ASPECTS OF USING THE SENSITIVITY THEORY IN ENERGETICAL OPTIMIZATION PROBLEMS

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INTRODUCTION

For different technical decisions taken practically it is useful and necessary to know manner how different parameters influence decision. In the optimization problems, it is interesting the sensitivity of the objective function of the process control parameters. This is given by the gradient of the objective function reported to the control parameters. For the optimization problems with constrain, this is given by the gradient reduced.

Pseudo-optimization is the optimization of the objective function based on the sensitivity theory modifying only some control parameters.

For the optimization problems with constrain, it is interesting sensibility of constrains touched reported to the control parameters, too. This is necessary because one control variable must be transformed into one dependent variable when it is touched one constrain. The dependent variables values result from solving equation system given by the equality constrains.

1. ANALITICAL RELATIONS OF THE SENSITIVITY

1.1. Sensitivity of the objective function

In the damages situations, when process parameters are out of the admissible limits, must be intervene rapidly for secure process stability. It must be chose that control variable which, modifying, has a maximum efficiency, so straightening rapidly. Knowledge this control variable can be realized on base sensitivity theory.

The expert systems use massive the sensitivity theory.

Using this theory is also due to low precision of input date from process. In this case, a high precision of calculus is useless.

It is consider a general form of one optimization problem:

$$\begin{aligned} &F(z_1, z_2, .., z_{TOT}) = min\,!, \quad TOT = I + D \\ &f_{ai}(z_1, z_2, .., z_{TOT}) = 0, \quad i = 1,2,..,D \\ &f_{li}(z_1, z_2, .., z_{TOT}) \le 0, \quad i = 1,2,..,L \end{aligned} \tag{1}$$

where $z_1,z_2,...,z_{TOT}$ are problem variables, f_{ai}, $i=1,2,..,D$ are equality- type constrains (active constrains) that involve a number D of dependent variable, and f_{li}, $i=1,2,..,L$ are inequality- type constrains (free constrains). I is number of independent variables (control variables).

If variables $z_1,z_2,...,z_{TOT}$ are divide in independent control variables $x_1,x_2,..,x_I$ and dependent variables $y_1,y_2,..,y_D$, then sensitivity of the objective function reported to the control variables is given by the gradient reduced of the objective function [2]:

$$\begin{aligned} [S_I] &= \left[\frac{\partial F_r}{\partial x}\right] = \\ &= \left[\frac{\partial F}{\partial x}\right] - \left[\frac{\partial f_a}{\partial x}\right]_T \cdot \left[\frac{\partial f_a}{\partial y}\right]_T^{-1} \cdot \left[\frac{\partial F}{\partial y}\right] \end{aligned} \tag{2}$$

where F_r is the objective function reduced to the independent variables.

The sensitivities can be also calculated directly by numerical derivate

$$\begin{aligned} &\frac{\partial F_r}{\partial x_i} = \\ &= \frac{F(x_1, x_2, .., x_i + \Delta x_i, .., x_I, y_1, y_2, .., y_D)}{\Delta x_i} - \\ &- \frac{F(x_1, x_2, .., x_i, .., x_I, y_1^0, y_2^0, .., y_D^0)}{\Delta x_i} \\ &i = 1,2,..,I \end{aligned} \tag{3}$$

where variables y^0_1 , y^0_2 ,..., y^0_D are determined by solving equations system given by equality- type constrains with independent variables unmodified; y_1, y_2 ,..., y_D are determined by solving equations system given by equality - type constrains with variable x_i modified.

In the optimization problems not included constrains, the second term dissolves from (2) and sensitivity become equal with simple gradient of the objective function

It must be precise that sensitivity is a punctual characteristic, valid for an operation point of the process. Its extended for the other operation areas must be analyzed

1.2. Sensitivity of the inequality-type constrains

Sensitivity of the inequality- type constrains reported to the independent variable (control variables) it is determine deriving constrains equality-type and inequality-type. It must be taking account that dependent variables [Y] depend to independent variables [X] through constrains system:

$$\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{x}}\right]+\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{y}}\right]\cdot\left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}\right]=0 \quad \Rightarrow$$
$$\left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}\right]=-\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{y}}\right]^{-1}\cdot\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{x}}\right] \tag{4}$$

$$\left[\frac{\partial \boldsymbol{f_{lr}}}{\partial \boldsymbol{x}}\right]=\left[\frac{\partial \boldsymbol{f_l}}{\partial \boldsymbol{x}}\right]+\left[\frac{\partial \boldsymbol{f_l}}{\partial \boldsymbol{y}}\right]\cdot\left[\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}}\right]=$$
$$=\left[\frac{\partial \boldsymbol{f_l}}{\partial \boldsymbol{x}}\right]-\left[\frac{\partial \boldsymbol{f_l}}{\partial \boldsymbol{y}}\right]\cdot\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{y}}\right]^{-1}\cdot\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{x}}\right] \tag{5}$$

where through notation $\left[\partial f_{lr}/\partial x\right]$ it is taking account that derivative it is calculated in the present equality- type constrains. Relation (5) gives calculus manner of the sensitivity of inequality-type constrains reported to the independent variables.

