METHODOLOGY OF MATRIX REPRESENTATION AND ANALYSIS OF TENSOR OF ELASTICITY CONSTANTS OF EIGHT ORDER

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INTRODUCTION

With eight order tensors we meet in the study of behavior of anisotropic materials. The governing equation in case of reversible processes is written

$$t_{ij} = c_{ijnm} d_{nm} + c_{ijnmpq} d_{nm} d_{pq} + c_{ijnmpqkl} d_{nm} d_{pq} d_{kl} \tag{1}$$

The matrix presentation of forth c_{ijnm} and six c_{ijnmpq} order tensors were presented in papers [1,2].

Further we will refer to matrix representation of eight order tensor $c_{ijnmpqkl}$ and will analyze the number of independent components in function of symmetry of stress, strain tensor and those symmetry elements which results from thermodynamic principles and material symmetry.

1. THE MATRIX REPRESENTATION OF EIGHT ORDER TENSOR

Will pass from two indexes notations at one single index after Voight convention, starting from stress strain tensors symmetry [3] $11 \sim 1, 22 \sim 2, 33 \sim 3, 23 \sim 4,\ 13 \sim 5,\ 12 \sim 6$.

The eight order tensor will present like composed matrix, adopting this convention

$$c_{ijnmpqkl} = C_{KMFL} ,$$

were small letters have values 1,2,3, big letters 1,2,...,6. In base of thermodynamic principles can be proved that composed matrix c_{KMFL} is total symmetric

$$C_{KMFL} = C_{MKFL} = C_{KMLF} = C_{MKLF} = C_{LFKM} =$$
$$= c_{LFMK} = c_{FLKM} = c_{FLMK} = c_{FKLM} = \ldots$$

Thus, the elasticity constants of third order can be presented like composed matrix

$$C_{KMFL} \Rightarrow (C_{KM})_{FL} . \tag{2}$$

The composed matrix $(C_{KM})_{FL}$ has 1296 components. From symmetry

$$(C_{KM})_{FL} = (C_{MK})_{FL} . \tag{3}$$

the number of independent constants is reduced at 756, but from symmetry

$$(C_{KM})_{FL} = (C_{KM})_{LF} , \tag{4}$$

at 441.

From that mentioned that composed matrix is total symmetric one relation take place

$$(C_{KM})_{FL} = (C_{FM})_{KL} . \tag{5}$$

The number of independent elasticity constants is reduced under 125, which will represent under column matrix – column 125x1. In that case the matrix of elasticity constants as developed will be (6)

