THE ITERATIVE ALGORITHM OF TUNING CONTROLLERS TO THE MODEL OBJECT WITH INERTIA IDENTICAL ELEMENTS, TIME DELAY AND NONMINIMAL PHASE

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Abstract – *This paper proposes an iterative algorithm of tuning the typical controllers PI, PID to the model objects with inertia identical elements, time delay and non minimal phase. The proposed algorithm is using the maximal stability degree method for tuning controllers. As the result of this study the algorithm of tuning controllers and the procedure of determining the system's performance in dependence of maximal stability value is proposed.*
Keywords – *the iterative algorithm, the maximal stability degree method, tuning of controllers.*

1. INTRODUCTION

During the automation of many slow technological processes the mathematical objects' models of control process are represented as the models with inertia identical elements, time delay and non minimal phase [1,2,3,4].
The procedure of tuning controllers to the model object with inertia identical elements, time delay and non minimal phase becomes difficult [1,2,3,4]. This paper analyses the model object (fixed part) with inertia identical elements, time delay and non minimal phase with transfer function, which is presented in the follow form

$$H_{PF}(s) = \frac{ke^{-\tau s}}{(T_1 s - 1)(T_2 s + 1)^n}, \qquad (1)$$

where k is the transfer coefficient, T_1, T_2, τ – time constants and time delay, and n is the number of identical elements of the model.
It is proposed to tune the standard controllers PI and PID using the maximal stability degree (M.S.D) method [3,5,6] to the model object (1) with known parameters k, T_1, T_2, τ, n and to analyse the dynamic of control system for the case when it varyies the object's parameters from the nominal values keeping the tuning parameters of controllers PI and PID. The maximal stability degree method permits to alocate the roots of characteristic ecuation as far as imaginary axes and in the result the transition process of the control system become shorter (control system will process beter the reference signal and the diviation will be quickly throw which apearer as the result of perturbation actions).

2. THE ITERATIVE ALGORITHM OF TUNING CONTROLLERS

We assume that the control system is formed of an object with transfer function $H_{PF}(s)$, which is presented in relation (1), and transfer function of controller $H_R(s)$ with typical control laws PI, PID, Fig.1.

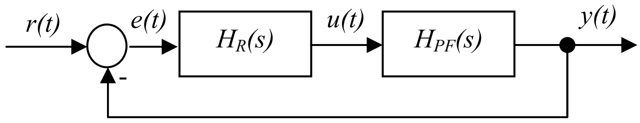

Fig.1 - Structure scheme of control system

It will tune the typical algorithms of tuning PI and PID for the model object with known parameters, using the M.S.D. method..
For the tuning of the PI controller using the M.S.D. method the algebraic expressions [5,6] were applied, which are the analytical expressions

$$k_p = \frac{e^{-\tau s}}{k}(1 - T_2 J)^{n-1}(d_0 J^3 - d_1 J^2 + d_2 J + d_3), \qquad (2)$$

where $d_0 = \tau T_1 T_2$, $d_1 = T_1 T_2(n+2) + \tau(T_1 - T_2)$,
$d_2 = 2T_1 - T_2(n+1) - \tau$, $d_3 = 1$,

$$k_i = -\frac{e^{-\tau s}}{k} J(T_1 J + 1)(1 - T_2 J)^n + k_p J. \qquad (3)$$

The optimal values of parameters k_p and k_i of PI controller were determined from expressions (2) and (3).
In the case of tuning parameters of PID controller using the M.S.D. method the algebraic expressions [5,6] were applied, which are the analytical expressions

$$k_d = \frac{e^{-\tau s}}{2k}(1 - T_2 J)^{n-2}(-d_0 J^4 + d_1 J^3 - d_2 J^2 + d_3 J - d_4), \qquad (4)$$

where $d_0=\tau^2 T_1 T_2^2$, $d_1=\tau^2(2T_1T_2 - T_2^2)+2\tau T_1 T_2^2 (n+2)$,
$d_2=\tau^2(T_1-2T_2)+\tau(2T_1T_2(n+4)-2(n+1)\ T_2^2)+$
$+T_1 T_2^2\ (n^2+3n+2)$,
$d_3=-\tau^2-\tau(4T_1-2T_2(n+1))+4T_1T_2(n+1)-\ T_2^2\ n(n+1)$,
$d_4=2T_1-\ 2(n+1)T_2-\ 2\tau$,

$$k_p = \frac{e^{-\tau s}}{k}(1 - T_2 J)^{n-1}(d_0 J^3 - d_1 J^2 + d_2 J + d_3) + 2\ k_d J, \qquad (5)$$

where $d_0 = \tau T_1 T_2$, $d_1 = T_1 T_2(n+2) + \tau(T_1 - T_2)$,
$d_2 = 2T_1 - T_2(n+1) - \tau$, $d_3 = 1$,

$$k_i = \frac{e^{-\tau s}}{k} J(-T_1 J - 1)(1 - T_2 J)^n - k_d J^2 + k_p J. \quad (6)$$

From expressions (4), (5) and (6) were determined the optimal values of parameters k_p, k_i and k_d of PID controller.

