MATHEMATICAL MODELING AND ELABORATION OF THE SIMULATION SOFTWARE DURING THE REAL TIME ON THE COMPUTER OF THE MECHANICS OSCILLATIONS PROBLEMS

N. Balmuş [1], **I. Balmuş** [2]

[1] Pedagogical University of Moldova, Chisinau, Moldova
[2] Technical University of Moldova, Chisinau, Moldova

Abstract: In the paper the problems of numeric experiments realization during the real period are treated. Applications realized in computer are used for learning process and permit quantative study of oscillating phenomena in mechanic systems.

Mathematical modeling in used in science and technology for a long period of time. Simultaneously with coming of the last generation computers with extinguished operative memory volume and high calculations speed, some conditions for perfect mathematical models and numeric experiments achievement were setup. On the basis of the latter some important information is get, information vital for fundamental sciences and technology. Lately mathematical modeling is widely applied in technical university studying process[1].

The present paper deals with some aspects of mathematical modeling of oscillating mechanic systems and problems of numeric experiments applications during real periods of time.

Usually, while dealing with mathematical modeling of oscillating mechanic systems, some structures of differential ordinal equations of the second rank are got:

$$\frac{d^2 q_i(t)}{dt^2} = F_i(t, q_1, q_2, \ldots q_n, \dot{q}_1, \dot{q}_2, \ldots \dot{q}_n), \quad i = 1 \ldots n \qquad (1)$$

Where q and $\dot{q}$ are generalized coordinates and speeds;

n – number of the free degrees.

For such kind of phenomena and its simulation on the computer during real time some numeric methods and algorithms are needed. Such numeric methods and algorithms must be effective in numeric solution of differential equations of the systems of the kind presented above (1) and operational control of the calculation errors while a period of time t compared to observational period in a real experiment. Such algorithm presents a combination of methods Runge-Kutta and preacher-corrector adopted for solving equation systems of the 1st degree.

Below two applications are doing to be presented concisely realized in the computer in the Delphi´s programming medium.

1. DOUBLE MATHEMATICAL PENDULUM

Equations of motion for such kind of conservative system with free degrees are deduced simply on the basis of Langrange´s formalism.

$$(m_1+m_2)l_1^2\ddot{\theta}+m_2 l_1 l_2\ddot{\varphi}\cos(\varphi-\theta)-m_2 l_1 l_2\dot{\varphi}\left(\dot{\varphi}-\dot{\theta}\right)\sin(\varphi-\theta)- \\ m_2 l_1 l_2\dot{\theta}\dot{\varphi}\sin(\varphi-\theta)+(m_1+m_2)g l_1\sin(\theta)=0 \qquad (2)$$

$$m_2 l_2^2\ddot{\varphi}+m_2 l_1 l_2\ddot{\theta}\cos(\varphi-\theta)-m_2 l_1 l_2\dot{\theta}\left(\dot{\varphi}-\dot{\theta}\right)\sin(\varphi-\theta)+ \\ m_2 l_1 l_2\dot{\theta}\dot{\varphi}\sin(\varphi-\theta)-m_2 g l_2\sin(\varphi)=0 \qquad (3)$$

On the basis of equations (2) and (3) an application was realized on the computer in the Delphi´s programming medium, which simulates during the real period oscillations produces by double mathematical pendulum for all types of values of the parameters, m_1, m_2, l_1, l_2 shown in the figure number 1.

