

# Membrane Petri Nets Model Optimization using Methods of Functional Locations and Dynamic Priorities

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**Abstract** — To simplify visual membrane Petri nets models and to minimize the computation time we propose two methods: a method of continuous functional locations and a method of dynamic priority functions for transitions involved in an structural conflict place transition systems. The method of functional locations allows a substantial reduction of the number of components of 3D Visual Membrane Petri Nets models. The method of dynamic marking-dependent priorities allows optimizing models that describe basic cellular functions (including degradation processes of proteins, enzymes, gene copies, etc.). As examples, a model of membrane oscillations and the model of SOS cellular response to environmental stimuli are optimized by using new proposed methods.

**Index Terms** — auto-regulation, modeling, P systems, Petri Nets, 2D and 3D graphycs.

## I. INTRODUCTION

New enhanced extensions of Petri nets, called 2D and 3D Visual Membrane Petri nets (VMPN) are introduced in [1, 2]. These approaches are elaborated according to the original Petri nets theory that was introduced by Carl Adam Petri in 1962 [3] and are based on today's different extensions of Petri nets such as discrete Petri nets, continuous Petri nets and hybrid discrete-continuous Petri nets [4, 5]. On the other hand, the theory of VMPN is based on the concept of membrane systems, P systems [6].

It was observed that models of P systems can be simulated by some extensions of Petri nets. In [7] is presented a direct connection between P systems models and Petri Nets models. Further, various extensions of PN, which use the concept of P Systems and can "express" modular (compartmental, membrane) structure, have been proposed. In the last years new directions of PN extensions, which are based on the concept of P Systems, have been developed. Reconfigurable PN was developed [5]. In addition, another direction, represented by 2D and 3D Membrane Petri (VMPN) nets was developed [1, 2]. In the structure of VMPN the membrane structure, denoted by  $\mu$ , was added as a basic component. In this way, the 2D and 3D VMPN parallel software tool accepts the behavior of parallel and distributed membrane systems, values for discrete and continuous places and transitions, continuous transitions of the net that are fired with speeds that are piecewise constants or marked-dependent variable, discrete and continuous arcs, guard functions, priorities, etc. Relevant aspects of auto-regulatory processes in cells and more complex systems such as the human body can be modeled by 2D and 3D VMPN. Thus, Patients Specific models that allow to elucidate the key role of compensatory mechanisms, which evolve in blood and target tissues such as: hepatic, kidney, heart and periphery is elaborated using 2D or 3D VMPN.

In the framework of the VMPN approach, we proposed new two methods that allow model's optimizing reducing the number of model components and the model computation time.

In sections II and III a new type of prioritized Petri nets – Petri Nets with dynamic marking-dependent priorities for transitions that are in structural conflict place transition systems. As a result, dynamic property method implementation will simplify considerably models and will reduce time of running of the models (it is known that mathematic formula calculation time is significant in calculating/running time of whole model).

The method of continuous functional locations applied to 3D Membrane Petri nets models is described in section IV.

## II. 2D MEMBRANE PETRI NETS WITH DYNAMIC PRIORITIES

In this section, we describe the concept of 2D visual membrane Petri nets (2D VMPN) [1]. Timed Petri nets are used to model numerous types of large complex systems, especially computer architectures, communication networks, economic and biological systems. However, simulation's computational requirements can be massive, especially on the large complex models that defeat analytic methods. One way of meeting these requirements is by executing some modules of the complex Petri nets model in parallel.

A 2D membrane Petri net is a structure:

$MPN = \langle P, T, Pre, Post, Test, Inh, \mu, G, Pri, Kp, M_0, \theta \rangle$ , where:

- P is a finite set of discrete and continuous places.
- T is a finite set of discrete and continuous transitions.  $P \cap T = \emptyset$ .
- $\forall p \in P$ , Pre, Test,  $Inh: P \times T \rightarrow Bag(P)$  and  $Post: T \times P \rightarrow Bag(P)$  are the forward, test, inhibition and backward functions in the multi-sets of P, which

define the set of arcs  $A$  with marking-dependent multiplicity of the input and output arcs, connecting transitions and places. For discrete components: the set  $A$  is partitioned into three subsets: direct normal arcs, inhibitory arcs and test arcs. For continuous components of 2D VMPN, the set  $A$  is partitioned into three subsets: continuous flow arcs, inhibitory, test and linkflow arcs. The default value of all these arcs is 1.