The tuning parameters of PI and PID controller - k_p , k_i and k_d are the function of known parametrs of control object and of the unknown value J stability degree of control system: $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ (see relations (2), (3), (4), (5) and (6)). Based on these relations in the case of known object's parameters and in the case of variation stability degree $J \geq 0$ in the strict limits, the respectivie calculations were made and the dependences $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for PI and PID controllers were obtained. A simulation was made to verify the performance of control system. If the performance doesn't satisfy the imposed performance, the iterative process will choose other sets of values of controllers parameters from curves $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$, the procedure will repeat until the performance of system will be satisfied.

3. APPLICATION AND COMPUTER SIMULATION

To show the efficiency of the proposed algorithm for tuning the typical controllers PI, PID to the model of object (1), which has the following parameters

$$k = 12,3,\ T_1 = 10\ s,\ \ T_2 = 0,357\ s,\ n = 3$$

the procedure described above for tuning typical controllers will be used.

For the relations $k_p=f(J)$, $k_i=f(J)$ of PI controller (see relations (2), (3)) with the known parameters values of object and at the variation of stability degree J in the respectively limits the iterative calculations were made and the obtained results are presented in the Fig. 2.

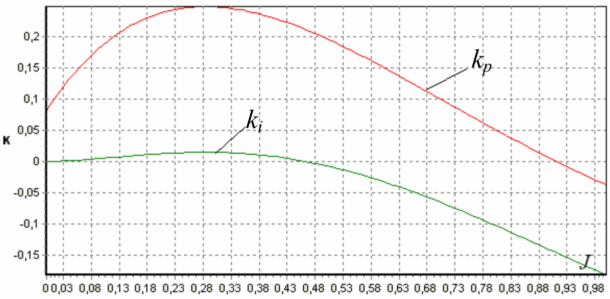

Fig. 2 - The dependence of PI controller parameters of the stability degree value

The iterative calculations were made for the relations $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ of PID controllers (see relations (4), (5) and (6)) with the known parameters values of object and at the variation of stability degree J in the respectively limits and the obtained results are presented in the Fig. 3.

The sets of values J - k_p, k_i for the PI controller (in Table 1) and J - k_p , k_i, k_d for the PID controller (in Table 3) were chosen to analyse the set of performance of control system with PI and PID controllers from Fig. 2 and 3.

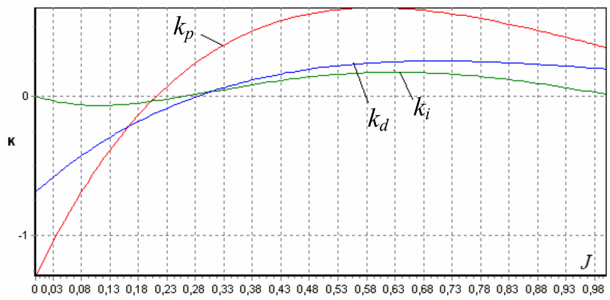

Fig. 3 - The dependence of PID controller parameters of the stability degree value

To verify the obtained results in case of tuning controllers PI, PID to the model object (1) using the M.S.D. method a computer simulation of the control system in MATLAB was made (see Fig. 4).

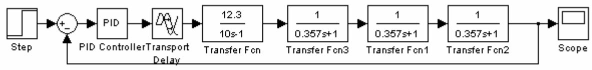

Fig. 4 - Simulation diagrams of the control system

The obtained results of the tuning of PI controller, using M.S.D. method are presented the transient processes of control systems with PI controller in the figure 5, where curve 1 – was obtained for the values presented in Table 1, row 1; curve 2 – was obtained for the values presented in Table 1, row 2; curve 3 – was obtained for the values presented in Table 1, row 3; curve 4 – was obtained for the values presented in Table 1, row 4.

For the case of tuning PI controller using Ziegler-Nichols method to the model object (1) it was obtained the negative results – the control system is not stabile for the tuning values presented in Table 1, row 5 (the parameters of critic regime are the oscillation time T_{cr}=6,62 s and k_{cr}=0,9155).