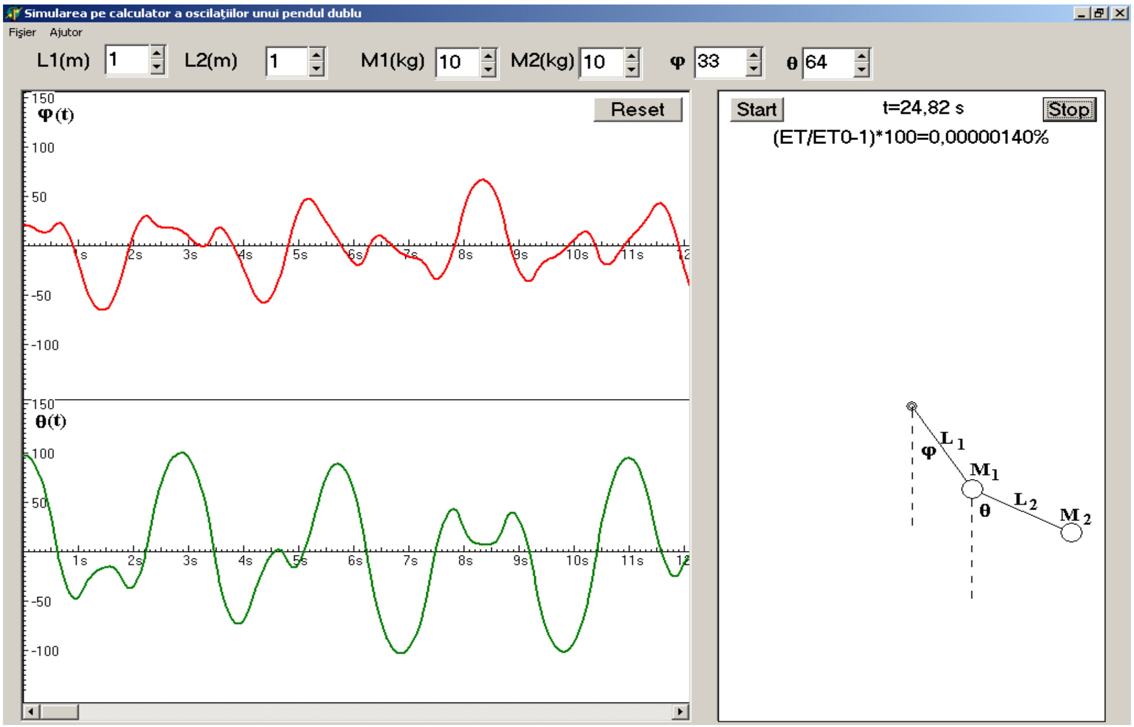

Fig.1. Computer´s monitor while simulation oscillations of the Double mathematical pendulum.

The correctness of the numeric solution of the above systems of differential equations (2) and (3) is verified indirectly by determining of the relative deviation of the total energy,

$$E = \frac{1}{2}(m_1 + m_2)l_1^2 \dot{\theta}^2 + \frac{1}{2}m_2 l_2^2 \dot{\varphi}^2 + m_2 l_1 l_2 \dot{\theta}\dot{\varphi}\cos(\varphi - \theta) + (m_1 + m_2)g l_1(1 - \cos(\theta)) + m_2 g l_2 (1 - \cos(\varphi)) \quad (4)$$

from the total energy of the system at the initial moment E_0 (the law of preserving energy).

This value is shown on the computer monitor continuously. With the time, the eroor is proceeding slowly and in two hours period to the 0,0015% value.

It has to be mentioned that such kind of application functions steadily as, for small though big oscillations, while pendulum´s deviations from the position of equilibrium are about the 180^0 value. From the graphics shown on the computer monitor an absolute non-periodic character of the double pendulum´s oscillations is seen.

Regarding small oscillations, the application can be used for theory conclusions checking up with reference to the existence of two own frequencies.

If $m_1 \gg m_2$, $l_1 = l_2$, $\varphi_o = 0$ and $Q_o \ll 0$, regarding theory [2] a phenomenon of puls is observed (fig.2).

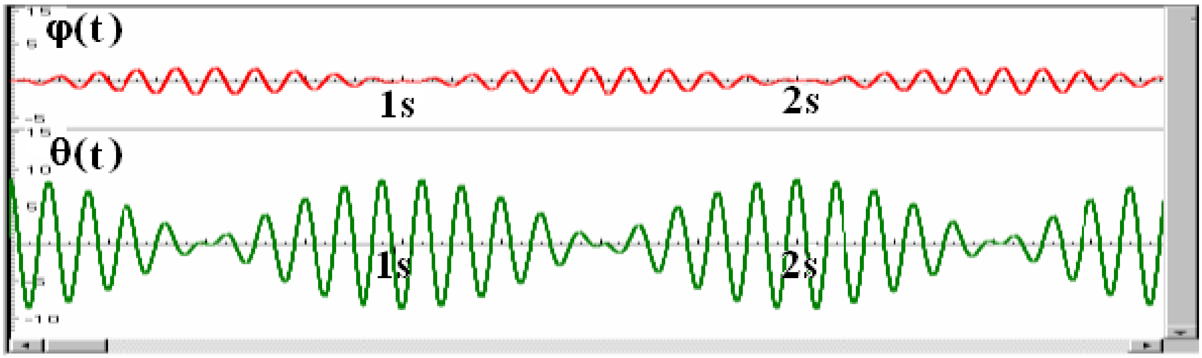

Fig.2. The puls of double pendulum oscillations

2. A PUSH-CART WITH SUSPENDED MASS

Equations with the motion for such kind of conservative system containing two free degrees are deduced simply on the basis of Langrange´s formalism:

