)}}{2h\nu\gamma}, \tag{31}$$

where

$$T_0^{'} = T_0 - \frac{3}{8}\tau. \tag{32}$$

To be mentioned that in formula (31) the solution obtained using the sign "-" (minus) before the square root doesn't have sense (the value of $N_{1\max}(T_0)$ would be negative).

In a similar mode, from the condition that the load of media should not exceed ρ_0 (see constraint (1)), taking into account formula (29), one has

$$N\gamma \left(N\gamma h + \frac{\nu}{V} \right) \leq \rho_0 \tag{33}$$

or

$$N_{1\max}(\rho_0) = \frac{-\nu + \sqrt{\nu^2 + 4h\rho_0 V^2}}{2\gamma h V}. \tag{34}$$

As in the case of formula (31), in formula (34) the solution obtained using the sign "-" (minus) before the square root doesn't have sense, because the value of $N_{1\max}(\rho_0)$ would be negative.

Combining expressions (31) and (34) with regard to the maximum number of stations in the network, one has

$$N_{1\max} = \min \left\{ \frac{-\nu + \sqrt{\nu^2 + 4h\rho_0 V^2}}{2\gamma h V}; \frac{hV + T_0^{'}\nu + \sqrt{(hV + T_0^{'}\nu)^2 + 4h\nu(\nu - T_0^{'}V)}}{2h\nu\gamma} \right\}. \tag{35}$$

4 Networks of configuration 2

4.1 An analytic model.

Let's consider a network with one switch to which $m$homogenous fragments-collision domains of n stations each are connected. The mean delay time T_2 of a packet in such a network is determined as

$$T_2 = \frac{\lambda_{2int} T_2^{'} + \lambda_{2ext} 2T_2^{'}}{\lambda_{2int} + \lambda_{2ext}} = T_2^{'} \frac{\lambda_{2int} + 2\lambda_{2ext}}{n\gamma}, \tag{36}$$

where:

$T_2^{'}$ - the mean delay time of a packet in a fragment-collision domain;

λ_{2int} - the rate of packets flow generated by the n stations of the same collision domain and destined to stations belonging to it;

λ_{2ext} - the rate of packets flow generated by the n stations of the same collision domain and destined to another $N-n$ stations (stations that belong to the other collision domains) of the network. Obviously,

$$\lambda_{2int} + \lambda_{2ext} = n\gamma. \tag{37}$$

In relation (37), the product $n\gamma$ determines the rate of requests flow generated by the n stations of the investigated collision domain. Taking into account that the rate of packets flow generated by one of the stations and destined to a different concrete station is $\gamma/(N-1)$ the following relations take place:

$$\lambda_{2int} = n\frac{\gamma}{N-1}(n-1) = n\gamma\frac{n-1}{N-1}; \tag{38}$$

$$\lambda_{2ext} = n\frac{\gamma}{N-1}(N-n) = n\gamma\frac{N-n}{N-1}. \tag{39}$$

The quantitative comparison of models 1a and 1b for networks of configuration 1 described in subsections 3.1 and 3.2 show (see section 6.2) that, at reasonable loads of the transmission media, these do not differ significantly. Therefore, in models for networks of configurations 2 and 3, the effects caused by signals collisions shall not be taken into consideration, nor will the duration of signal propagation through the transmission media. So, like in formula (9), the following relation takes place

$$T_2^{'} = \frac{1}{\mu - \lambda_2}, \tag{40}$$

where the rate λ_2 of packets flow in the transmission media of one collision domain is calculated according to formula

$$\lambda_2 = n\gamma + (N-n)\frac{\gamma}{N-1}n = n\gamma\frac{2N-n-1}{N-1}. \tag{41}$$

In expression (41), product $n\gamma$ determines, as was mentioned above, the rate of the flow of requests generated by the n stations of this collision domain, and expression $n\gamma(N-n)/(N-1)$ – the rate of the flow of requests generated by the other $N-n$ stations of the network and addressed to stations of the investigated collision domain (external requests). From (41) it is easy to notice that

$$\lambda_2 < 2n\gamma, \tag{42}$$

and at $n << N$ or, more exactly, at $m \to \infty$ relation (41) turns to

$$\lim_{m\to\infty} \lambda_2 = 2n\gamma, \tag{43}$$

meaning that λ_2 depends, practically, not on the total number N of stations in the network, but on the number $n = N/m$ of stations in a collisions domain.

From (38), (39) and (41) one can notice that the following relation takes place

$$\lambda_2 = \lambda_{2int} + 2\lambda_{2ext}, \tag{44}$$

therefore and, also, taking into consideration relation (40), formula (36) can be replaced with

$$T_2 = \frac{1}{\mu - \lambda_2} \cdot \frac{\lambda_2}{n\gamma} = \frac{2N-n-1}{\mu\,(N-1) - n\gamma\,(2N-n-1)}. \tag{45}$$

To be mentioned that at $m = 1$ the equality $n = N$ takes place and formula (45) for T_2 is reduced to formula (9) for T_1; thus

$$T_2|_{m=1} = T_1 \tag{46}$$

and, respectively, see (3),

$$\theta_2|_{m=1} = \theta_1. \tag{47}$$

As far as it concerns load ρ_2 of the transmission media, it is determined as

$$\rho_2 = \frac{\lambda_2}{\mu} = \frac{n\nu\gamma\,(2N-n-1)}{V\,(N-1)}. \tag{48}$$

In a manner similar to (46), at $m = 1$, the equality $n = N$ takes place, and formula (48) for ρ_2 is reduced to formula (12) for ρ_1, therefore

$$\rho_2|_{m=1} = \rho_1. \tag{49}$$

So, T_2 is calculated according to the expressions (45) and (48) (taking into account (3), and $\theta_2 = \theta$) and, respectively, ρ_2 which constitutes, from the viewpoint of the research aspects proposed in section 2, the analytic model of networks of configuration 2.

