COMPUTATION OF THE ADDED MASSES OF SHIP'S HULL VERTICAL VIBRATIONS IN SHALLOW WATERS

M. Pricop, V. Chiţac, V. Oncica, N. M. Zus
Academia Navală "Mircea Cel Bătrân" Constanţa

INTRODUCTION

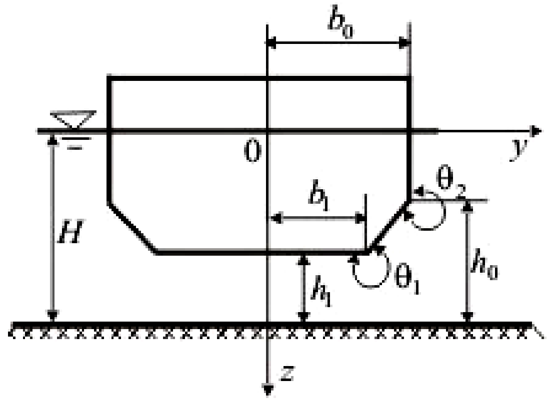
In the study of ship's hull vibratory motions, the presence of the fluid medium causes the increase of the ship's mass with the added mass value of the fluid which moved together with the vessel. This depends on the flow around the ship's hull, the ship's shape, the free surface, the vibration modes and the depth beneath keel. In present days the computation of ship's hull vibrations it's realized based on the added mass determined at infinite depth, with the Lewis Method [2] help which was for the first time applied in 1927.

This paper presents the computation of the added masses of ship's hull vertical vibrations in shallow waters, using the Schwarz-Christoffel transform [1] for determination of the cross section of the hull in the bilge area and the Newman method [3]. The results obtained show differences between this method and the Lewis one, and also the added mass influence on the natural frequencies of a bulk carrier seen as a continuous girder [4][5].

1. THE COMPUTATIONAL METHOD

The fluid in which the ship floats, at the transversal section ***i*** (pict. 1), can be divided in 3 domains [8]:

- the internal domain, D_1, extends in height between the bottom of the water and the bottom of the ship $(z \in [H - h_1, H])$, and in breadth between the ship's symmetrical plane and the longitudinal-vertical plane which contains the intersection point between the ship's bottom line and the bilge line $(y \in [0, b_1])$;
- the intermediary domain, D_2, between $y \in (b_1, b_0]$ and $z \in [H - h_0, H - h_1]$;
- the external domain, D_3, between $y \in (b_0, \infty)$ and $z \in [0, H - h_0]$.



Picture 1. Cross section geometrical parameters.

In the vertical vibrations with high amplitudes of the ship's hull, in which the speed is described by:

$$v = V \cos \omega t \qquad (1)$$

assuming that the flow is irotational, the fluid's potential is given by:

$$\Phi(x, y, z) = \varphi(y, z) \cos \omega t \qquad (2)$$

in which **ω** represents the angular frequency of the vertical vibrations, and V represents the speed's amplitude of the ship's vertical motions. In any point **(*y,z*)** of the studied fluid domain, the speed potential must concur with the next conditions:

1. $\nabla^2 \varphi = 0$;
2. $\dfrac{\partial \varphi}{\partial z} = 0$ for ***z=H***;
3. $\dfrac{\partial \varphi}{\partial z} - \upsilon \varphi = 0$, where $\upsilon = \dfrac{\omega^2}{g}$ at the free surface ***(z=0)***;
4. $\dfrac{\partial \varphi}{\partial n} = V_n$, on the perpendicular direction on ship's hull;
5. $\lim\limits_{y \to \infty} \mathrm{Re}\left(\dfrac{\partial \varphi}{\partial y} - i \upsilon \varphi \right) = 0$.

The speed potential in the D_1 domain, in accord with the first ***4*** conditions and for $h_1 \to 0$, will have the next expression:

$$\varphi_1(y) = -\frac{V}{2h_1} y^2 + A_0 \qquad (3)$$

in which can be noticed that in vertical direction the flow can be neglected.