$$
C := \begin{bmatrix}
\begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\ \cdot & c_7 & c_8 & c_9 & c_{10} & c_{11} \\ \cdot & \cdot & c_{12} & c_{13} & c_{14} & c_{15} \\ \cdot & \cdot & \cdot & c_{16} & c_{17} & c_{18} \\ \cdot & \cdot & \cdot & \cdot & c_{19} & c_{20} \\ \cdot & \cdot & \cdot & \bullet & \cdot & c_{21} \end{pmatrix} &
\begin{pmatrix} c_2 & c_7 & c_8 & c_9 & c_{10} & c_{11} \\ \cdot & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ \cdot & \cdot & c_{27} & c_{28} & c_{29} & c_{30} \\ \cdot & \cdot & \cdot & c_{31} & c_{32} & c_{33} \\ \cdot & \cdot & \cdot & \cdot & c_{34} & c_{35} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{36} \end{pmatrix} &
\begin{pmatrix} c_3 & c_8 & c_{12} & c_{13} & c_{14} & c_{15} \\ \cdot & c_{23} & c_{27} & c_{28} & c_{29} & c_{30} \\ \cdot & \cdot & c_{37} & c_{38} & c_{39} & c_{40} \\ \cdot & \cdot & \cdot & c_{41} & c_{42} & c_{43} \\ \cdot & \cdot & \cdot & \cdot & c_{44} & c_{45} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{46} \end{pmatrix} &
\begin{pmatrix} c_4 & c_9 & c_{13} & c_{16} & c_{17} & c_{18} \\ \cdot & c_{24} & c_{28} & c_{31} & c_{32} & c_{33} \\ \cdot & \cdot & c_{38} & c_{41} & c_{42} & c_{43} \\ \cdot & \cdot & \cdot & c_{47} & c_{48} & c_{49} \\ \cdot & \cdot & \cdot & \cdot & c_{50} & c_{51} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{52} \end{pmatrix} &
\begin{pmatrix} c_5 & c_{10} & c_{14} & c_{17} & c_{19} & c_{20} \\ \cdot & c_{25} & c_{29} & c_{32} & c_{34} & c_{35} \\ \cdot & \cdot & c_{39} & c_{42} & c_{44} & c_{45} \\ \cdot & \cdot & \cdot & c_{48} & c_{50} & c_{51} \\ \cdot & \cdot & \cdot & \cdot & c_{52} & c_{53} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{54} \end{pmatrix} &
\begin{pmatrix} c_6 & c_{11} & c_{15} & c_{18} & c_{20} & c_{21} \\ \cdot & c_{26} & c_{30} & c_{33} & c_{35} & c_{36} \\ \cdot & \cdot & c_{40} & c_{43} & c_{45} & c_{46} \\ \cdot & \cdot & \cdot & c_{49} & c_{51} & c_{52} \\ \cdot & \cdot & \cdot & \cdot & c_{53} & c_{54} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{55} \end{pmatrix} \\
\cdot &
\begin{pmatrix} c_7 & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ \cdot & c_{56} & c_{57} & c_{58} & c_{59} & c_{60} \\ \cdot & \cdot & c_{61} & c_{62} & c_{63} & c_{64} \\ \cdot & \cdot & \cdot & c_{65} & c_{66} & c_{67} \\ \cdot & \cdot & \cdot & \cdot & c_{68} & c_{69} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{89} \end{pmatrix} &
\begin{pmatrix} c_8 & c_{23} & c_{27} & c_{28} & c_{29} & c_{30} \\ \cdot & c_{57} & c_{61} & c_{62} & c_{63} & c_{64} \\ \cdot & \cdot & c_{71} & c_{72} & c_{73} & c_{74} \\ \cdot & \cdot & \cdot & c_{75} & c_{76} & c_{77} \\ \cdot & \cdot & \cdot & \cdot & c_{78} & c_{79} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{80} \end{pmatrix} &
\begin{pmatrix} c_9 & c_{24} & c_{28} & c_{31} & c_{32} & c_{33} \\ \cdot & c_{58} & c_{62} & c_{65} & c_{66} & c_{67} \\ \cdot & \cdot & c_{72} & c_{75} & c_{76} & c_{77} \\ \cdot & \cdot & \cdot & c_{81} & c_{82} & c_{83} \\ \cdot & \cdot & \cdot & \cdot & c_{84} & c_{85} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{86} \end{pmatrix} &
\begin{pmatrix} c_{10} & c_{25} & c_{29} & c_{32} & c_{34} & c_{35} \\ \cdot & c_{59} & c_{63} & c_{66} & c_{68} & c_{69} \\ \cdot & \cdot & c_{73} & c_{76} & c_{78} & c_{79} \\ \cdot & \cdot & \cdot & c_{82} & c_{84} & c_{85} \\ \cdot & \cdot & \cdot & \cdot & c_{87} & c_{88} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{89} \end{pmatrix} &
\begin{pmatrix} c_{11} & c_{26} & c_{30} & c_{33} & c_{35} & c_{36} \\ \cdot & c_{60} & c_{64} & c_{67} & c_{69} & c_{70} \\ \cdot & \cdot & c_{74} & c_{77} & c_{79} & c_{80} \\ \cdot & \cdot & \cdot & c_{83} & c_{85} & c_{86} \\ \cdot & \cdot & \cdot & \cdot & c_{88} & c_{89} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{90} \end{pmatrix} \\