4.2 An asymptotic analytic model.

The analytic model determined by relations (45) and (48) can be considerably simplified at $n << N$ or, more exactly, at m $\to \infty$. Indeed, from (45) it is easy to notice that

$$T_2 < \frac{2}{\mu - 2n\gamma}, \tag{50}$$

and at $m \to \infty$ relation (45) turns to an asymptotic, simpler one

$$\lim_{m\to\infty} T_2 = \frac{2}{\mu - 2n\gamma}, \tag{51}$$

meaning that T_2 depends, practically, not on the total number N of stations in the network, but only on the number $n = N/m$ of stations in a collision domain.

In a similar way, from (48) it is easy to notice that

$$\rho_2 < \frac{2n\gamma}{\mu} \tag{52}$$

and at $m \to \infty$ expression (48) is reduced to a simpler one

$$\lim_{m\to\infty} \rho_2 = \frac{2n\gamma}{\mu} = \frac{2n\nu\gamma}{V}, \tag{53}$$

meaning that ρ_2 depends, practically, not on the total number N of stations in the network, but only on the number $n = N/m$ of stations in the collision domain.

Thus, since usually $n << N$, both T_2 (see formula (51)) and ρ_2 (see formula (53)) depend, practically, not on the total number N of stations in the network, but only on the number $n = N/m$ of stations in the collision domain. Therefore it is reasonably to calculate the maximum number n_{2max} of stations in the collision domain, imposed by restrictions (1) and (4).

4.3 Calculation of n_{2max}.

Taking into account relation (45), the condition (4) of not exceeding the mean delay time T_0 of packets in the network takes, for this case, the following form

$$\frac{2N - n - 1}{\mu\,(N-1) - n\gamma\,(2N - n - 1)} \leq T_0, \tag{54}$$

whence, in the result of some transformations,

$$\gamma T_0 n^2 - [\gamma T_0\,(2N-1) - 1]n - [\mu T_0\,(N-1) + 2N - 1] \geq 0. \tag{55}$$

From this square inequation it is easy to obtain the solution for the maximum number $n_{2max}(T_0)$ of stations in a collision domain, calculated taking into account separately just the restriction (4) regarding the limit mean time $T_2 = T_0$ of a packet in a network of configuration 2

$$\begin{aligned} n_{2\max}(T_0)_{1,2} &= N - \frac{1}{2} - \frac{1}{2\gamma T_0} \pm \\ &\pm \sqrt{\left[N - \frac{1}{2} - \frac{1}{2\gamma T_0}\right]^2 + \frac{\mu T_0\,(N-1) + 2N - 1}{\gamma T_0}}. \end{aligned} \tag{56}$$

In a similar way, taking into account relation (48), the condition (1) that the load ρ_2 of the transmission media of the respective collision domain mustn't exceed the limit value ρ_0 takes, for this case, the form

$$\frac{n\nu\gamma\,(2N - n - 1)}{V\,(N-1)} \leq \rho_0 \tag{57}$$

or, after some simple transformations,

$$\nu\gamma n^2 - \nu\gamma(2N-1)n + \rho_0 V(N-1) \geq 0. \qquad (58)$$

From here it is easy to obtain the expression for the maximum number $n_{2max}(\rho_2)$ of stations in a collision domain, calculated taking into account just the restriction (1) regarding the limit load $\rho_2 = \rho_0$ of the transmission media in this domain,

$$n_{2\max}(\rho_0)_{1,2} = N - \frac{1}{2} \pm \sqrt{\left[N - \frac{1}{2}\right]^2 - \frac{\rho_0 V(N-1)}{\nu\gamma}}. \qquad (59)$$

Combining solutions (56) and (59), one can obtain the expression for the maximum number n_{2max} of stations in the collision domain calculated taking into account restriction (1) regarding the limit load $\rho_2 = \rho_0$ of the transmission media in this domain, as well as restriction (4) regarding the limit mean time $T_2 = T_0$ of the packet transfer in the network

$$n_{2\max} = N - \frac{1}{2} + \\ + \min\left\{ -\frac{1}{2\gamma T_0} \pm \sqrt{\left[N - \frac{1}{2} - \frac{1}{2\gamma T_0}\right]^2 + \frac{\mu T_0\,(N-1) - 2N + 1}{\gamma T_0}}; \right. \\ \left. \pm\sqrt{\left[N - \frac{1}{2}\right]^2 - \frac{aV(N-1)}{\nu\gamma}} \right\}. \qquad (60)$$

Obviously, upon using formulas (56), (59) and (60) there should be taken into account the direct relation among parameters n and N (see relation (6)).

In a similar way, to asymptotic solutions for T_2 and ρ_2 (see relations (51) and (53), respectively) correspond asymptotic expressions for $n_{2max}(T_0)$, $n_{2max}(\rho_0)$ and n_{2max}.

At $m \to \infty$, condition (4), considering relation (50), takes the form

$$\frac{2}{\mu - 2n\gamma} \leq T_0, \qquad (61)$$

from which one can obtain the asymptotic, simpler expression for $n_{2max}(T_0)$

$$\lim_{m\to\infty} n_{2max}(T_0) = \frac{\mu T_0 - 2}{2\gamma T_0} \tag{62}$$

Likewise, at $m \to \infty$, condition (1), considering relation (53), takes the form

$$\frac{2n\nu\gamma}{V} \le \rho_0, \tag{63}$$

from which one can obtain the asymptotic, simpler expression for $n_{2max}(\rho_0)$

$$\lim_{m\to\infty} n_{2\max}(\rho_0) = \frac{\rho_0 V}{2\nu\gamma}. \tag{64}$$

Combining solutions (62) and (64), one can obtain the asymptotic expression for the maximum number n_{2max} of stations in a collision domain

$$\lim_{m\to\infty} n_{2\max} = \min\left\{\frac{\mu T_0 - 2}{2\gamma T_0}; \frac{\rho_0 V}{2\nu\gamma}\right\}. \tag{65}$$

4.4 Calculation of N_{2max}.

In many practical cases, it is necessary to know, in addition to n_{2max}, the maximal number N_{2max} of stations that can be in the network. Examples of such cases include: creating a new network; expanding the number of stations in a functioning network; determining the cause of low efficiency of a functioning network, etc.