To determine the speed potential in the intermediary domain must be established the form of the transversal section in the ship's hull in the bilge area by the θ_1 and θ_2 angles (picture 1):

$$\theta_1 = \pi + \frac{\pi}{n}\,;\ \theta_2 = \frac{3\pi}{2} - \frac{\pi}{n} \tag{4}$$

in which n is arbitrary, and also transform the points from the real Z plane in source points from ζ plane, using the Schwarz-Christoffel transform:

$$Z = b_1 - \frac{ih}{\pi\beta^{\frac{1}{2}-\frac{1}{n}}}\int_1^{\zeta}\frac{(\zeta-1)^{\frac{1}{n}}(\zeta-\beta)^{\frac{1}{2}-\frac{1}{n}}}{\zeta}d\zeta + i(h_1 - H) \tag{5}$$

In relation (5) β is the point in ζ plane which corresponds to the intersection point between the bilge and the side plating from the Z plane, and was determined with:

$$\pi\beta^{\frac{1}{2}-\frac{1}{n}}t_0 = h_1\int_1^{\beta}\frac{(\zeta-1)^{\frac{1}{n}}(\beta-\zeta)^{\frac{1}{2}-\frac{1}{n}}}{\zeta}d\zeta \tag{6}$$

in which t_0 means the length of the bilge (between $(b_1, H-h_1)$ and $(b_0, H-h_0)$):

$$t_0 = \sqrt{(b_0-b_1)^2 + (h_0-h_1)^2} \tag{7}$$

The complex potential in ζ plane is:

$$W(\zeta) = \frac{Q}{2\pi}\ln\zeta + C \tag{8}$$

in which Q represents the source intensity, and C is a constant who can be determined from the boundary conditions. For $\zeta \to 0$ relation (5) becomes:

$$Z = b_1 + \frac{h_1}{\pi}(\ln\zeta + K) \tag{9}$$

with the speed potential given by:

$$\varphi_2^i = \mathrm{Re}\left\{W(\zeta)\big|_{\zeta\to 0}\right\} = \frac{Q}{2h_1}\left(y - b_1 - \frac{h_1}{\pi}K\right) + C \tag{10}$$

and for $\zeta \to 0$ relation (5) becomes:

$$Z = -\frac{2ih_1\zeta^{\frac{1}{2}}}{\pi\beta^{\frac{1}{2}-\frac{1}{n}}} \tag{11}$$

with the speed potential given by:

$$\varphi_2^e = \mathrm{Re}\left\{W(\zeta)\big|_{\zeta\to\infty}\right\} = \frac{Q}{\pi}\ln\frac{\pi r\beta^{\frac{1}{2}-\frac{1}{n}}}{2h_1} + C \tag{12}$$

where $r = \sqrt{(y-b_0)^2 + (z-H)^2}$.

In relations (9) and (10) K represents the integration constant and has the expression:

$$K = \left(\frac{1}{2} - \frac{1}{n}\right)\cdot\frac{1}{\beta} + \frac{1}{n} \tag{13}$$

In D_3 domain, the vertical vibration of the ship doesn't directly influence the fluid flow, but indirectly, caused by the movement of fluid masses between D_2 and D_3 domains, in both ways, movement which can be described as a q source in (b_0, H)point. The speed potential in D_3 domain, obtained by Wehausen and Laitone, accordingly with the stated conditions, is simplified neglecting the free surface effect. In these conditions the speed potential in D_3 domain, in the vicinity of D_2 is:

$$\varphi_3^i = \frac{q}{\pi}\left[\ln\left(\frac{\pi r}{H}\right) - 2\ln 2\right] \tag{14}$$

Constants A_0, q, Q şi C are obtained from the continuity conditions between the domains:

$$\varphi_3^i = \varphi_2^e\,;\varphi_2^i = \varphi_1^e\,;\frac{\partial\varphi_2^1}{\partial y} = \frac{\partial\varphi_1^e}{\partial y} \tag{15}$$

