CHARACTERISTIC OF PARALLELING LIMITED CAPACITY VOLTAGE SOURCES

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Abstract. *The droop current- sharing method for voltage sources of the limited capacity is considered. Influence of load resistor is investigated on uniform distribution of relative value of currents when the actual loading corresponds to the capacity of a concrete source. The novel concepts for quantitative representation of circuit characteristic are entered with use of projective geometry approach.*

Key-Words: *droop method, current sharing, paralleling, projective geometry, voltage sources.*

I. Introduction

The paralleling of lower-power voltage sources (converter modules) offers well-known advantages over a single, high power source. The basic problem of such power supply system is the load-current sharing among the paralleled modules. Various approaches of the current distribution are known [1]. In the most traditional droop method the equalizing resistors are used [2], [3], [4], including lossless passive elements [5]. Usually, equality of parameters of modules is provided, i.e. open circuit voltage and internal resistance. Therefore, the distribution of currents means the equality of these currents.

On the other hand, a scatter of module parameters, possible cases of use of primary voltage sources with different capacity determines the non-uniformity of currents distribution. Therefore, it is natural to understand uniform loading of sources in relative sense when the actual loading corresponds to the capacity of the source**.**

The analysis of such power supply system by the method of projective geometry has led to introduction of concept for quantitative representation of circuit characteristic [6]. In the present paper recently elaborated approach is developed.

II. Analysis of the paralleling voltage sources

Let us consider voltage sources E_1, E_2, presented in Fig.1. Resistors R_{i1}, R_{i2} are internal resistance of these voltage sources; equalizing resistors R_{e1}, R_{e2} provide current distribution for the given load resistance R_0.

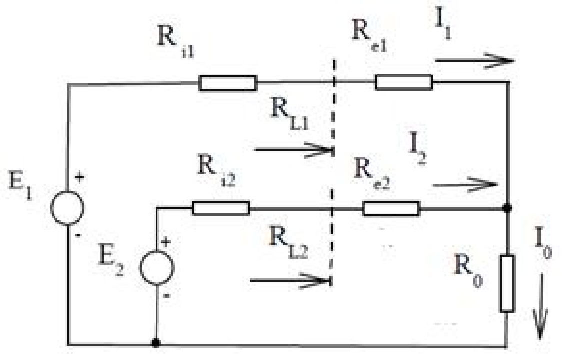

Fig.1. Paralleling of voltage sources

A circuit in Fig.1 is described by the following system of equations:

$$\begin{cases} E_1 = I_1 R_{i1} + I_1 R_{e1} + V_0 \\ E_2 = I_2 R_{i2} + I_2 R_{e2} + V_0 \\ V_0 = I_0 R_0 = (I_1 + I_2) R_0 \end{cases} \tag{1}$$

The normalized parameters of a loading regime of the sources look like

$$m_1 = \frac{R_{L1}}{R_{i1}} = \frac{I_1^M}{I_1} - 1\,,\ m_2 = \frac{R_{L2}}{R_{i2}} = \frac{I_2^M}{I_2} - 1\,, \tag{2}$$

where $I_1^M = E_1 / R_{i1},\ I_2^M = E_2 / R_{i2}$ are the short circuit currents.

Let us write expressions which associate the parameters of sources loading in the form $m_2(m_1)$. From (1) it follows

$$E_1 - E_2 = I_1 (R_{i1} + R_{e1}) - I_2 (R_{i2} + R_{e2})\,.$$

Keeping in mind that

$$I_1 = \frac{I_1^M}{m_1 + 1} = \frac{E_1}{(m_1 + 1) R_{i1}}\,, \qquad I_2 = \frac{E_2}{(m_2 + 1) R_{i2}}\,,$$

we obtain

$$m_2 = \frac{-\left(\frac{E_1}{E_1 - E_2} + \frac{E_2}{E_1 - E_2}\frac{R_{e2}}{R_{i2}}\right) m_1 + \left(\frac{E_1}{E_1 - E_2}\frac{R_{e1}}{R_{i1}} - \frac{E_2}{E_1 - E_2}\frac{R_{e2}}{R_{i2}}\right)}{m_1 - \left(\frac{E_2}{E_1 - E_2} + \frac{E_1}{E_1 - E_2}\frac{R_{e1}}{R_{i1}}\right)} = \frac{-a m_1 + (d - a + 1)}{m_1 - d}\,. \tag{3}$$

The expression (3) corresponds to the hyperboles in Fig.2. The desirable operating regime corresponds to the straight lines on this plot, $m_2 = m_1$.

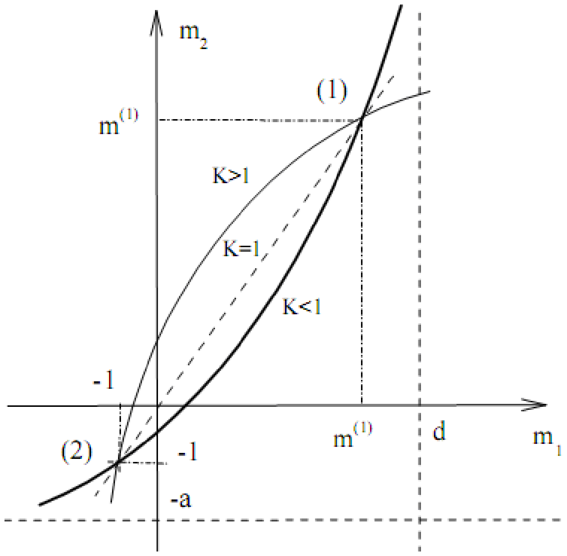

Fig.2. Correlated changes of the sources loading parameters as a family of the hyperboles $m_2(m_1)$ for various values of non-uniformity factor of loading K

The crossing of this straight line with the hyperbole plot gives two points of equal loading of sources, $m^{(1)}, m^{(2)}$. The working area (a load consumes energy) corresponds to the first point $m^{(1)}$. The second point corresponds to a condition when the voltage sources relatively equally consume energy. The points of equal loading are fixed points of projective transformation $m_1 \to m_2$, as it is shown in Fig.3.