\bullet & \bullet &
\begin{pmatrix} c_{12} & c_{27} & c_{37} & c_{38} & c_{39} & c_{40} \\ \cdot & c_{61} & c_{71} & c_{72} & c_{73} & c_{74} \\ \cdot & \cdot & c_{91} & c_{92} & c_{93} & c_{94} \\ \cdot & \cdot & \cdot & c_{95} & c_{96} & c_{97} \\ \cdot & \cdot & \cdot & \cdot & c_{98} & c_{99} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{100} \end{pmatrix} &
\begin{pmatrix} c_{13} & c_{28} & c_{38} & c_{41} & c_{42} & c_{43} \\ \cdot & c_{62} & c_{72} & c_{75} & c_{76} & c_{77} \\ \cdot & \cdot & c_{92} & c_{95} & c_{96} & c_{97} \\ \cdot & \cdot & \cdot & c_{101} & c_{102} & c_{103} \\ \cdot & \cdot & \cdot & \cdot & c_{104} & c_{105} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{106} \end{pmatrix} &
\begin{pmatrix} c_{14} & c_{29} & c_{39} & c_{42} & c_{44} & c_{45} \\ \cdot & c_{63} & c_{73} & c_{76} & c_{78} & c_{79} \\ \cdot & \cdot & c_{93} & c_{96} & c_{98} & c_{99} \\ \cdot & \cdot & \cdot & c_{102} & c_{104} & c_{105} \\ \cdot & \cdot & \cdot & \cdot & c_{107} & c_{108} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{109} \end{pmatrix} &
\begin{pmatrix} c_{15} & c_{26} & c_{30} & c_{33} & c_{35} & c_{36} \\ \cdot & c_{60} & c_{64} & c_{67} & c_{69} & c_{70} \\ \cdot & \cdot & c_{94} & c_{97} & c_{99} & c_{100} \\ \cdot & \cdot & \cdot & c_{103} & c_{105} & c_{106} \\ \cdot & \cdot & \cdot & \cdot & c_{108} & c_{109} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{110} \end{pmatrix} \\
\bullet & \bullet & \bullet &
\begin{pmatrix} c_{16} & c_{31} & c_{41} & c_{47} & c_{48} & c_{49} \\ \cdot & c_{65} & c_{75} & c_{81} & c_{82} & c_{83} \\ \cdot & \cdot & c_{95} & c_{101} & c_{102} & c_{103} \\ \cdot & \cdot & \cdot & c_{111} & c_{112} & c_{113} \\ \cdot & \cdot & \cdot & \cdot & c_{114} & c_{115} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{116} \end{pmatrix} &
\begin{pmatrix} c_{17} & c_{32} & c_{42} & c_{48} & c_{50} & c_{51} \\ \cdot & c_{66} & c_{76} & c_{82} & c_{84} & c_{85} \\ \cdot & \cdot & c_{96} & c_{102} & c_{104} & c_{105} \\ \cdot & \cdot & \cdot & c_{112} & c_{114} & c_{115} \\ \cdot & \cdot & \cdot & \cdot & c_{117} & c_{118} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{119} \end{pmatrix} &
\begin{pmatrix} c_{18} & c_{33} & c_{43} & c_{49} & c_{51} & c_{52} \\ \cdot & c_{67} & c_{77} & c_{83} & c_{85} & c_{86} \\ \cdot & \cdot & c_{97} & c_{103} & c_{105} & c_{106} \\ \cdot & \cdot & \cdot & c_{113} & c_{115} & c_{116} \\ \cdot & \cdot & \cdot & \cdot & c_{118} & c_{119} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{120} \end{pmatrix} \\
\bullet & \bullet & \bullet & \bullet &
\begin{pmatrix} c_{19} & c_{34} & c_{44} & c_{50} & c_{52} & c_{53} \\ \cdot & c_{68} & c_{78} & c_{84} & c_{87} & c_{88} \\ \cdot & \cdot & c_{98} & c_{104} & c_{107} & c_{108} \\ \cdot & \cdot & \cdot & c_{114} & c_{117} & c_{118} \\ \cdot & \cdot & \cdot & \cdot & c_{121} & c_{122} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{123} \end{pmatrix} &
\begin{pmatrix} c_{20} & c_{35} & c_{45} & c_{51} & c_{53} & c_{54} \\ \cdot & c_{69} & c_{79} & c_{85} & c_{88} & c_{89} \\ \cdot & \cdot & c_{99} & c_{105} & c_{108} & c_{109} \\ \cdot & \cdot & \cdot & c_{115} & c_{118} & c_{119} \\ \cdot & \cdot & \cdot & \cdot & c_{122} & c_{123} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{124} \end{pmatrix} \\
\bullet & \bullet & \bullet & \bullet & \bullet &
\begin{pmatrix} c_{21} & c_{36} & c_{46} & c_{52} & c_{54} & c_{55} \\ \cdot & c_{70} & c_{80} & c_{86} & c_{89} & c_{90} \\ \cdot & \cdot & c_{100} & c_{106} & c_{109} & c_{110} \\ \cdot & \cdot & \cdot & c_{116} & c_{119} & c_{120} \\ \cdot & \cdot & \cdot & \cdot & c_{123} & c_{124} \\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{125} \end{pmatrix}
\end{bmatrix} \tag{6}
$$