The added mass given on a unit length, at the vertical vibration of the ship in the considered transversal section, is computed based on linear Bernoulli equation:

$$\overline{m_a} = \frac{2\rho}{V}\left[\int_0^{b_1}\varphi_1 dy + \int_{b_1}^{b_0}\frac{\varphi_2(b_0-b_1)}{t_2}dy\right] \tag{16}$$

from which:

$$\overline{m_a} = \overline{Cm_a}\cdot\frac{\rho\pi b^2}{2} \tag{17}$$

in which the added mass coefficient is:

$$\overline{Cm_a} = \frac{4b_1^2}{\pi b^2}\left[\frac{b_1}{3h_1} + \frac{K}{\pi} - \frac{2}{\pi}\left(1 + \frac{t_0}{b_1}\right)\ln\frac{h_1}{2(T+h_1)\beta^{\frac{1}{2}-\frac{1}{n}}}\right] \tag{18}$$

The terms b_{1i} and t_{0i}, for $i = \overline{1,20}$, has been determined from the ship's lines drawing.

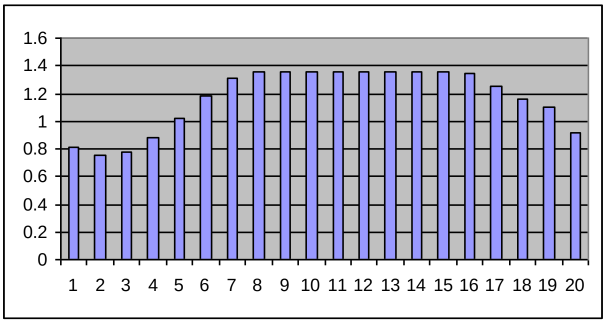
2. NUMERICAL RESULTS AND CONCLUSIONS

For the numerical computations has been utilized a bulk carrier (length L_{CWL}=296.33m, breadth B=46m, draft d=18m) divided in 20 segments of different lengths, with constant geometrical and mechanical characteristics [4].

The added masses have been determined, neglecting the 3D effect, with the classical Lewis method for infinite water depth and with the method presented above.

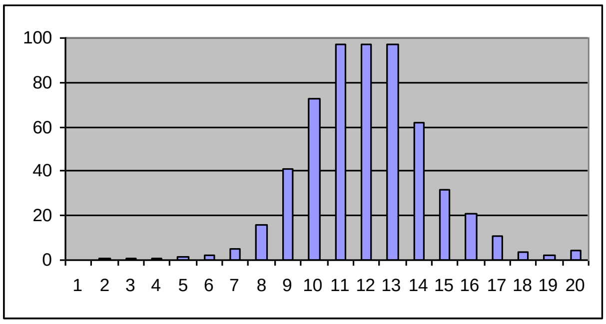
At the determination of expression (18), with $\frac{h_1}{b_{1i}} \leq \mathbf{0.3...0.4}$, in order to respect the imposed conditions, for the ***U*** shape transversal sections has been chosen **5** significant depths beneath the keel: **0.09*m***, **0.2*m***, **0.5*m***, **0.9*m***, **1.8*m***.

Using the Lewis method, has been realized the ***MASADL*** program [5], the obtained results being showed in the picture 2.

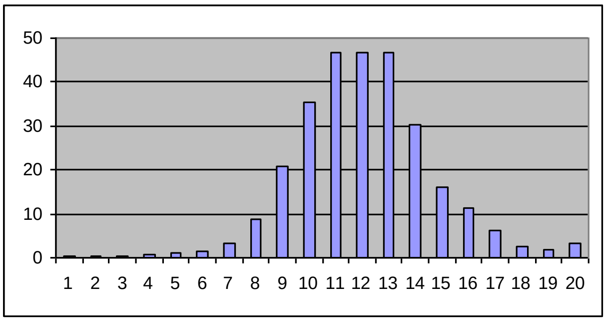


Picture 2. $\overline{Cm_a}$ Lewis.