Taking into consideration relation (6) among N, n and m, restriction (54) for the value of T_2 can be turned, after some simple transformations, to the form:

$$\gamma T_0\left(2 - \frac{1}{m}\right) N^2 - [\gamma T_0 + 1 - m\,(2 - \mu T_0)]N + m[\mu T_0 - 1] \le 0, \tag{66}$$

whence

$$N_{2\max}(T_0)_{1,2} = \frac{m}{2\gamma T_0\,(2m-1)}\,[\gamma T_0 + 1 - m(2-\mu T_0) \pm$$
$$\pm\sqrt{[\gamma T_0 + 1 - m\,(2-\mu T_0)]^2 - 4\gamma T_0\,(2m-1)\,(\mu T_0 - 1)}\,\Big]. \tag{67}$$

In a similar way, considering relation (6), restriction (57) for ρ_2, can be turned, after some simple transformations, to the form:

$$\nu\gamma\left(2-\frac{1}{m}\right)N^2-(\nu\gamma+\rho_0 mV)N+\rho_0 mV\leq 0, \tag{68}$$

whence

$$N_{2\max}(\rho_0)_{1,2}=\frac{m}{2\nu\gamma(2m-1)}\left[\nu\gamma+\rho_0 mV\pm\right.$$
$$\left.\pm\sqrt{(\nu\gamma+\rho_0 mV)^2-4\rho_0\nu\gamma V(2m-1)}\right]. \tag{69}$$

Combining solutions (67) and (69), one can obtain

$$N_{2\max}=\min\{N_{2\max}(T_0);N_{2\max}(\rho_0)\}. \tag{70}$$

To be mentioned that at $m=1$ expressions (67) and (69) can be reduced to (14) and (16) ones for networks of configuration 1, therefore:

$$N_{2\max}(T_0)|_{m=1} = N_{1\max}(T_0)=\frac{\mu T_0-1}{\gamma T_0}=\frac{VT_0-\nu}{\nu\gamma T_0}; \tag{71}$$

$$N_{2\max}(\rho_0)|_{m=1} = N_{1\max}(\rho_0)=\frac{\rho_0\mu}{\gamma}=\frac{\rho_o V}{\nu\gamma} \tag{72}$$

and

$$N_{2\max}|_{m=1}=\min\{N_{1\max}(T_0);N_{1\max}(\rho_0)\}=$$
$$=\min\left\{\frac{\rho_0 V}{\nu\gamma};\frac{VT_0-\nu}{\nu\gamma T_0}\right\}. \tag{73}$$

5 A model for networks of configuration 3

Let's consider a network with two switches, to each of which m homogeneous fragments-collision domains of n stations each are connected. The mean time T_3 of a packet delay in such a network is determined as

$$T_3=\frac{\lambda_{3int}T_3^{'}+\lambda_{31}T_{31}+\lambda_{32}T_{32}}{\lambda_{3int}+\lambda_{31}+\lambda_{32}}=\frac{\lambda_{3int}T_3^{'}+\lambda_{31}T_{31}+\lambda_{32}T_{32}}{n\gamma}, \tag{74}$$

where:

$T_3^{'}$ - the mean time of a packet delay in a fragment-collision domain;

T_{31} - the mean delay time of a packet transferred between two stations connected to different collision domains of the same switch. Obviously,

$$T_{31} = 2T_3^{'}; \tag{75}$$

T_{32} - the mean delay time of a packet transferred between two stations connected to collision domains of different switches. Of course

$$T_{32} = 2T_3^{'} + T_3^{''}, \tag{76}$$

where $T_3^{''}$ is the mean delay time of a packet in the fragment-collision domain that interconnects the two switches;

λ_{3int} - the rate of packets flow generated by the n stations of a collision domain and destined to stations of the same domain;

λ_{31} - the rate of packets flow generated by the n stations of a collision domain and destined to the other $N/2 - n$ stations (stations connected to the other collision domains) of the same switch;

λ_{32} - the rate of packets flow generated by the n stations of a collision domain and destined to the $N/2$ stations connected to the another switch. Obviously,

$$\lambda_{3int} + \lambda_{31} + \lambda_{32} = n\gamma. \tag{77}$$

Let λ_{3c} is the rate of packets flow in media c that interconnects the two switches. Taking into account that the rate of packets flow generated by one station and destined to certain another station of the network is $\gamma/(N-1)$, the following relations take place:

$$\lambda_{3int} = n\frac{\gamma}{N-1}(n-1) = n\gamma\frac{n-1}{N-1} = \lambda_{2int}; \tag{78}$$

$$\lambda_{31} = n\frac{\gamma}{N-1}\left(\frac{N}{2} - n\right) = n\gamma\frac{N-2n}{2(N-1)}; \tag{79}$$

$$\lambda_{32} = n\frac{\gamma}{N-1}\frac{N}{2} = n\gamma\frac{N}{2(N-1)}; \tag{80}$$

$$\lambda_{3c} = 2\frac{N}{2}\cdot\frac{\gamma}{N-1}\cdot\frac{N}{2} = \gamma\frac{N^2}{2(N-1)} \approx \frac{\gamma N}{2}. \tag{81}$$

Relation (81) shows that the rate of flow through media c that interconnects the two switches is approximately equal to half of the rate of the summary packets flow generated by all N stations. Thus, regarding the restriction not to exceed the load ρ_0 of the transmission media, c is the critical segment. At the same time this segment can be easily used in full duplex mode and then the ρ_0 value can be bigger in comparison with the one for the collision domains; in this case, for segment c it is sufficient to consider only restriction (4) regarding the value of $T = T_3$.