2. THE RATIONAL FORM OF RELATION AMONG STRESS AND STRAIN

In base of matrix representation of tensor of 2,4,6,8 order the (1) expression can be presented

$$\boldsymbol{t_I} = \boldsymbol{A_{IN}} \boldsymbol{d_N} + (\boldsymbol{B_I})_{\boldsymbol{NM}} \boldsymbol{d_N} \boldsymbol{d_M} + + (\boldsymbol{C_{IK}})_{\boldsymbol{NM}} \boldsymbol{d_N} \boldsymbol{d_M} \boldsymbol{d_K}, \tag{7}$$

were

$$\boldsymbol{t}_1 = \boldsymbol{t}_{11}, \boldsymbol{t}_2 = \boldsymbol{t}_{22}, \boldsymbol{t}_3 = \boldsymbol{t}_{33},$$

$$\boldsymbol{t}_4 = \boldsymbol{t}_{32}, \boldsymbol{t}_5 = \boldsymbol{t}_{13}, \boldsymbol{t}_6 = \boldsymbol{t}_{12}; \tag{8}$$

$$\boldsymbol{d}_1 = \boldsymbol{d}_{11}, \boldsymbol{d}_2 = \boldsymbol{d}_{22}, \boldsymbol{d}_3 = \boldsymbol{d}_{33},$$

$$\boldsymbol{d}_4 = 2\boldsymbol{d}_{32}, \boldsymbol{d}_5 = 2\boldsymbol{d}_{13}, \boldsymbol{d}_6 = 2\boldsymbol{d}_{12}; \tag{9}$$

Matrixes $\boldsymbol{A_{IN}}$, $(\boldsymbol{B_I})_{\boldsymbol{NM}}$, $(\boldsymbol{C_{IK}})_{\boldsymbol{NM}}$ are expressed from:

- 21 of independent constants of elasticity $\boldsymbol{a}_1, \boldsymbol{a}_2, ..., \boldsymbol{a}_{21}$ of fourth order tensor 4;