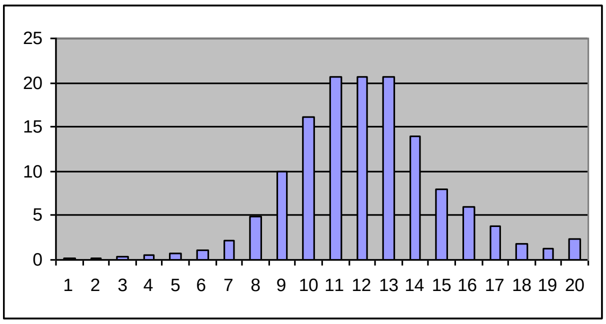
For the method presented in the second part of the article, has been realized the ***MASADN*** program [5]. Pictures 3÷7 represent the variations of the added mass coefficients in the 20 segments of the ship, accordingly with the different depths beneath the keel.



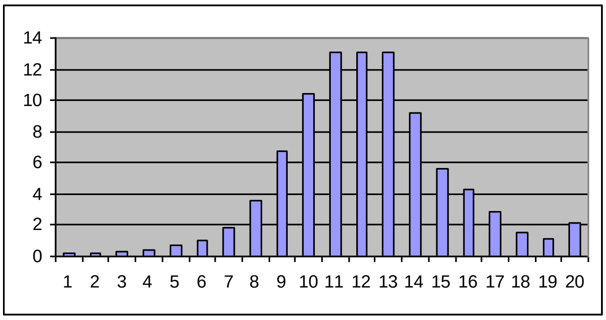
Picture 3. $\overline{Cm_a}$ Newman $h_1 / d = 0.005$.



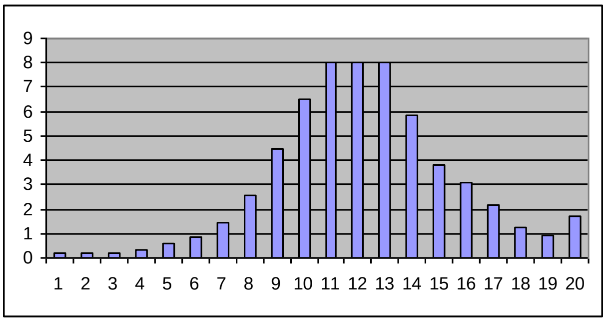
Picture 4. $\overline{Cm_a}$ Newman $h_1 / d = 0.011$.



Picture 5. $\overline{Cm_a}$ Newman $h_1 / d = 0.028$.



Picture 6. $\overline{Cm_a}$ Newman $h_1 / d = 0.05$.



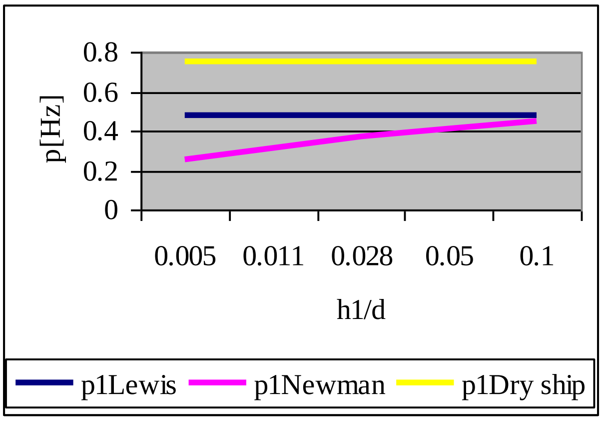
Picture 7. $\overline{Cm_a}$ Newman $h_1 / d = 0.1$.

Drawing a parallel between the above results, it can be easily observed that the $\overline{Cm_a}$ values are far bigger in shallow waters (especially amidships), and also a modification in the $\overline{Cm_a}$ longitudinal distribution across the ship's length.

