SCIENTIFIC METHODOLOGICAL CALCULATIONS FOR HYDRAULIC BEHAVIOR OF NON–NEWTONIAN FLUIDS IN MACHINERY CHANNELS

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Annotation. On the basis of the research made, the method of calculating the hydraulic characteristics of non–Newtonian fluid flows in the processing equipment with basic geometry channels.

Key words: non–Newtonian fluids, mathematical modeling, hydraulic properties, flow, channel, equipment.

Main processes in food industry are associated with complex disperse systems, most which are non–Newtonian fluids. Low viscosity fluid flows are used as intermediate coolant in high–temperature devices. Typically, the fluid motion of a High viscosity fluids usually flow in working chamber channels of various manufacturing machines and depends on numerous parameters: pressure, losses, flow shear velocity, temperature, degree of mixing, dispersion etc. Thus, the information about flow structure and flow rates play an important role in the organization of technological processes and can influence their power efficiency by establishing rational values of hydrodynamic, thermal, mass transfer, and other parameters.

Nonlinear material flow in complex geometry channels can be described by developing a theory–based three–dimensional mathematical model of non–Newtonian fluid flow in basic geometry channels to further obtain most optimal and energy–saving technological parameters of the processes, chemical and food industry machinery.

The problem of flows in basic geometry channels is theoretically grounded and calculated in the work [1] for a three–dimensional case. The longitudinal flow under the influence of pressure difference at the channel ends was separately examined. Besides, the additional driving force is the movement of channel boundaries which are not restricted by any speed limits. The obtained mathematical models allow us to calculate basic macrodynamic and macrokinetic flow behavior at each point in the channel.

It should be noted that the obtained mathematical solutions for the models of non–Newtonian material flows must have sufficiently reliable results. It is known that under real conditions the object of research cannot be completely adequate to the built model as its making used different approximations and agreements. Therefore, the best way to justify the validity of the derived theoretical solutions is to make experimental studies and to use them as a basis to estimate adequacy of the obtained models.

Taking into account the above said, calculation methods for verifying experimental data with theoretical results of the flow patterns in the base geometry channels are proposed.

To study the flow of non–Newtonian fluid in a flat channel, there was designed and manufactured a test bench for measuring hydraulic behavior (see Pic. 1). It made it possible to study and adequately estimate the pressure, temperature, losses and power. They are mostly important for effective technological processes in food and chemical industry [2].

Materials with properties which make it possible to more fully evaluate the above mentioned parameters (minced beef and fish, low–fat cheese and processed cheese, creamed curds, candy praline Kara–Kum) were chosen as objects for studying non–Newtonian fluid flow.

The method of calculating hydraulic characteristics of non–Newtonian fluid flow in basic geometry channels was designed for processing the experimental data.

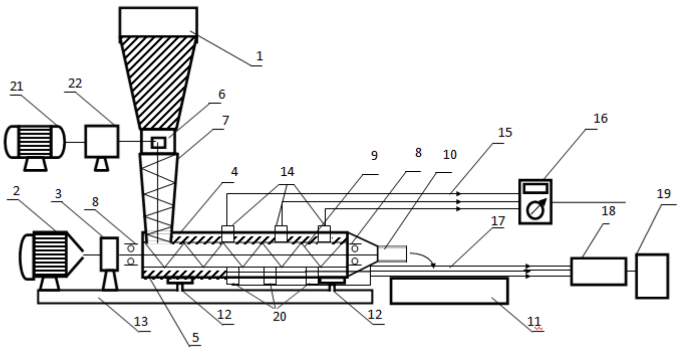

Fig. 1 Device for measuring hydraulic characteristics of non–Newtonian fluid flows:
1 – load hopper; 2 – motor; 3 – reducer; 4 – screw supercharger; 5 – cooling jacket;
6 – pipe fittings; 7 – feed dispenser; 8 – bearing units; 9 – worm; 10 – nozzle; 11 – receiving tank;
12 – supports; 13 – supporting frame; 14 – thermocouple; 15 – thermocouple wiring;
16 – voltmeter; 17 – wires; 18 – tensor amplifier; 19 – recording device; 20 – tensor sensors;
21 – motor; 22 – reducer

While choosing geometrical dimensions of the worm channel, this channel may be approximated as nearest to flat. The influence of the second pair of walls can be neglected [3]. In this case, the expression for determining boundaries coordinates is as follows:

$$\gamma^{\pm} = \delta\gamma \pm \gamma; \qquad \gamma = \frac{\tau_0}{\frac{dP}{d\zeta}}; \qquad \delta\gamma = \frac{\mu(W^{+} - W^{-})}{2h\tau_0} \qquad (1)$$

where $\gamma^{\pm}$ – boundaries;
δ – channel width, m;
τ_0 – taper tension;
μ – viscosity;
h – channel height.