To be mentioned that the rate of packets flow in the transmission media of a collision domain and, respectively, the load of this media, are the same as for the network of configuration 2 (see formula (41)). Therefore

$$T_3^{'} = T_2^{'}. \tag{82}$$

We will consider that in the transmission through the c media, the duplex operation mode is used. Then, in a similar way as (9), the following takes place

$$T_3^{''} = \frac{1}{\mu - \frac{\lambda_{3c}}{2}} = \frac{2}{2\mu - \lambda_{3c}} \tag{83}$$

and, considering relation (69), one can obtain

$$T_3^{''} = \frac{4(N-1)}{4(N-1)\mu - \gamma N^2}. \tag{84}$$

Thus, considering relations (75), (76), (78)-(82) and (84), the formula (74) can be replaced with

$$T_3 = \frac{T_2^{'}(n\gamma + \lambda_{31} + \lambda_{32}) + T_3^{''}\lambda_{32}}{n\gamma} = T_2 + T_3^{''}\frac{N}{2(N-1)}. \tag{85}$$

So, the use of configuration 3 instead of configuration 2, at preserving the total number of collision domains, results in the growth of mean delay time by value

$$T_3^{''}\frac{N}{2(N-1)} = \frac{2N}{4\mu(N-1) - \gamma N^2} \approx \frac{2}{4\mu - \gamma N} \tag{86}$$

in the case of full duplex operation mode in the c transmission media and - by value

$$T_3^{''} \frac{N}{2(N-1)} = \frac{N}{2\mu(N-1) - \gamma N^2} \approx \frac{1}{2\mu - \gamma N} \qquad (87)$$

in the case of duplex operation mode in the c transmission media.

Substituting T_2 and $T_3^{''}$ in formula (85) with expressions, (45) and (84), respectively, one can obtain

$$T_3 = \frac{2N - n - 1}{\mu(N-1) - n\gamma(2N - n - 1)} + \frac{2N}{4\mu(N-1) - \gamma N^2}. \qquad (88)$$

Condition (4) not to exceed the mean delay time T_0 of packets in the net for the full duplex operation mode in the c media, taking into account relation (70), takes the form

$$\frac{2N - n - 1}{\mu(N-1) - n\gamma(2N - n - 1)} + \frac{2N}{4\mu(N-1) - \gamma N^2} \leq T_0 \qquad (89)$$

At the same time it should be mentioned that the condition not to exceed load ρ_0 of the transmission media of a collision domain is the same as for configuration 2 (see (57)). Obviously, it is supplementary required the observance of the restriction

$$\rho_{3c} < 1, \qquad (90)$$

but this is, indirectly, taken into account by condition (88) for the T_3 value.

6 Calculations concerning the performance of some networks

6.1 Initial data.

In the aim of practical research of dependences among the parameters of networks of different configurations, calculations were made for a concrete set of initial data, some of which are the following:

$V = 10$ Mbps;

$\beta = 0,01$ mes/s;
$L = 1$ Mbit;
$\nu = 0,006392$ Mbits;
$s = 0,000208$ Mbits;
$\rho_0 = 0,3$;
$a = 0$.

The average length v of a packet is calculated, in concordance with the specifications of the standard IEEE802.3 [6], as (12784 + 576)/2 bits = 6392 bits, where 12784 bits is the maximum size and 576 bits - the minimum size of an Ethernet packet. The control information per an Ethernet packet constitutes $(18 + 8) \cdot 8$ bits = 208 bits.

6.2 Comparison of models 1a and 1b for networks of configuration 1.

In calculations, for models described in subsections 3.1 and 3.2 the following supplementary initial data are used:
$\tau = b = 285$ bt;
$\theta_0 = 0,35$ s.

As criteria for the comparison of the two models, the following relative differences are used: of mean time of a packet delay δT_1, of load $\delta\rho_1$ of the transmission media and of maximum number δN_{1max} of stations in the net:

$$\delta T_1 = \frac{T_{1b} - T_{1a}}{T_{1b}} \cdot 100\%; \tag{91}$$

$$\delta\rho_1 = \frac{\rho_{1b} - \rho_{1a}}{\rho_{1b}} \cdot 100\%; \tag{92}$$

$$\delta N_{1\,\max} = \frac{N_{1a\,\max} - N_{1b\,\max}}{N_{1a\,\max}} \cdot 100\% \tag{93}$$

The results of calculations of these indicators, using the respective expressions from subsections 3.1 and 3.2, are systemized in Table 1.

From this table one can notice that by these three indicators models 1a and 1b differ only slightly from each another. The biggest difference refers to the mean time T_1 of a packet delay, but it doesn't exceed 1,7%.

Table 1. Dependence of parameters δT_1, $\delta\rho_1$ and δN_{1max} on the number N of stations

N, stations	Message size 1000 bits			Message size 1 Mbit		
	$\delta\rho$, %	δT, %	δN_{max}, %	$\delta\rho$, %	δT, %	δN_{max}, %
50	2.73E-06	1.644	0.0158	0.0027	1.563	0.0158
100	5.46E-06	1.644	0.0158	0.0055	1.482	0.0158
150	8.19E-06	1.644	0.0158	0.0082	1.400	0.0158
200	1.09E-05	1.644	0.0158	0.0109	1.317	0.0158
250	1.36E-05	1.644	0.0158	0.0136	1.235	0.0158
300	1.64E-05	1.644	0.0158	0.0164	1.151	0.0158
350	1.91E-05	1.644	0.0158	0.0191	1.068	0.0158
400	2.18E-05	1.644	0.0158	0.0218	0.984	0.0158
450	2.46E-05	1.644	0.0158	0.0246	0.899	0.0158
500	2.73E-05	1.644	0.0158	0.0273	0.814	0.0158

This confirms the opportunity of using, at low loads of transmission media, a reductive model without taking into consideration the time of signal propagation through media and the supplementary delay of packets caused by possible collisions.