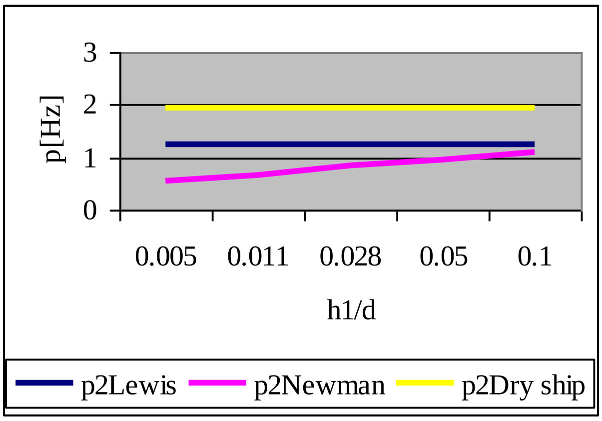
The natural frequencies of the continuous girder which represents the 170000tdw bulk carrier, has been computed using a transmission dynamic matrices method [6][7] for the vertical vibrations, in the next hypothesis:

- the ship's hull is considered as a continuous girder of variable transversal section, free at the extremities, leaned on an elastic medium;
- each segment of the girder is modeled using the Euler girder theory, without being taken in consideration the rotational inertia and the shearing deformations;

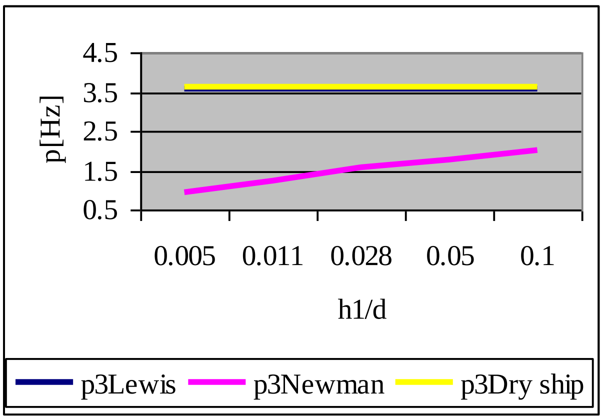
- the vertical vibration is considered irrespective of the other general vibration types of the ship's hull.



Picture 8. The natural frequencies in the first vibration way.



Picture 9. The natural frequencies in the second vibration way.



Picture 10. The natural frequencies in the third vibration way.

In the pictures 8, 9, 10 are represented by comparison the first three natural frequencies of the ship's hull, with and without the effect of added masses computed with the two methods presented in the second part of the article.

From the representations can be easily seen that in the first vibration way (pict. 8), computed for shallow waters, the frequency falls by maximum 50% from the frequency in infinite waters, also falls by approximately 70% from the frequency of the ship's hull without the effect of the added mass. These decreases in frequency become more prominent with the increase of the vibration mode. In this way, in the third vibration way, the frequency computed for shallow waters decreases by 75% from the frequency in infinite waters, also by 85% from the frequency of the ship's hull without the effect of the added mass.

References

1. Korn, G.A., Korn, T.M., *Mathematical Handbook for Scientists and Engineers.*
2. Lewis, F.M., *Vibration and Engine Balance in Diesel Ships, Trans. SNAME 35, 1927.*
3. Newman, J.N., Scortland, B., Vinje, T. *Added mass and Damping of Rectangular Bodies Close to the Free surface, J.S.R. 28, 1984.*
4. Oncică, V., Pricop, M., Zus, N.M. *Influenţa formelor navei şi a adâncimii navei sub chilă asupra vibraţiilor verticale ale grinzii navă de secţiune constantă, Lucrările celei de-a XIX-a Sesiuni de comunicări ştiinţifice cu participare internaţională NAV MAR. EDU 2005, Secţiunea Inginerie Mecanică, Academia Navală "Mircea cel Bătrân", Constanţa, 2005.*
5. Oncică, V. *Contribuţii la studiul dinamicii structurilor navale sub acţiunea instalaţiilor de propulsie, Teză de doctorat, Bucureşti, 2005.*
6. Posea, N. *Calculul dinamic al structurilor, Editura Tehnică, Bucureşti, 1991.*
7. Pricop, M. *Cercetări privind statica şi dinamica structurii corpurilor navelor vrachiere, Teză de Doctorat, 1999.*
8. Zong, Z., Lam, K.Y. *Hydrodynamic Influence on Ship-hull Vibration Close to Water Bottom, Journal of Engineering Mathematics 37, 2000.*

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