To describe losses for a flow in a flat channel, we obtained the expression:

$$\dot{V}_1 = (W^+ + W^-)h - \left(W^+ - W^-\right)h\delta\gamma_1 - \\ -\frac{2}{3}\frac{h^2}{\mu_0}\frac{dP_1}{d\zeta_1}(1-\frac{3}{2}\gamma_1+\frac{1}{2}\gamma_1^3-\frac{3}{2}\gamma_1\left(\delta\gamma_1\right)^2-3\left(\delta\gamma_1\right)^2). \tag{2}$$

Losses for a flow in a circular pipe is calculated using the following formula:

$$\dot{V}_2 = \frac{\pi r_0^3}{8\mu_0}\frac{dP_2}{d\zeta_2}\left(1-\frac{4}{3}\gamma_2+\frac{1}{3}\gamma_2^4\right) \tag{3}$$

where $\dot{V}_1, \dot{V}_2$ – losses, sm³/sec

It should be noted that in the formulas (2) and (3) for distinguishing characteristics of a flat channel flow, indices "1" and "2" are used respectively. The system of equations to calculate flow losses is as follows:

$$\dot{V}_1 = \dot{V}_2 \qquad \frac{dP_1}{dZ_1}L_1 + \frac{dP_2}{dZ_2}L_2 = 0 \tag{4}$$

where L_1 – flat channel length, m;

L_2 – pipe length, m.

With regard to the worm equipment, it was decided that the value L_1 is the length of a spiral channel turns, L_2 is the length of the nozzle, r_0 is the radius of the nozzle. Numerical analysis showed that the following simplification is possible without big losses in accuracy:

$$1-\frac{3}{2}\gamma_i+\frac{1}{2}\gamma_i^3 \approx 1-\gamma_i; \\ 1-\frac{4}{3}\gamma_2+\frac{1}{3}\gamma_2^4 \approx 1-\gamma_2; \tag{5}$$

The solution of equation (4) leads to the following results:

$$\frac{dP_1}{d\zeta_1} = \frac{3\mu_0\left(W^+ - W^-\right)}{2h_1}\left\{1+\frac{\mu_0\left(W^+ - W^-\right)}{2h\tau_0}\times\frac{\gamma_1}{\left(1-\gamma_1\right)^2}\right\}\times\frac{1}{1+\xi\frac{3\pi r_0^3}{s16a_1h_1^2}\times\frac{1-\gamma_1/\xi_s}{1-\gamma_1}}$$

$$\dot{V}_1 = \dot{V}_2 = \dot{V} = \left(W^+ + W^-\right) a_1 h_1 \left\{ 1 + \frac{\mu_0 \left(W^+ - W^-\right)}{2h\tau_0} \times \frac{\gamma_1}{\left(1-\gamma_1\right)^2} \right\} \times$$
$$\times \frac{\xi_s (\pi r_0^3 / 16 a_1 h_1^2) \times (1 - \gamma_1 / \xi_s)}{1 + \xi_s (\pi r_0^3 / 16 a_1 h_1^2) \times (1 - \gamma_1 / \xi_s)}, \quad (6)$$

$$\xi_s = \frac{L_1}{L_2} \times \frac{r^0}{h}$$

In the case of helical channel in a coordinate system which revolves with a worm, velocity W_1 should be considered zero. For value $\gamma^{\pm}$, the following expression may be appropriate:

$$\gamma_1 \approx \frac{2h\tau_0}{3\mu_0 W_1^+} \times \left(1 + \frac{2}{3} \times \frac{3\pi r_0^3}{16 a_1 h_1^2} \xi_s \right) \quad (7)$$

The study was carried out using two types of worms: the worm having screw pitch $t_B = 86 мм$ and channel depth $2h = 7 мм$ and the worm with screw pitch $t_B = 132 мм$ and channel depth $2h = 14 мм$.

Features flat channel defined by the following relations:

$$L_1 = n \times \sqrt{t_B^2 + \pi^2 D^2}, \quad tg\varphi_B = {t_B}/{\pi D},$$
$$a_1 = t_B \cos\varphi_B, \quad (8)$$

where t_B – worm pitch, mm;

n – number of turns;

φ_B – worm ridge clearance angle, degrees.