- $\mu$  is a membrane structure of 2D VMPN model, consisting of 2D labeled elementary membranes, maps a membrane structure  $\mu$  of a model. The skin membranes of 2D VMPN models are usually not represented.
- $G: T \times IN_+^{|P|} \rightarrow \{0,1\}$  is a *guard function* for each transition as a Boolean function (default value is *true*);
- $Pri: T \rightarrow IN_+$  defines the priority functions for the firing of each transition and dynamic marking-dependent priorities for transitions that are in structural conflict place transition systems;
- $K_p: P \rightarrow IN_+$  is the capacity of places, and by default being infinite value;

The complete marking (state) of a net is described by a vector-column  $M = (m_i, p_i, m_i \geq 0, \forall p_i \in P)$ , where  $M: P \rightarrow IN_+$  are marking functions of places. The initial marking of net is  $M_0$ . The set of transitions  $T$  can be partitioned into a set  $T_0$  of immediate transitions and a set  $T_\tau$  of timed transitions,  $T = T_0 \cup T_\tau = \emptyset$ . An immediate transition  $t_j \in T_0$  is drawn as a black thin bar and a timed transition  $t_k \in T_\tau$  – as a black rectangle,  $Pri(T_0) > Pri(T_\tau)$ ;

- $\theta: T \times N^+ \rightarrow R^+$  is a weight function that maps transitions into real numbers  $R^+$  (delay time or weight speeds). It can be marking dependent. The delay time  $\theta(t_i, M) = di(M)$  defines the transition firing duration for each timed transition of  $T_\tau$ . If several timed transitions  $t_j \in T(M)$  are enabled concurrently, then either fire in competition or independently. We assume that a race condition exists between them. Two or more enabled transitions will be executed in parallel mode. The degree of parallelism is determined by the number of enabled firing transitions with the same delay time.

So, a 2D Visual Membrane Petri nets system is a construct  $2D VMPN = \langle TPN, \mu \rangle$ , where: TPN is a hybrid timed Petri net considered in [4, 5] and  $\mu$  is a membrane structure, composed of 2D elementary membranes included in a skin membrane (Fig. 1).

Such a membrane structure is useful to model dynamics of molecular interactions, communication and molecular transport between different compartments of  $\mu$ . Compartments (in different models can represent cell, organelle, cell membranes, human tissues, organs) are mapped by 2D elementary membranes.

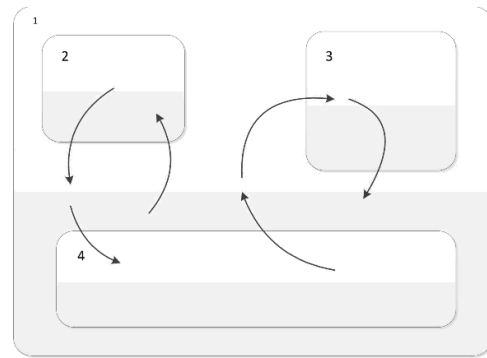


Fig. 1. Schematic representation of 2D VMPN software membrane structure  $\mu$ . Skin membrane is represented by the 2D membrane labeled 1; the 2D membranes labeled 2, 3, 4 are elementary membranes.

The skin membranes map cellular membranes (for living cell models [1, 8]) or human body (in the case of Patient Specific models [2]).



Fig. 2. The main window of the 3D VMPN application.

So, the skin membrane delimits an internal environment, which depicts cytoplasm in cell models or human body or circulatory system in Patient Specific models, respectively (Fig. 2).

All primitives of the 2D VMPN formalism are shown in Fig. 3.

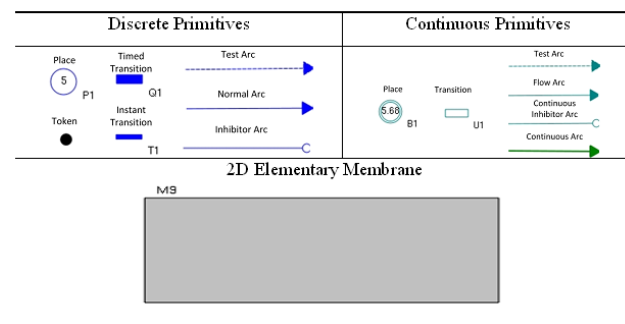


Fig. 3. Primitives of 2D VMPN

Within proposed formalism all components of the 2D VMPN (transitions, locations and arcs) are localized in labeled elementary membranes of the model membrane structure  $\mu$ .