6.3 Results of other calculations.

In this section, data regarding the networks of configuration 1 refer to model 1a. In Figure 2, for networks of configuration 1, there are shown some results regarding the dependence of the maximum number $N_{1max}(T)$ of stations in the network, calculated using constraint (12) regarding the limit mean time T_0 of a packet delay. One can notice that, although the value of $N_{1max}(T)$ grows with the increase of θ_0 value, this increase is not linear.

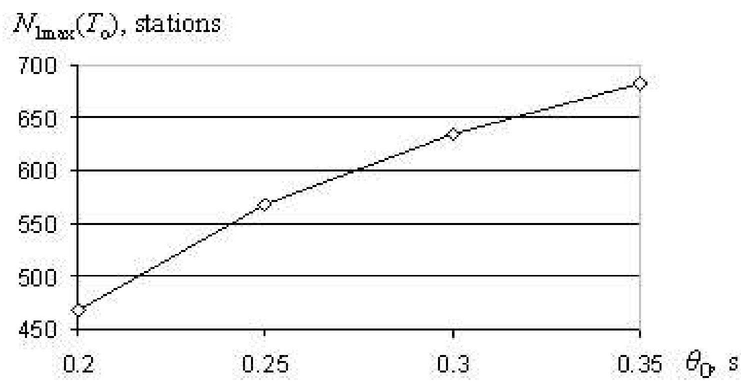

Figure 2. Dependence of $N_{1max}(T_0)$ on θ_0

Dependences of $N_{max}(T_0)$ and $N_{max}(\rho_0)$ on θ_0 and the number m of fragments-collision domains in the network, for configurations 1 and 2, are shown in Figure 3. Calculations were made according to formulas (67) and (69).

Obviously, $N_{max}(\rho_0)$ doesn't depend on θ_0. Depending on the case, there takes place $N_{max}(T_0) > N_{max}(\rho_0)$ or $N_{max}(T_0) < N_{max}(\rho_0)$. Already at relatively not big values of m, the respective dependences are almost linear. At the same time if in the case of $\theta_0 = 0,35$ s the value of $N_{max}(T_0)$ grows with the increase of m, then in cases $\theta_0 = 0,3$ s and $\theta_0 = 0,25$ s the value of $N_{max}(T_0)$ decreases at first and only afterwards it grows once with the increase of m. Moreover, in the case of $\theta_0 = 0,2$ s, the growth of the number m of fragments results in the decrease of the maximum number $N_{max}(T_0)$ of stations in the network. Thus, the fragmentation doesn't necessarily permit to increase the number of stations in the network; it may be even vice versa.

The two criteria $N_{max}(T_0)$ and $N_{max}(\rho_0)$ are combined into a single one by formula (70) for N_{max}. The dependence of N_{max} on θ_0 and m, at the same initial data as for cases from Figure 3, are shown in Figure 4.

It is important to estimate the dependence of the mean time θ of a message delay by the number N of stations and the number m

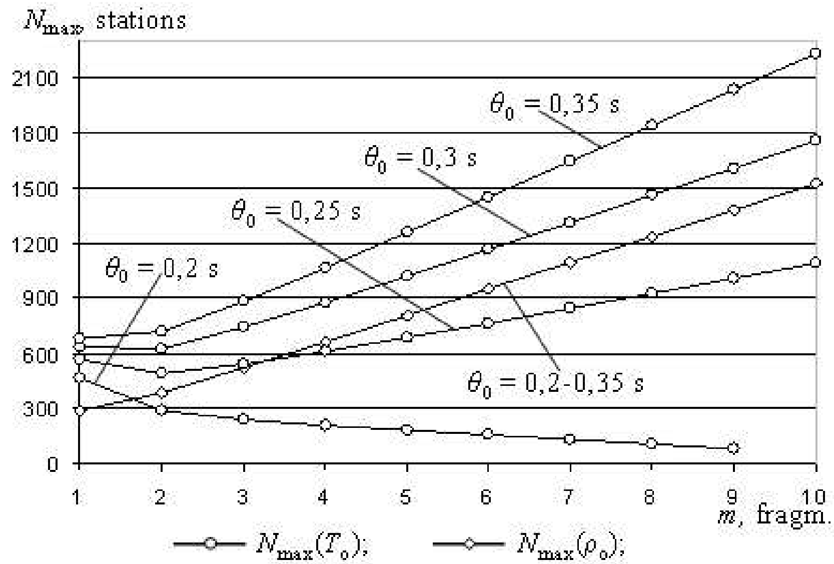

Figure 3. Dependence of $N_{max}(T_0)$ and $N_{max}(\rho_0)$ on θ_0 and m, configurations 1 and 2

of network fragments. This dependence, for configuration 2 (at $m = 1$, configuration 1) for cases encompassed by initial data described in subsection 6.1, can be traced from Figure 5. The respective calculations were made according to formula (3), in which T was substituted by T_2 which, in turn, was determined according expression (45).

From Figure 5 there can be noticed that, although at a small number m of fragments the θ value differ considerably for networks of different number N of stations, with the increase of m value these differences gradually decrease. Also, there are cases, for which the increase of m results in the increase of the mean time θ of messages delay – see, for example, the case for $N = 200$ from Figure 5. Thus, the increase of the number of network fragments doesn't necessarily result in the decrease of the mean delay time of messages transferred among stations.