Practically, when in an iteration it goes to the optimal solution on the direction *[D]* with a step α , it is interesting sensitivity of the constrains reported to this step. Noting *[X⁰]* current value of the independent variables, then new value is:

$$[\boldsymbol{X}]=\left[\boldsymbol{X}^0\right]+\alpha\cdot[\boldsymbol{D}] \tag{6}$$

and sensitivity of the constrains reported to step α results from (6) and (5):

$$\left[\frac{\partial \boldsymbol{f_{lr}}}{\partial \alpha}\right]=\left[\frac{\partial \boldsymbol{f_{lr}}}{\partial \boldsymbol{x}}\right]\cdot\left[\frac{\partial \boldsymbol{x}}{\partial \alpha}\right]=$$
$$=\left[\frac{\partial \boldsymbol{f_l}}{\partial \boldsymbol{x}}\right]\cdot[\boldsymbol{D}]-\left[\frac{\partial \boldsymbol{f_l}}{\partial \boldsymbol{y}}\right]\cdot\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{y}}\right]^{-1}\cdot\left[\frac{\partial \boldsymbol{f_a}}{\partial \boldsymbol{x}}\right]\cdot[\boldsymbol{D}] \tag{7}$$

Based on the relation (7) it can decide introducing an independent variable in the base. It is choose that variable for which constrain attained has maxim sensitivity.

2. USING SENSITIVITY THEORY IN ENERGETIC

In this paragraph we present three possibilities for using sensitivity in energetic with example for the optimization problem voltage – reactive, [1]. For the numerical results we use o network 220/110 kV with 48 nodes in areas Bacau and Neamt.

2.1. Pseudo-optimization

Practically, optimization is usefully if it is taken with little maneuvers in the installations. Otherwise, optimization is difficult, even that through objective function it can be quantify risks caused by maneuvers. For this reason, recognizing contribution every control action, it can be give up to these lessees efficiently.

It is consider an optimization problem voltage – reactive, [1]:

$$\begin{aligned}
&\boldsymbol{F}([U],[\delta],[W])=\\
&=\Delta \boldsymbol{P}([U],[\delta],[W])+\sum_{\boldsymbol{i}\in \boldsymbol{q}} \boldsymbol{CPT_i}(\boldsymbol{Q_{p_i}})+\\
&+\frac{\boldsymbol{l}}{\boldsymbol{T}}\sum_{\boldsymbol{i},\boldsymbol{j}\in \boldsymbol{r}} \boldsymbol{C_{PLOT_{ij}}}(\Delta \boldsymbol{W_{ij}})=\boldsymbol{min}\\
&\boldsymbol{P_i}([U],[\delta],[W])-\boldsymbol{P_i^{imp}}=0,\quad \boldsymbol{i}\in \boldsymbol{n}\setminus \boldsymbol{e}\\
&\boldsymbol{Q_i}([U],[\delta],[W])-\boldsymbol{Q_i^{imp}}=0,\quad \boldsymbol{i}\in \boldsymbol{c}\\
&\boldsymbol{U_i^{min}}\le \boldsymbol{U_i}\le \boldsymbol{U_i^{max}}\quad ,\boldsymbol{i}\in \boldsymbol{n}\\
&\boldsymbol{Q_i^{min}}\le \boldsymbol{Q_i}([U],[\delta],[W])\le \boldsymbol{Q_i^{max}}\quad ,\boldsymbol{i}\in \boldsymbol{q}\\
&\boldsymbol{W_{ij}^{min}}\le \boldsymbol{W_{ij}}\le \boldsymbol{W_{ij}^{max}}\quad ,\boldsymbol{i},\boldsymbol{j}\in \boldsymbol{r}\\
&\boldsymbol{I_{ij}}([U],[\delta],[W])\le \boldsymbol{I_{ij}^{max}}([U],[\delta],[W]),\quad \boldsymbol{i},\boldsymbol{j}\in \boldsymbol{n}
\end{aligned} \tag{8}$$