- 56 of independent constants of elasticity $\boldsymbol{b}_1, \boldsymbol{b}_2, ..., \boldsymbol{b}_{56}$ of six order tensor and
- 125 of independent constants of elasticity $\boldsymbol{c}_1, \boldsymbol{c}_2, ..., \boldsymbol{c}_{125}$ of eight order tensor;

$$\boldsymbol{A} = \begin{pmatrix} a_1 & a_4 & a_6 & a_5 & a_3 & a_2 \\ . & a_{16} & a_{18} & a_{17} & a_{13} & a_9 \\ . & . & a_{21} & a_{20} & a_{15} & a_{11} \\ . & . & . & a_{19} & a_{14} & a_{10} \\ . & . & . & . & a_{12} & a_8 \\ . & . & . & . & . & a_7 \end{pmatrix}, \quad (10)$$

$$B := \begin{bmatrix} \begin{pmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \\ . & b_7 & b_8 & b_9 & b_{10} & b_{11} \\ . & . & b_{12} & b_{13} & b_{14} & b_{15} \\ . & . & . & b_{16} & b_{17} & b_{18} \\ . & . & . & . & b_{19} & b_{20} \\ . & . & . & . & . & b_{21} \end{pmatrix} \\ \begin{pmatrix} b_2 & b_7 & b_8 & b_9 & b_{10} & b_{11} \\ . & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ . & . & b_{27} & b_{28} & b_{29} & b_{30} \\ . & . & . & b_{31} & b_{32} & b_{33} \\ . & . & . & . & b_{34} & b_{35} \\ . & . & . & . & . & b_{36} \end{pmatrix} \\ \begin{pmatrix} b_3 & b_8 & b_{12} & b_{13} & b_{14} & b_{15} \\ . & b_{23} & b_{27} & b_{28} & b_{29} & b_{30} \\ . & . & b_{37} & b_{38} & b_{39} & b_{40} \\ . & . & . & b_{41} & b_{42} & b_{43} \\ . & . & . & . & b_{44} & b_{45} \\ . & . & . & . & . & b_{46} \end{pmatrix} \\ \begin{pmatrix} b_4 & b_9 & b_{13} & b_{16} & b_{17} & b_{18} \\ . & b_{24} & b_{28} & b_{31} & b_{32} & b_{33} \\ . & . & b_{38} & b_{41} & b_{42} & b_{43} \\ . & . & . & b_{47} & b_{48} & b_{49} \\ . & . & . & . & b_{50} & b_{51} \\ . & . & . & . & . & b_{52} \end{pmatrix} \\ \begin{pmatrix} b_5 & b_{10} & b_{14} & b_{17} & b_{19} & b_{20} \\ . & b_{25} & b_{29} & b_{32} & b_{34} & b_{35} \\ . & . & b_{39} & b_{42} & b_{44} & b_{45} \\ . & . & . & b_{48} & b_{50} & b_{51} \\ . & . & . & . & b_{53} & b_{54} \\ . & . & . & . & . & b_{55} \end{pmatrix} \\ \begin{pmatrix} b_6 & b_{11} & b_{15} & b_{18} & b_{20} & b_{21} \\ . & b_{26} & b_{30} & b_{33} & b_{35} & b_{36} \\ . & . & b_{40} & b_{43} & b_{45} & b_{46} \\ . & . & . & b_{49} & b_{51} & b_{52} \\ . & . & . & . & b_{54} & b_{55} \\ . & . & . & . & . & b_{56} \end{pmatrix} \end{bmatrix} \quad (11)$$

The number of constants will describe, if material has some symmetry elements. The law of components transformation of system of coordinate's rotation is written

$$\boldsymbol{C}'_{KMFL} = \boldsymbol{R}_{KI}\boldsymbol{R}_{MG}\boldsymbol{R}_{FT}\boldsymbol{R}_{LU}\boldsymbol{C}_{IGTU}, \qquad (12)$$

in which the R matrix expresses by rotation matrix [4]