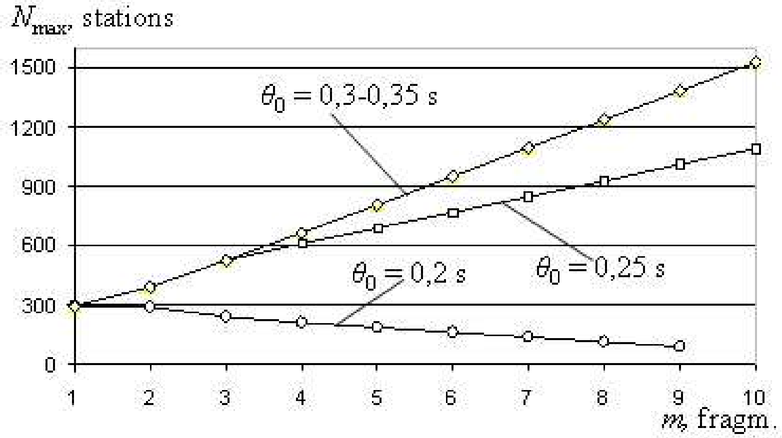

Figure 4. Dependence of N_{max} on θ_0 and m, configurations 1 and 2

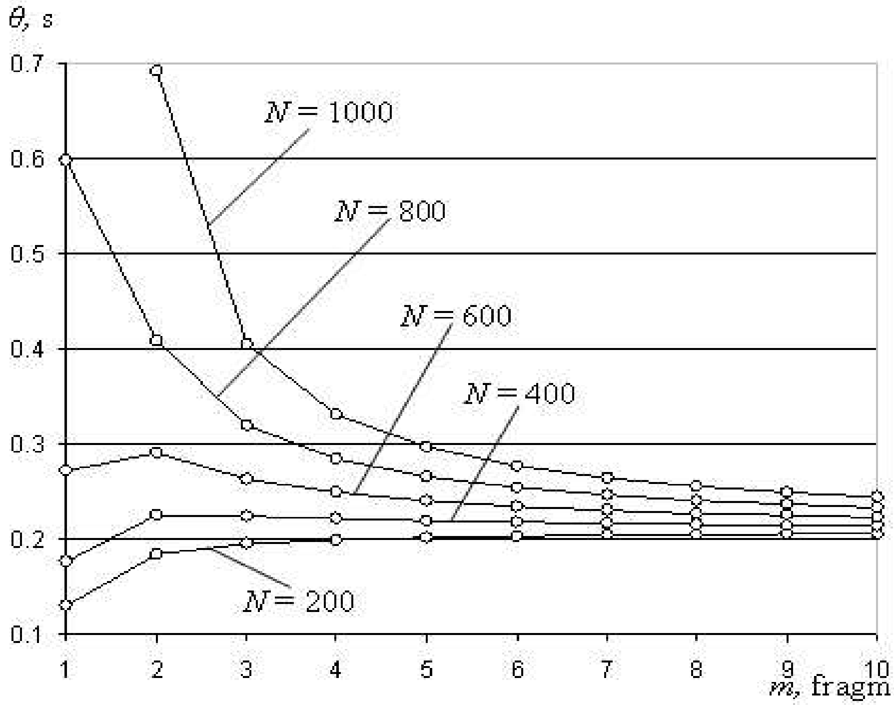

Figure 5. Dependence of θ on N and m at $\theta_0 = 0,25$s, configurations 1 and 2

Data similar to those from Figure 5, but for networks of configuration 3, are shown in Figure 6. The respective calculations are made according to formula (3), taking into account that, in this case, $T = T_3$ and T_3 is determined according to expression (88).

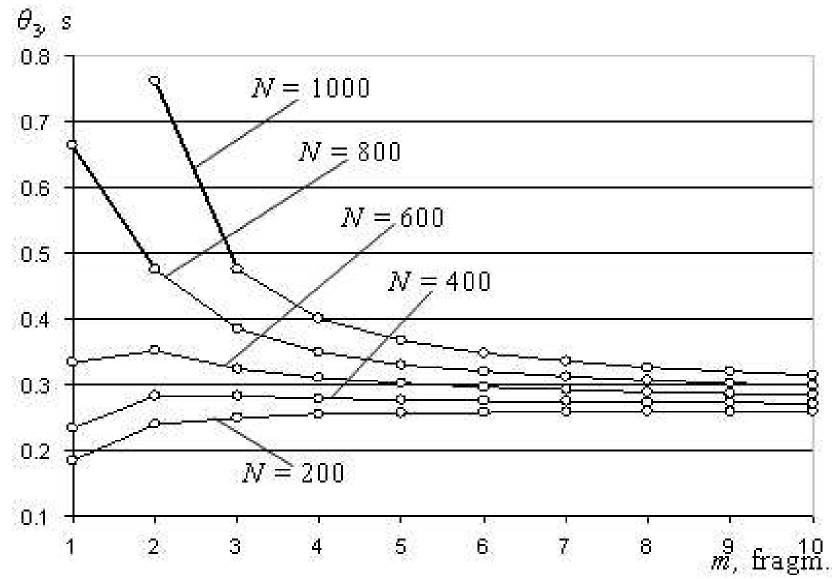

Figure 6. Dependence of θ_3 on N and m at $\theta_0 = 0,25$s, configurations 3

Graphics from Figure 6 show that these dependences have the same character with those for the respective networks of configuration 2 (see Fig. 5), only the value of the mean time θ_3 of messages delay is bigger. So, if the adding of switches results in the decrease of the load of some separate fragments, then the time of messages delay can be, at the same time, longer.

Of special interest is the comparison of configurations 1 and 2 in terms of the mean time θ of messages delay. For this purpose calculations were made for the relative increase $\delta\theta_{12}$ of the mean delay time θ_1

of messages in networks of configuration 1 comparatively to the mean delay time θ_2 of messages in networks of configuration 2:

$$\delta\theta_{12} = \frac{\theta_1 - \theta_2}{\theta_1} \cdot 100\%. \qquad (94)$$

The results obtained are shown in Figure 7. Data from Figure 7 show that the difference between the θ_1 and θ_2 values can be considerable. At a small number of stations in the network, the mean time of message transfer is smaller for networks of configuration 1 comparatively to those of configuration 2 (in Figure 7, at $N = 200$ stations and $N = 400$ stations), and at a relatively large number of stations in the network (in Figure 7, at $N = 600$ stations and $N = 800$ stations) vice versa: $\theta_1 > \theta_2$. Also, in some cases (in Figure 7, at $N = 200$ stations) the increase of the number of fragments results with the decrease of the parameter $\delta\theta_{12}$, and in other cases (in Figure 7, at $N = 400$ stations, $N = 600$ stations and $N = 800$ stations) on the contrary leads to the growth of parameter $\delta\theta_{12}$.