where $F([U],[\delta],[W])$ is the objective function, [MW]; $[U]$ –vector of the voltages between phases in the network nodes, [kV]; $[\delta]$ – vector of the voltages phase differences in the network nodes reported to the reference node (equilibration node), [rad]; $[W]$ – vector of the plots of the transformers with long-transversal control, t – its crowd; $CPT_i(Q_{pi})$ – expenses, [MW], realized in the network node i for producing reactive power Q_{pi} , [MVAR]; $C_{PLOTij}(\Delta W_{ij})$ – expense for modification plot of the transformer connected between node i and node j with amount ΔW_{ij}, , [MWh]; P_i^{imp}, Q_i^{imp} – active power, reactive power, imposed in node i by the conditions from this node; the value is given by the algebraically sum between power produced and power consumed for the nodes were there are production groups connected, [MW], [MVAR]; $P_i([U],[\delta],[W])$, $Q_i([U],[\delta],[W])$ – active power, reactive power in node i determine function by the regime date of the network, [MW], [MVAR]; $I_{ij}([U],[\delta],[W])$ – current on the side ij , [A]; n – network nodes crowd; e – equilibration ode; c – consumer nodes crowd; q – control voltage nodes crowd; t – long-transversal control transformers crowd.

Sensitivity of the objective function reported to the independent variables is given by the value of the gradient reduced in the initial point. Its components, for a network 220/110 kV with 48 nodes in zone STD Bacau, are presented in table 1.

Table 1. Sensitivity of the objective function reported to the control variables. Network with 48 nodes

Variable	$U_{STEJARU}$	$W_{GUTINAS}$	$W_{DUMBRAVA}$	$W_{STEJARU}$	W_{BC_SUD}
Gradient reduced	-0.05659 [MW/kV]	0.00522 [MW/plot]	0.04628 [MW/plot]	0.03977 [MW/plot]	-0.06358 [MW/plot]

From table 1 it is found that objective function has maxim sensitivity reported to the plot of autotransformer in station Bacău Sud (the objective function decreases when plot it is increased).

Table 2 shows results for successive optimization reported to the problem variables, in the succession indicated by the sensitivities and in the succession modified.

In variant 1 variable succession is chosen in the sensitivity order from table 1 and in variant 2 the order is modified. It is find that by pseudo-optimization final result differ with 0.91% from value obtained by simultaneous optimization of 5 variables (3.059 MW). That notes a decrease of objective function less 11 %. Changing of optimization succession leads to a bigger value of the objective function (deviation 2.06 %). That notes a decrease of objective function less 24 %.

Table 2. Successive optimization reported to the one variable. Network with 48 nodes

Variant	**Stage**	**Variable modified**	**Value**		**Objective function**	
			Initial	**Final**	**[MW]**	**Contribution [%]**
1	**I**	W_{BACAU_SUD}	9	12	3.2001	51.7
	II	$Q_{STEJARU}$	7.8	20	3.1487	21.9
	III	$W_{DUMBRAVA}$	12	12	3.1487	0
	IV	$W_{STEJARU}$	11	11	3.1487	0
	V	$W_{GUTINAS}$	12	14	3.0869	26.4
2	**I**	$Q_{STEJARU}$	7.8	20	3.2797	20.8
	II	$W_{DUMBRAVA}$	12	11	3.2775	1.1
	III	W_{BACAU_SUD}	9	12	3.1621	57.9
	IV	$W_{STEJARU}$	11	10	3.1553	3.4
	V	$W_{GUTINAS}$	12	3	3.1220	16.7

2.2. Sensitivity of constrains reported to control variables

In problem (8) it is used a sensitivity matrix, [S]:

$$[S] = \begin{bmatrix} \left[\partial \boldsymbol{Q_q}/\partial \boldsymbol{U_q}\right] & \left[\partial \boldsymbol{Q_q}/\partial \boldsymbol{W}\right] \\ \left[\partial \boldsymbol{U_c}/\partial \boldsymbol{U_q}\right] & \left[\partial \boldsymbol{U_c}/\partial \boldsymbol{W}\right] \\ \left[\partial \boldsymbol{I}/\partial \boldsymbol{U_q}\right] & \left[\partial \boldsymbol{I}/\partial \boldsymbol{W}\right] \end{bmatrix} \qquad (9)$$

It was calculated using numerical methods, with relation (3). Some values of sensitivities are given in table 3.

Sensitivity values show how constrain it is modified when it is altered control variable corresponding. For network analyzed it is find zone influence of control voltage. For example node Stejaru 110 KV modifies voltages in area Piatra-Neamţ - Roman. In this problem, activate constrain – reactive power in node STEJARU 110 kV, leads to introducing this node voltage in base. When this voltage it is blocked to the maxim limit, it is introduced in base plot AT 5-46 because reactive power (and voltage) has maxim sensitivity.