Table 1 - The Values of the PI Controller's Parameters

Item	J	k_p	k_i
1	0.2	0.236	0.012
2	0.25	0.246	0.014
3	**0.28**	**0.247**	**0.0144**
4	0.35	0.242	0.0125
5	ZN	0,412	0,1888

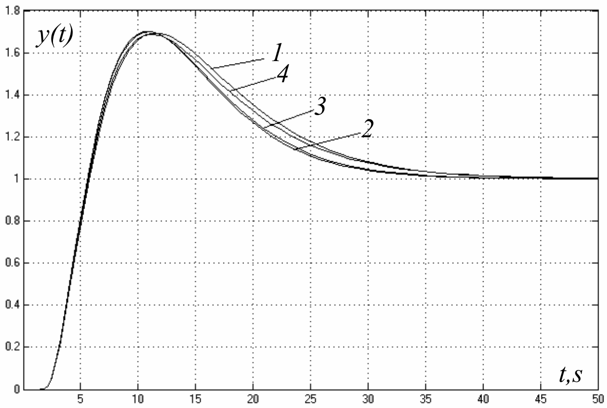

Fig. 5 - The transient processes of control systems with PI controller

The performance of the transient processes of control systems with PI controller presented in Table 2.

3

Table 2 - The Performance of Control Systems with PI Controller

The type of model	The tuning method	ε, %	t_r, s	σ, %	t_c, s	λ
1	GMS	5	31,95	70	4,57	1
2	GMS	5	28,57	70	4,41	1
3	**GMS**	**5**	**27,83**	**71**	**4,39**	**1**
4	GMS	5	31,67	69	4,47	1

Analysing the performance of the transient processes of control systems with controller PI Fig. 5 and Table 2 it was observed that the optimal performances were obtained for curves 3 with J_{opt}=0,28, σ=71% and t_r=27,83 s, but for the other transitional processes the tuning times are: for curve 1 – t_r=31,95 s, curve 2 – t_r=28,57 s.

The obtained results of the tuning PID controller, using M.S.D. method are presented the transient processes of control systems with PID controller in the figure 6, where curve 1 – was obtained for the values presented in Table 3, row 1; curve 2 – was obtained for the values presented in Table 3, row 2; curve 3 – was obtained for the values presented in Table 3, row 3; curve 4 – was obtained for the values presented in Table 3, row 4;. curve 5 – was obtained for the values presented in Table 3, row 5;. curve 6 – was obtained for the values presented in the Table 3, row 6;. curve 7 – was obtained for the values presented in Table 3, row 7;.

For the case of tuning PID controller using the Ziegler-Nichols method to the model object (1) it was obtained the results: the critic regime - the oscillation period T_{cr}=6,62 s and k_{cr}=0,9155 and the tuning values are presented in Table 3, row 7.

Table 3 - The Values of the PID Controller's Parameters

Item	J	k_p	k_i	k_d
1	0,9	0,461	0.0854	0,226
2	**0,93**	**0,432**	**0,0667**	**0,218**
3	0,94	0,422	0,0602	0,215
4	0,95	0,412	0,0537	0,212
5	0,96	0,402	0,047	0,21
6	1,0	0,36	0,0191	0,197
7	ZN	0,549	0,2518	0,662

The performance of the transient processes of control systems with PID controller (curve -1, 2, 3, 7) presented in Table 4.

Analysing the performance of the transient processes of control systems with PID controller from Fig. 6 and in Table 4 it was observed that the optimal performances were obtained for curves 2 with J_{opt}=0,93, t_r=11,24 s and σ=66 %.

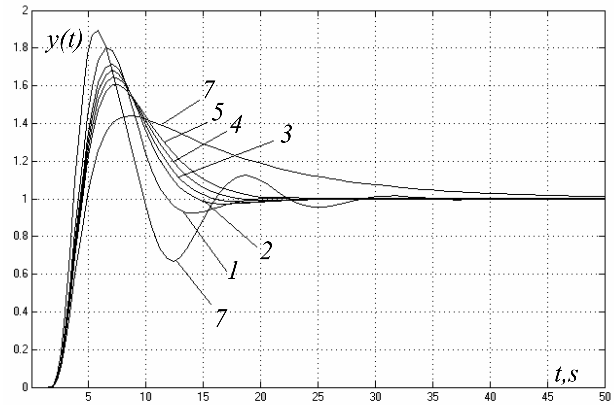

Fig. 6 - The transient processes of control systems with PID controller

Table 4 - The Performance of Control Systems with PID Controller

The type of model	The tuning method	ε, %	t_r, s	σ, %	t_c, s	λ
1	GMS	5	14,92	74	2,62	1
2	**GMS**	**5**	**11,24**	**66**	**2,73**	**1**
3	GMS	5	16,75	54	2,94	1
7	ZN	5	24	72	1,9	4

In the Fig. 7 an overlapping of the processes is presented for the following cases: curve 1 - transient processes in the case of the tuning PI controller for the optimal values, curve 2 - transient processes in the case of the tuning PID controller for the optimal values.