### III. METHOD OF DYNAMIC PRIORITIES

In this section a new type of prioritized Petri nets – Petri Nets with dynamic marking-dependent priorities is

described. The difference between dynamically prioritized Petri nets from traditional Petri nets lies in the introduction of priority into output transitions involved in structural conflict place transition systems (Fig. 4). In this case, the output transitions ( $t_j, t_k \in p_i^\bullet$ ) with a common preplace ( $p_i$ ) must not fire in the same step, because they are treated unequally at transitions. The priorities of this transitions are not equal, i.e.  $O(t_j) \neq O(t_k)$ .

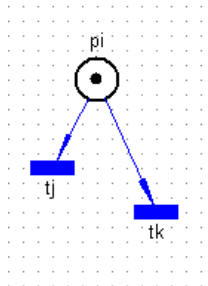


Fig. 4. Transitions that are in structural conflict place transition systems.

Traditional prioritized Petri nets add priorities to transitions, whereby a transition cannot fire, if a higher-priority transition is enabled (i.e. can fire). Thus, transitions are in priority groups, e.g. priority group 3 can only fire if all transitions are disabled in groups 1 and 2. Within a priority group, firing is still non-deterministic [9].

In the models which describe basic cellular functions (including degradation processes of proteins, enzymes, and gene copies, etc.) we frequently encounter (see Fig. 4, red highlights) the problem of reducing the marker numbers (for discrete Petri nets locations) or the level of continuous locations (for continuous Petri nets) to zero values. Traditionally, to model the decreasing of marker numbers in one location up until minimal/zero values, two transitions, guard functions (test arcs, inhibitory arcs), etc. are typically used (Fig. 5). To simplify and optimize this process we propose to apply the dynamic marking-dependent priorities functions to these two output transitions (that are in structural conflict). In this case, output transitions will change their states from activated to inactivated (or vice versa) in dependence on the preplace marking.

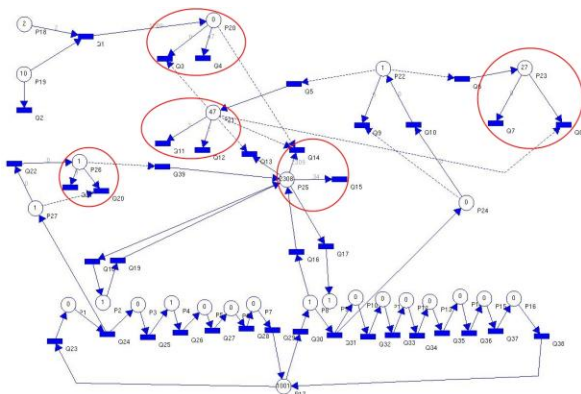


Fig. 5. Model of SOS cellular responses to environmental stimuli [1, 8].

So, we consider that a minimum priority  $O(t_j) = 0$  has the transition  $t_j$ , which is located nearer the input location  $p_i \in {}^\bullet t$  than transition  $t_k$ . In this case, transition  $t_j$  will be inactive. Thus, only transition  $t_k$  will be initially validated, because it is located at a longer distance of input location  $p_i$ . The priority of this transition is  $O(t_k), O(t_k) = 1$ . The transition  $t_j$  will be inactivated until the second transition  $t_k$  is active (as long as the marking of the input location  $p_i, M(p_i)$  will be greater or equal to the weight of the input arc  $W(p_i, t_k)$ :

$$M(p_i) \geq W(p_i, t_k) \quad (1)$$

Since the condition (1) will no longer be valid, the priority of transition  $t_j$  will become equal to 1 ( $O(t_j)=1$ ), and the priority of transition  $t_k$  will become 0,  $O(t_k)=0$ . As a result, transition  $t_j$  will become activated and transition  $t_k$  will become inactivated.

Thus, the dynamic property method implementation will considerably simplify models reducing the number of components (guard functions, test arcs, inhibitory arcs and mathematical formulas by which some arcs weighs are calculated) and running time of the models (it is generally known that the calculation time of mathematical formula is significant in the calculating/running time of the whole model).

#### IV. METHOD OF FUNCTIONAL LOCATIONS

The method of continuous functional locations applied to 3D Membrane Petri nets models is described in this section. In Fig. 6 it is represented the model of membrane oscillations created using 3D VMPN application. This type of oscillations can occur in different types of living cells. This model can be simplified considerably and therefore, the calculation time of the model can be minimized by using the method of functional locations, implemented in the 3D VMPN application.