Table 3. Some values of sensitivities. Network with 48 nodes

Constrain	Independent variable	UM	Sensitivity
$Q_{STEJARU}$	$W_{GUTINAS}$	MVAR/plot	-3.0569924817
$Q_{STEJARU}$	$W_{STEJARU}$	MVAR/plot	-6.7334745218
$Q_{STEJARU}$	W_{BACAU_SUD}	MVAR/plot	-2.3605251821
$Q_{STEJARU}$	$U_{STEJARU}(110)$	MVAR/kV	9.8189869439
$U_{MĂRGINENI}$	$W_{GUTINAS}$	kV/plot	0.7628456434
$U_{MĂRGINENI}$	$W_{BACĂU\_SUD}$	kV/plot	0.5293106261
$U_{MĂRGINENI}$	$W_{STEJARU}$	kV/plot	-0.0617648732
$U_{MĂRGINENI}$	$U_{STEJARU}(110)$	kV/kV	0.2053868741
$I_{AT4GUTINAS}$	$W_{GUTINAS}$	A/plot	23.229075788
$I_{ATBACAU\_SUD}$	$W_{GUTINAS}$	A/plot	-10.04957672

In this paper it was also studied how modifying of operating regime influences sensitivity matrix [S] in the optimization process. For simplify, it was considered initial regime where are modified some variable. It was find that, although regime analyzed is much far off to initial regime (power loss is up 60 %) voltage sensitivity in consumer nodes it is modified, generally, with value to 10 %. Sensitivity of site current reported to plots it is modified in large limits (to 70 %).

It is considered that using sensitivities calculated in reference regime for optimization some regimes is satisfactory. That is because in calculus procedure it is interesting, usually, only sensitivity order of different variable reported to control variables and, practically, real regimes are very close to optimal regimes. For optimization problem, where sensitivity it is using only for choosing variables that enter in base and do not for control strictly speaking, modifying of sensitivity is even less important. At the same time, it must be take account that exceeding site current is unusually, practically, in normal regimes.

2.3. Voltage control in damage regimes using sensitivity matrix

In case damage, important decrease of network voltage it must be annulled quickly, [2]. Voltage sensitivity in node (or in nodes) with low voltage reported to control variable is characteristic which allow taken decisions quickly. If it is consider a node $i \in c$, where we want increase voltage with value ΔU_i, the correction necessary is:

$$\begin{aligned}\Delta \boldsymbol{U_i} &= \sum_{j \in q} \frac{\partial \boldsymbol{U_i}}{\partial \boldsymbol{U_j}} \cdot \Delta \boldsymbol{U_j} + \\ &+ \sum_{i,j \in r} \frac{\partial \boldsymbol{U_i}}{\partial \boldsymbol{W_{ij}}} \cdot \Delta \boldsymbol{W_{ij}},\ \boldsymbol{i} \in \boldsymbol{c}\end{aligned} \tag{10}$$

The control can be realized throw one or many means. It is recommended that used little means. For example, we consider that station 220/110 kV Stejaru, including AT and lines 220 kV and 110 kV, turn out due to a damage. Evidently, it must be used sensitivities calculated for this operating scheme or one close by it due to operating scheme is modified significantly. Control possibilities are plots of AT in stations Dumbrava, Gutinas or Bacau_Sud. In this regime, voltages in some nodes in Piatra-Neamt and Roman are affected. We consider voltage control in node Ciritei 110 kV, which has value 106.2 KV in this regime. Sensitivities of this voltage reported to plots of AT are presented in table 4.

Table 4. Voltage sensitivities in node Ciritei reported to plots of AT

Plot of AT	$W_{GUTINAS}$	$W_{DUMBRAVA}$	W_{BACAU_SUD}
Sesitivity [kV/plot]	0.456	1.097	0.353

If it is imposed increasing of voltage up 109 KV, it must be modified plot of AT in station Dumbrava (because voltage in this node has maxim sensitivity) with value:

$$\Delta W_{DUMBRAVA} = ROUND\left[\frac{109 - 106.2}{1.097}\right] = 3 \quad (11)$$

Voltage estimated is 106.2 + 3×1.097 = 109.49 KV. After calculus of regime with plot of AT modified from 12 to 15, results voltage value in node Ciritei - 109.55 KV, so very close by the value estimated with sensitivity.

So, for control voltage in damage regime, it is necessary to have sensitivity coefficients calculated for operating scheme close by damage scheme.

CONCLUSIONS

Using sensitivity theory in energetic is a necessity because it must be a rapid response in the optimization problems or in the damage regimes.

For problem of inequality- type constrains control, this method offers the best solution for introducing an independent variable in base.

Values calculated for sensitivities are valid for a large domain of regimes in case of voltages in consumer nodes.

Using sensitivities for establishing succession in pseudo-optimization process leads to the final results better than in case aleatory order.

In the damage regimes, when we want to return quickly of some parameters in the admissible domain, this method leads to the very well results if sensitivities are calculated for regimes close it.

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