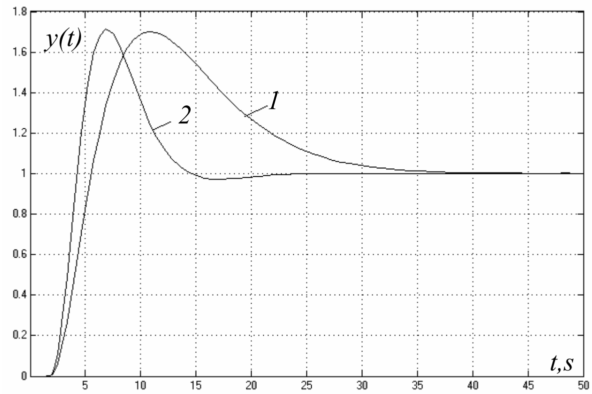

Fig. 7- The transient processes of control systems with PI, PID controller

In the Fig. 8 presents the domination poles: 1 – for the control system with PI controller tuning by M.S.D. method, the minimal pole has the value - 0,14; 2 - for the control system with PID controller tuning by M.S.D. method, the minimal pole has the value - 0,343; 3 - for the control system with PID controller tuning by Ziegler-Nichols method, the minimal pole has the value - 0,0583.Analysing the distribution of poles of characteristic equations of control system with PI, PID controllers tuning by M.S.D. and Ziegler-Nichols method, it can be observed that the relative stability of the control system with PI controllers tuning by M.S.D.

method has the reserve of stability much higher than the reserve of stability of the control system with PID controller tuning by M.S.D. method.

It follows that the robustness of control system with PID controller tuning by M.S.D. method in the case of the variation of parameters of the control object (1) is higher than the robustness of control system with PI controller tuning by M.S.D. method.

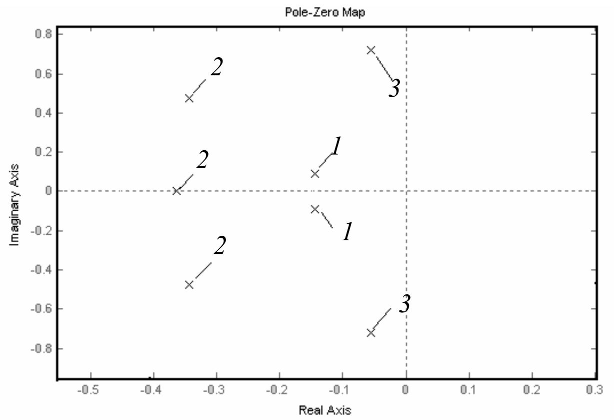

Fig. 8 - The distribution of the characteristic equation's poles

4. CONCLUSIONS

As a result of the study, the following conclusion can be made: The grafo-analitical method of tuning PI, PID controllers to the model objects with inertia identical elements, time delay and non minimal phase, which permitted to obtain the settled performance, is proposed.

For the control system with PI controller tuning by M.S.D. method, the transition process of system is oscillatory and optimal for the given values of object (see the curve 1, Fig. 7). The robustness of control system with PID controller tuning by M.S.D. method in the case of the variation of parameters of the control object (1) is higher than the robustness of control system with PI controller tuning by M.S.D. method.

For the control system with PID controller tuning by M.S.D. method the transition process of system is also oscillatory for the given values of the object (view the curve 2, Fig. 7), but with lesser relative stability than control system with PI controller.

REFERENCES

[1] R. K. Dorf, R. X. Bishop. Sovremennîe sistemî upravlenia (Modern Control Systems). Moskva: Laboratoria Bazovîh Znanii, 2004, 832 s.

[2] Ş. Preitl, R. E. Precup. Introducere în ingineria reglării automate, Timişoara: Editura Politehnica, 2004., 334 p.

[3] G. I. Zagarii, A. M. Shubladze. Sintez system upravlenia na osnove criteria maximalnoi stepeni ustoichivosti. (The Synthesis of the Control System According to the Maximal Stability Degree), Moskva: Energoatomizdat, 1998, 198 s.

[4] N. Tan, D. P. Atherton. Design of stabilizing PI and PID controllers.In: International Journal of Systems Science, Vol. 37 Issue 8, 2006, pp. 543-554.

[5] B. Izvoreanu, I. Fiodorov. The Synthesis of Linear Regulators for Aperiodic Objects with Time Delay According to the Maximal Stability Degree Method.//In Preprints the Fourth IFAC Conference on System Structure and Control, Bucureşti: Editura Tehnică, 1997, pp. 449 – 454.

[6] B. Izvoreanu, I. Fiodorov, F. Izvoreanu. The Tuning of Regulator for Advance Delay Objects According to the Maximal Stability Degree Method / In: Proceedings of the 11th International Conference on Control Systems and Computer Science (CSCS-11), Bucureşti, V.I. , 1997, pp. 179-184.