$$R=\begin{pmatrix} r_{11}^2 & r_{12}^2 & r_{13}^2 & r_{12}r_{13} & r_{13}r_{11} & r_{11}r_{12} \\ r_{21}^2 & r_{22}^2 & r_{23}^2 & r_{22}r_{23} & r_{23}r_{21} & r_{21}r_{22} \\ r_{31}^2 & r_{32}^2 & r_{33}^2 & r_{32}r_{33} & r_{33}r_{31} & r_{31}r_{32} \\ 2r_{21}r_{31} & 2r_{22}r_{32} & 2r_{23}r_{33} & r_{22}r_{33}+r_{23}r_{32} & r_{23}r_{31}+r_{21}r_{33} & r_{21}r_{32}+r_{22}r_{31} \\ 2r_{31}r_{11} & 2r_{32}r_{12} & 2r_{33}r_{13} & r_{32}r_{13}+r_{33}r_{12} & r_{33}r_{11}+r_{31}r_{13} & r_{31}r_{12}+r_{32}r_{11} \\ 2r_{11}r_{21} & 2r_{12}r_{22} & 2r_{13}r_{23} & r_{12}r_{23}+r_{13}r_{22} & r_{13}r_{21}+r_{11}r_{23} & r_{11}r_{22}+r_{12}r_{21} \end{pmatrix}. \qquad (13)$$

The rotation matrix has the following components if material has the symmetry plan $\boldsymbol{x}_1\boldsymbol{x}_2$

$$\boldsymbol{r}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

For this case the rotation matrix will be

$$\boldsymbol{R}=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Will establish that the number of independent constants is reduced to 81 from the values of R matrix in (12) express.

If material has one more $\boldsymbol{X}_1\boldsymbol{X}_3$ symmetry plan, that rotation matrix will be

$$\boldsymbol{r}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

We obtain the components of R matrix in base of r matrix components and (13) express

$$\boldsymbol{R}=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}.$$

Taking into account those values in (12) relation, we will describe the number of independent constants of third order to 47.

If the eight order tensor is total symmetric, the following relations take place

$$c_{1123}=c_{1144}=c_{1255}=c_{1244}=c_{1233}=c_{1234}=c_{1235}=$$
$$=c_{1236}=c_{1366}=c_{1346}=c_{1344}=c_{1346}=c_{1456}=c_{1444}=$$
$$=c_{1445}=c_{1446}=c_{2255}=c_{2366}=c_{2355}=c_{2356}=c_{2456}=$$
$$=c_{2455}=c_{2445}=c_{2556}=c_{3366}=c_{3456}=c_{3566}=c_{3666}=$$
$$=c_{4366}=c_{4455}=c_{4456}=c_{4466}=c_{4556}=c_{4566}=c_{5566}$$

In that case the number of elasticity constants is reduced to 33, (14) relation.

If material has one symmetry axes of forth order the rotation matrix will be (15)

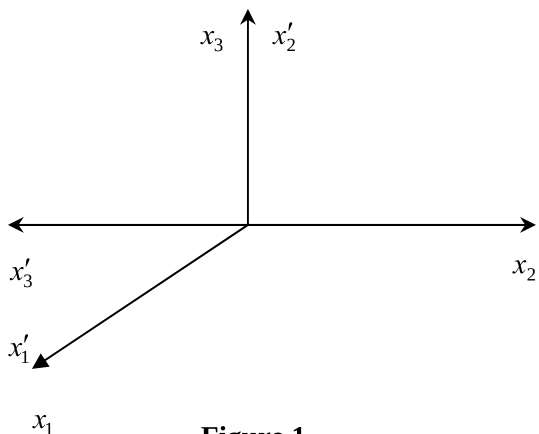

Figure 1.