Substitution of the calculation results of formula (8) into (6) leads to the following expressions for determining worm device loss values:

$$\dot{V} = 33,2 \times N \left\{ 1 + \frac{0,63\gamma}{(1-\gamma)^2} \right\} \times \frac{0,60\left(1 - \gamma / 7,14\right)}{1 + 0,60 \times \left(1 - \gamma / 7,14\right) / \left(1 - \gamma\right)},$$
$$\gamma = \frac{1,31}{\mu_0 W^+ / h\tau_0},$$
$$\dot{V} = 83,5 \times N \left\{ 1 + \frac{0,67\gamma}{(1-\gamma)^2} \right\} \times \frac{0,68\left(1 - \gamma / 5,80\right)}{1 + 0,68 \times \left(1 - \gamma / 5,80\right) / \left(1 - \gamma\right)},$$
$$\gamma = \frac{1,31}{\mu_0 W^+ / h\tau_0} \quad (9)$$

The first formula (9) refers to the worm pitch $t_B = 86 мм$ with the channel depth

$2h = 7\textit{мм}$ and the second one to the worm pitch $t_B = 132\textit{мм}$ with the channel depth $2h = 14\textit{мм}$.

Overall worm unit capacity is a sum of capacities which include idling, material displacement, worm channel heat dissipation and hea, and nozzle tube heat distance. The expression for these capacities is as follows:

$$W_{\text{displ.}} = \frac{3\mu_0 L_1 \dot{V}^2}{2a_1 h_1^3 \left(\frac{3\pi r_0^3}{16 a_1 h_1^2} \xi_s \right) \times \frac{1-\gamma_1/\xi_s}{1-\gamma_1}} \tag{10}$$

Substituting the geometrical characteristics for the worm set in the expression (10) leads to the following results:

$$W_{\text{displ.}} = \frac{\mu_0 \dot{V}^2}{0,60 \times \frac{1-\gamma_1/7,14}{1-\gamma_1}} \cdot 5,25 \cdot 10^{-4} \tag{11}$$

$$W_{\textit{worm.dissip.}} = 5,94 \cdot 10^{-4} \frac{1}{\mu_0} \left\{ \frac{\mu_0 \dot{V}}{0,60 \times \frac{1-\gamma_1/7,14}{1-\gamma_1}} \right\}^2 \times \times \left[\left(1-\gamma_1^+\right)^3 + \left(1+\gamma_1^-\right)^3 \right]. \tag{12}$$

$$W_{\text{nozzle}\,\textit{dissip.}} = 8,67 \cdot 10^{-4} \frac{1}{\mu_0} \left\{ \frac{\mu_0 \dot{V}}{0,60 \times \frac{1-\gamma_1/7,14}{1-\gamma_1}} \right\}^2 \times 2\left(1-\gamma_1/7,14\right)^3 . \tag{13}$$

$$W_{\text{displ.}} = 0,36 \cdot 10^{-4} \frac{\mu_0 \dot{V}^2}{0,68 \times \frac{1-\gamma_1/5,80}{1-\gamma_1}} \tag{14}$$

$$W_{\textit{worm.dissip.}} = 0,19 \cdot 10^{-4} \frac{1}{\mu_0} \left\{ \frac{\mu_0 \dot{V}}{0,68 \times \frac{1-\gamma_1/5,80}{1-\gamma_1}} \right\}^2 \times \times \left[\left(1-\gamma_1^+\right)^3 + \left(1+\gamma_1^-\right)^3 \right] \tag{15}$$

$$W_{\text{nozzle}\,dissip.} = 0,22 \cdot 10^{-4} \frac{1}{\mu_0} \left\{ \frac{\mu_0 \dot{V}}{0,68 \times \frac{1-\gamma_1/5,80}{1-\gamma_1}} \right\}^2 \times 2\left(1-\gamma_1/5,80\right)^3 \quad (16)$$

Equations (11) – (16) compare obtained results of the experiment with the calculated data for experimental conditions, and the formulas are written in such a form and with such multipliers so that the value V could be calculated in sm^3/s to make comparison with experimental data simplier.

Formulas (9), (10), (11) – (16) were calculated for the two types of worms, measurements of shaft revolutions of the device were being made within: $0,5 \le N \le 4r/s$ and compared with experimental data for food materials.

The experimental data treatment used methods of variation statistics, regression and dispersion analysis method, and methods of testing statistical hypotheses. The analysis made showed that the theoretical results agree well with experimental data and are within the permissible error.

Thus, the proposed method of calculating hydraulic characteristics of non–Newtonian material flow in basic geometry channels with moving boundaries and pressure difference at the ends will allow us to further determine necessary parameters to improved energy efficiency of various processing equipment and to avoid the use of traditional methods of experimental studies, which demand significant amount of time and costs.

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