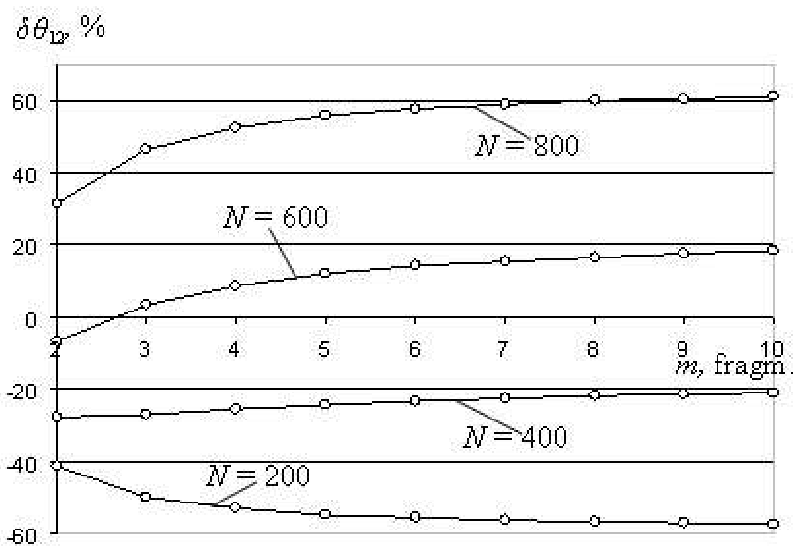

Figure 7. Dependence of $\delta\theta_{12}$ on N and m at $\theta_0 = 0,25$s

In a similar mode, it is of interest the comparison of configurations 2 and 3 by the mean time θ of message delay. For this purpose calculations were made for the relative difference $\delta\theta_{32}$ of the mean time θ_3 of message delay for configuration 3 comparatively to the mean time θ_2 of messages delay for configuration 2:

$$\delta\theta_{32} = \frac{\theta_3 - \theta_2}{\theta_3} \cdot 100\%. \tag{95}$$

The results obtained are shown in Figure 8. Data from Figure 8

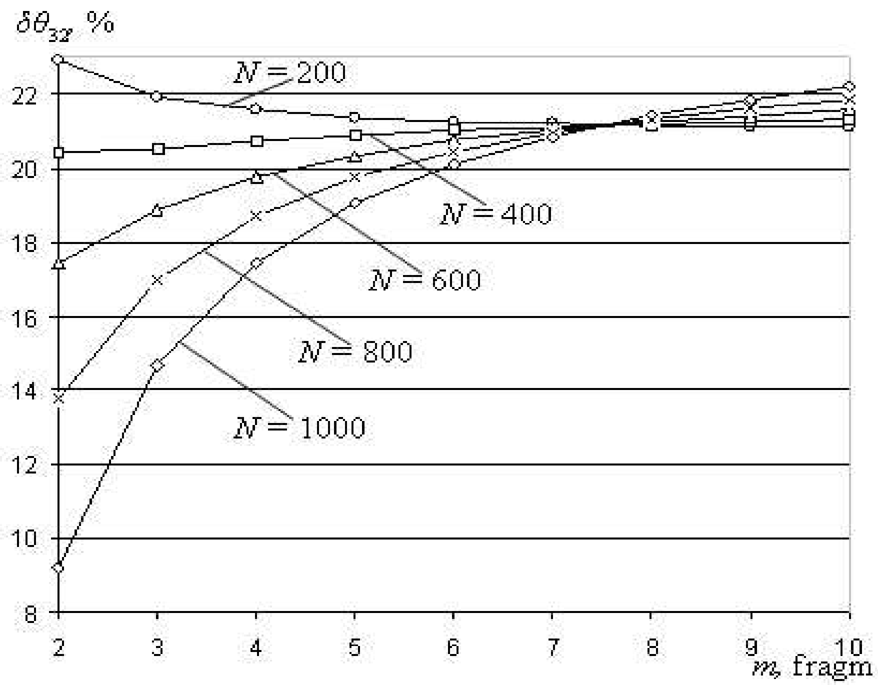

Figure 8. Dependence of $\delta\theta_{32}$ on N and m at $\theta_0 = 0,25$s

show that there exists an m value at which the value of parameter $\delta\theta_{32}$ doesn't depend of the number N of stations in the network (in Figure 8 this is $\approx 7 - 8$ fragments). The increase of the number of fragments results in the decrease of differences between the values of

θ_2 and θ_3 at a relatively small number of stations in the network (in Figure 8, at $N = 200$ stations) and in the increase of these differences at a relatively large number of stations in the network (in Figure 8, at $N \geq 400$ stations). At a certain number of stations in the network, the number m of fragments in the network doesn't influence or have little influence on the value of parameter $\delta\theta_{32}$ (in Figure 8, $N \approx 350$ stations).

The dependence of load ρ on N and m presents interest as well. Some results of the calculations referring to these dependences for the set of initial data described in subsection 6.1 are shown in Figure 9.

From Figure 9 there can be noticed, as expected, the growth of load ρ once with the increase of number N of stations and the decrease of this load once with the increase of the number m of collision domains in the network. The increase of m results also in the decrease of differences between load ρ for networks with a different number N of stations.