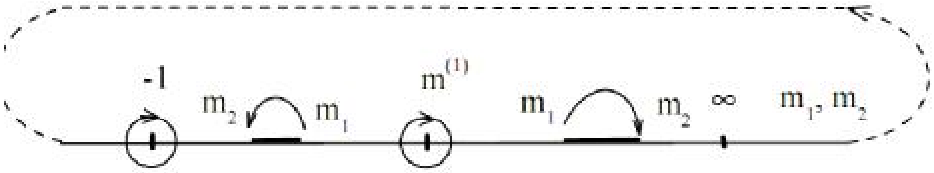

Fig.3. Display of projective transformation of the points $m_1 \to m_2$

In this case, the expression (3) leads to a quadratic one

$$m^2 - (d-a)m - (d-a+1) = 0$$

Solution gives two roots

$$m^{(1)} = d - a + 1, \quad m^{(2)} = -1. \tag{4}$$

For the second fixed point, the currents $I_2, I_1 \to \infty$. Though this case physically is not feasible, but its mathematical description allows enter a necessary characteristic of the circuit. Let us consider in details the geometrical interpretation of transformation (3) for different initial values of quantities m_1, m_2 at loading change. These quantities define a line segment, its length (in the usual sense of Euclidean geometry) or degree of difference of loading of sources is decreased at its approach to the fixed points. It is obvious that this length for different circuits will be various.

Thus, it is possible to enter a concept which defines the circuit: how much the loadings of sources can differ.

For introduction of such characteristic we use a number of concepts of projective geometry [7], [8], applied in the electric circuit theory [9], [10]. The characteristic of a relative positioning of two points concerning the chosen points (as the special case, it is fixed points $m^{(1)}, m^{(2)}$) is the cross ratio of these four points

$$(m^{(2)}\ \ m_2\ \ m_1\ \ m^{(1)}) = \frac{m_2 - m^{(2)}}{m_2 - m^{(1)}} \div \frac{m_1 - m^{(2)}}{m_1 - m^{(1)}}, \tag{5}$$

where the points $m^{(2)}, m^{(1)}$ are extreme or base. A cross ratio is generalization of a usual proportion. Also, it is known that the cross ratio concerning fixed points does not depend on values of running points m_1, m_2. Therefore, we accept for simplification of calculations $m_1 = \infty$. Then

$$(m^{(2)}\ \ m_2(\infty)\ \ \infty\ \ m^{(1)}) = \frac{m_2(\infty) - m^{(2)}}{m_2(\infty) - m^{(1)}} = \frac{a-1}{d+1} = K\ ,$$

$$K = \frac{E_2}{E_1} \cdot \frac{1 + \frac{R_{e2}}{R_{i2}}}{1 + \frac{R_{e1}}{R_{i1}}} \tag{6}$$

The obtained expression (6) is defined only by circuit parameters and characterizes ability of the circuit to equal loading of sources and corresponds to the entered concept. We name this expression as the ***factor of non-uniformity of loading*** K. If $m_2 \to m_1$, then $K \to 1$ for the given circuit. Generally, the factor of non-uniformity of loading $K \neq 1$. The equation (5), taking into account (6), allows expressing of dependence $m_2(m_1)$ using only two parameters of a circuit, such as $m^{(1)}$ and K. Let us present (5) as

$$K = \frac{m_2 + 1}{m_2 - m^{(1)}} \div \frac{m_1 + 1}{m_1 - m^{(1)}}\ .$$

From here

$$m_2 = \frac{-\frac{1 + Km^{(1)}}{1-K} m_1 + m^{(1)}}{m_1 - \frac{K + m^{(1)}}{1-K}} = \frac{-(1 + Km^{(1)})m_1 + (1-K)m^{(1)}}{(1-K)m_1 - (K + m^{(1)})}\ . \tag{7}$$

The dependence (7) for different values of K is presented in Fig.2.

Let us analyze the expression (6). Consider $E_1 = E_2$. Then

$$K = \frac{1 + \frac{R_{e2}}{R_{i2}}}{1 + \frac{R_{e1}}{R_{i1}}}.$$

The condition $K = 1$ leads to equality

$$\frac{R_{e2}}{R_{i2}} = \frac{R_{e1}}{R_{i1}}.$$

Therefore, it is quite possible to put $R_{e2} = R_{e1} = 0$. Thus, if at voltage sources identical open circuit voltage, they are equally loaded, and it is independent on their capacity.

III. Conclusions

The factor of non-uniformity of loading defines the circuit characteristic and determines the values of equalizing resistors.

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