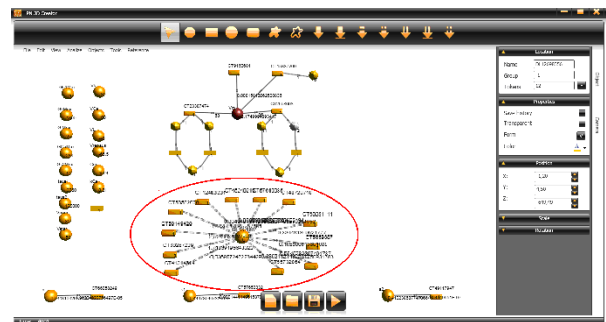


Fig. 6. Screenshot of the model of membrane oscillations created by 3D VMPN software application.

Some parts of the models by means of which the model parameters are calculated (using mathematical formulas) represent structures composed of continuous Petri nets components: locations, inhibitory (or test) arcs and transitions (see Fig. 6, red highlight). In this model 12 parameters should be calculated dynamically using mathematical formulas. So, to obtain values of 12 parameters a structure composed on 1 continuous location, 12 test arcs and 12 continuous transitions is created. The weights of the test arcs are calculated using mathematical

formulas. Such structures can be replaced by new type of locations, namely continuous functional locations. Marking values of these locations can be calculated on the basis of mathematical formulas. So, the number of Petri nets components involved in the parameters calculations is reduced considerably (from 25 to 12 components, in our case). Thus, the main advantage that it is offered by this method consists of reducing the number of components of the models and the calculation time.

The simplified model of membrane oscillations by means the method of functional locations is represented in Fig. 7.

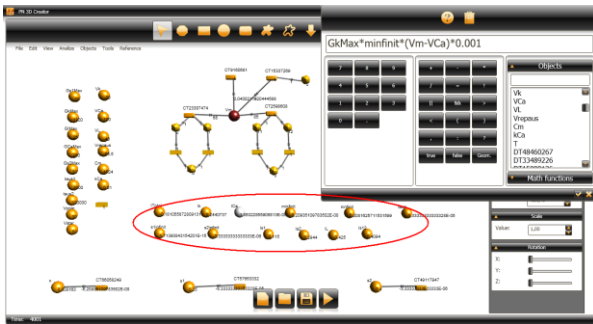


Fig. 6. Screenshot of simplified model of membrane oscillations by using the method of functional locations.

Some of simulation results of the model of membrane oscillations are represented in Fig. 7. The first chart represents the oscillations of the membrane potential, noted by  $V_m$ . The oscillations of the membrane potential appear as the result of oscillations of  $Ca$ ,  $K$  ion currents, noted in the model by  $I_{Ca}$ ,  $I_K$ . The last plot shows the oscillations of the total transmembrane ion current, noted as  $I_{TOTAL}$ .

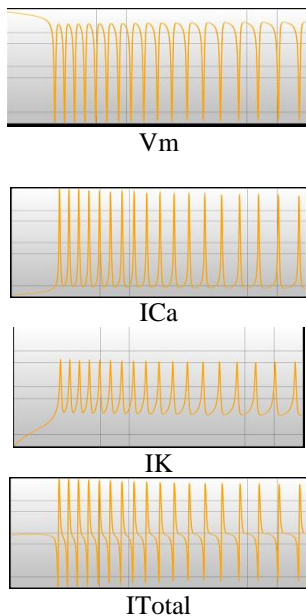


Fig.7. Simulation results of the membrane oscillation model using the 3D VMPN application.  $V_m$  – membrane potential,  $I_{Ca}$ ,  $I_K$ ,  $I_{total}$ - transmembrane ion currents.

The model of membrane oscillations can be considered as a part of an integrated Patient specific model [5].

## V. CONCLUSION

The 2D and 3D VMPN (new extensions of parallel and distributed software), is based on the concept of P Systems [6] and the Petri Nets theory [3, 4, 5]. We combine different fundamental characteristics into a single system for modeling behavioral properties of membrane system models and their visual interactive discrete-continuous simulation with emphasis on the relationship between spatial organization and function. These application offers graphic representation of 2D and 3D membrane structures and Petri nets components.

In this paper it shown that to simplify membrane Petri nets models and to minimize the computation time two new methods: a method of continuous functional locations and a method of dynamic priorities can be applied.

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