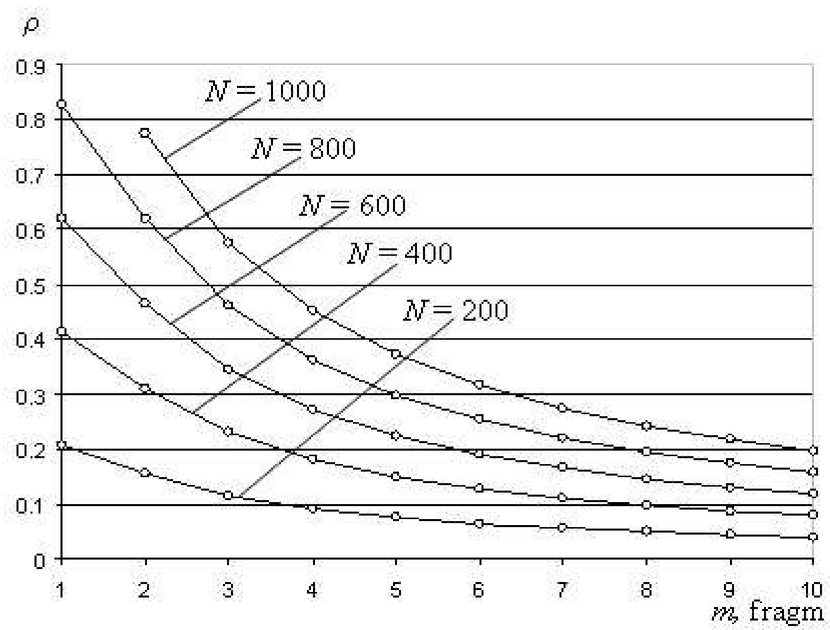

Figure 9. Dependence of ρ on N and m at $\theta_0 = 0,25$s

It is of interest the estimation of differences between load ρ_1 of the transmission media for networks of configuration 1 and load ρ_2 of the transmission media for networks of configuration 2 (for networks of configuration 3, excluding the case of the transmission media c between the two switches, the equality $\rho_3 = \rho_2$ takes place). In this purpose, there were made calculations for parameter $\delta\rho_{12}$, which is determined as

$$\delta\rho_{12} = \frac{\rho_1 - \rho_2}{\rho_1} \cdot 100\%. \qquad (96)$$

Some results of calculation of $\delta\rho_{12}$ are systemized in Table 2.

Table 2. The dependence of $\delta\rho_{12}$ on N and m, %

m, fragm.	N, stations			
	200	400	600	800
2	25,12	24,94	25,00	25,03
3	44,44	44,31	44,35	44,38
4	56,04	56,17	56,13	56,23
5	63,77	63,92	64,03	63,97
6	69,57	69,49	69,35	69,41
7	73,43	73,37	73,39	73,40
8	76,33	76,51	76,61	76,54
9	79,23	78,93	79,03	78,96
10	81,16	80,87	80,97	81,02

Data from Table 2 show that the value of $\delta\rho_{12}$ very little depends on the total number N of stations, but depends significantly on the number m of network fragments. In a graphic form, this dependence for the case of $N = 800$ stations is shown in Figure 10.

Calculations were made for Ethernet networks with data transfer speed of 10 Mbps, but the elaborated models and procedure can be applied to FastEthernet and Gigabit Ethernet networks, as well.

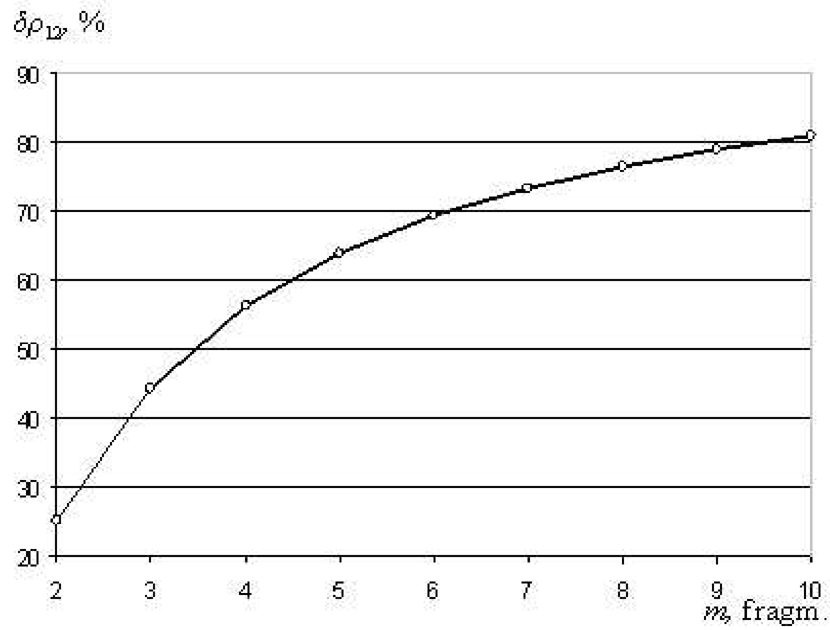

Figure 10. Dependence of $\sigma\rho_{12}$ on m at $N = 800$ stations and $\delta_0 = 0,25$s.

7 Conclusions

The analysis of performance features of different networks with frame memorization by switches confirms the necessity of case dependent special investigations on the configuration of concrete Ethernet networks. It was determined, for example, that:

1) the fragmentation doesn't necessarily permit the increase of the number of stations in the network, in may be even vice versa;

2) the increase of the number of network fragments doesn't necessarily results in the decrease of the mean time of messages transfer among stations;

3) if the adding of switches results in the load decrease for some fragments, then the mean time of messages transfer in the network

can be, at the same time, longer;

4) there exists a value of number m of fragments for which the relative difference $\delta\theta_{32}$ of the mean time θ_3 of messages delay in a network with two switches comparatively to the mean time θ_2 of messages delay in a network with one switch doesn't depend, practically, on the number N of stations in the network;

5) at certain number N of stations in the network, the number m of network fragments doesn't influence or little influences the value of parameter $\delta\theta_{32}$;

6) the relative difference $\delta\rho_{12}$ of load ρ_1 of the transmission media of a network with one collision domain from load ρ_2 of the transmission media of a network with one switch little depends on total number N of stations, but depends significantly on number m of network fragments.

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Ion Bolun, Victor Andronatiev
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Academy of Economic Studies of Moldova
59, Banulescu Bodoni str., off. 401
Chişinău, MD−2005, Moldova
Phone: (373+2) 22-27-65
E–mail: *bolun@ase.md*