$$
C:=\begin{bmatrix}
\begin{pmatrix} c_1 & c_2 & c_3 & 0 & 0 & 0\\ \cdot & c_4 & 0 & 0 & 0 & 0\\ \cdot & \cdot & c_5 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & c_7 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & c_8 \end{pmatrix} &
\begin{pmatrix} c_2 & c_4 & 0 & 0 & 0 & 0\\ \cdot & c_9 & c_{10} & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{11} \end{pmatrix} &
\begin{pmatrix} c_3 & 0 & c_5 & 0 & 0 & 0\\ \cdot & c_{10} & 0 & 0 & 0 & 0\\ \cdot & \cdot & c_{12} & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & c_{14} & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & c_6 & 0 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & c_{15}\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & c_7 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & c_{14} & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & c_8\\ \cdot & 0 & 0 & 0 & 0 & c_{11}\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} \\
\bullet &
\begin{pmatrix} c_4 & c_9 & c_{10} & 0 & 0 & 0\\ \cdot & c_{16} & c_{17} & 0 & 0 & 0\\ \cdot & \cdot & c_{18} & 0 & 0 & 0\\ \cdot & \cdot & \cdot & c_{20} & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{27} \end{pmatrix} &
\begin{pmatrix} 0 & c_{10} & 0 & 0 & 0 & 0\\ \cdot & c_{17} & c_{18} & 0 & 0 & 0\\ \cdot & \cdot & c_{23} & 0 & 0 & 0\\ \cdot & \cdot & \cdot & c_{24} & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{25} \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & 0 & 0 & c_{20} & 0 & 0\\ \cdot & \cdot & 0 & c_{24} & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & c_{26}\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & 0 & 0 & 0 & c_{21} & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & c_{11}\\ \cdot & 0 & 0 & 0 & 0 & c_{22}\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} \\
\bullet & \bullet &
\begin{pmatrix} c_5 & 0 & c_{12} & 0 & 0 & 0\\ \cdot & c_{18} & c_{23} & 0 & 0 & 0\\ \cdot & \cdot & c_{28} & 0 & 0 & 0\\ \cdot & \cdot & \cdot & c_{29} & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & c_{30} & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & c_{13} & 0 & 0\\ \cdot & 0 & 0 & c_{24} & 0 & 0\\ \cdot & \cdot & 0 & c_{29} & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & c_{14} & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & c_{30} & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} \\
\bullet & \bullet & \bullet &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & c_{20} & c_{24} & 0 & 0 & 0\\ \cdot & \cdot & c_{29} & 0 & 0 & 0\\ \cdot & \cdot & \cdot & c_{31} & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & c_{23} & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} \\
\bullet & \bullet & \bullet & \bullet &
\begin{pmatrix} c_7 & 0 & c_{14} & 0 & 0 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & c_{30} & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & c_{32} & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} &
\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ \cdot & 0 & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{pmatrix} \\
\bullet & \bullet & \bullet & \bullet & \bullet &
\begin{pmatrix} c_8 & c_{11} & 0 & 0 & 0 & 0\\ \cdot & c_{22} & 0 & 0 & 0 & 0\\ \cdot & \cdot & 0 & 0 & 0 & 0\\ \cdot & \cdot & \cdot & 0 & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & 0 & 0\\ \cdot & \cdot & \cdot & \cdot & \cdot & c_{33} \end{pmatrix}
\end{bmatrix}
\qquad (14)
$$

$$\boldsymbol{r} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \qquad (15)$$

respectively *R* matrix

$$\boldsymbol{R} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}. \qquad (16)$$

Taking into account the *R* matrix in (12) relation deduce that, the number of elasticity constants is reduced to 18. If material has two symmetry axes of forth order, the number of independent constants is reduced to 9.

3. CONCLUSIONS

The matrix representation of high order tensors considerably simplifies the analyze of nonlinear behavior of anisotropic materials. The forth order tensors are expressed from 81 of components, the six order tensors from 3^6=729, eight order tensor from 3^8 =6561 components.

The total number of components in calculus ramain unchanged, althought the number of independent components is reduced in base of symmetry relations.

The matrix reprezentation is simplified radically the posibilities of analyze of nonlinear behavior of anisotropic materials being accessible for engineers.

Was proved that eight order tensor can be presented like composed matrix 6x6, each element of which is represents the same a matrix 6x6. Was shown, that in case of lake of central interaction and material symmetry the number of independent constants is 125. For orthotropic materials the number of independent constants is reduced to 33. If material has two symmetry axes of forth order that number of independent constants is reduced to 9.

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