$$l\ddot{\varphi} + \ddot{x}\cos(\varphi) + g\sin(\varphi) = 0 \quad (5)$$

$$(M + m)\ddot{x} + ml\ddot{\varphi}\cos(\varphi) - ml\dot{\varphi}^2\sin(\varphi) = 0 \quad (6)$$

The parameters M, m, l, x and φ are specified in fig.3. On the basis of (5) and (6) equations system, oscillations are simulated taking in consideration for every initial conditions of the push-cart and suspended mass. From fig.3 push-cart oscillations φ(t) and suspended mass x(t) is seen that, generally speaking, non-sinusoid, but absolutely periodical.

It must be mentioned that for big oscillations do not exist any formulas on whose basis oscillation period could be estimated.

In case when initial deviation φ_0 of the suspended weight is small then push-cart oscillations and of suspended weight become harmonious with the period

$$T = 2\pi\sqrt{\frac{Ml}{g(M+m)}} \qquad (7)$$

Expression (7) is verified with precision using an extern chronometer, independent in the system clock of the computer. Such result shows that numeric experiment suits perfectly theory.

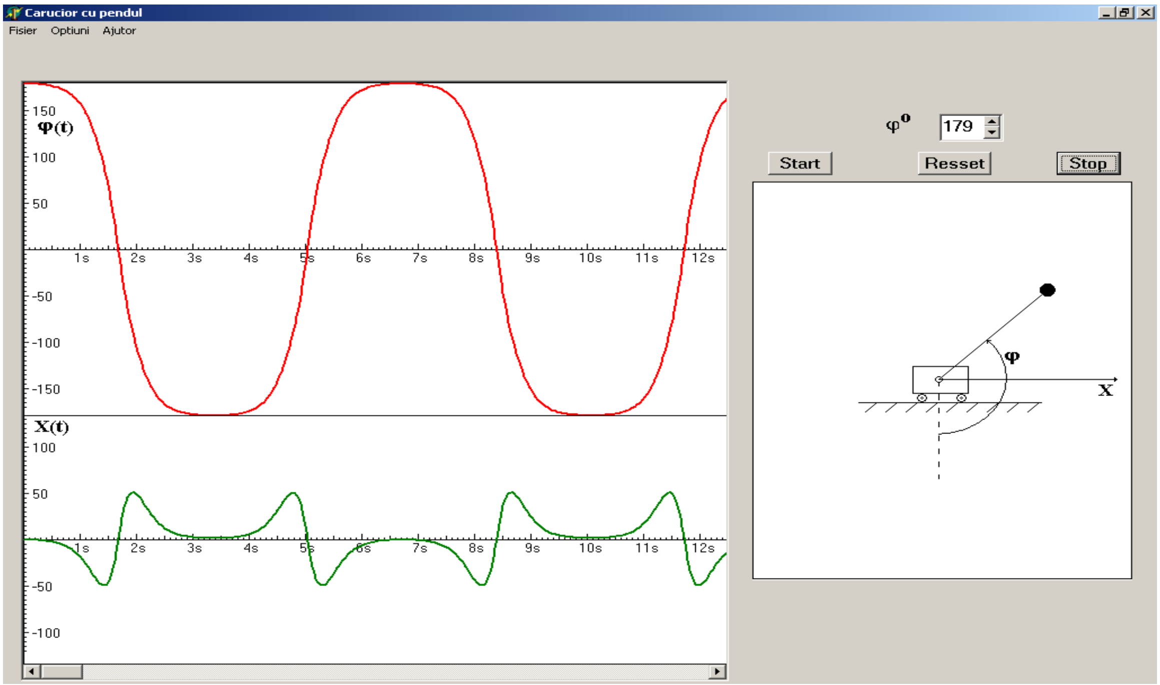

Fig.3. The monitor of the computer while the push-cart oscillations simulation with suspended mass

Other conservative and non-conservative mechanical systems are modeled with 1,2 and more free degrees. The program was willed for a system of 300 free degrees (coupled linear oscillators).

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