BOSE–EINSTEIN CONDENSATION OF TWO-DIMENSIONAL POLARITONS IN MICROCAVITY UNDER THE INFLUENCE OF THE LANDAU QUANTIZATION AND RASHBA SPIN–ORBIT COUPLING

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Abstract

The Bose-Einstein condensation (BEC) of the two-dimensional (2D) magnetoexciton–polaritons in microcavity, where the Landau quantization of the electron and hole states accompanied by the Rashba spin–orbit coupling plays the main role, was investigated. The Landau quantization levels of the 2D heavy holes with nonparabolic dispersion law and third-order chirality terms both induced by the external electric field perpendicular to the semiconductor quantum well and strong magnetic field B give rise to a nonmonotonous dependence of the magnetoexciton energy levels and the polariton energy branches on B. The Hamiltonian describing the Coulomb electron–electron and electron–radiation interactions was expressed in terms of the two-particle integral operators, such as density operators $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$ representing the optical and acoustical plasmons and magnetoexciton creation and annihilation operators $\Psi^{\dagger}_{ex}(\vec{k}_{\parallel}), \Psi_{ex}(\vec{k}_{\parallel})$ with in-plane wave vectors $\vec{k}_{\parallel}$ and $\vec{Q}$. Polariton creation and annihilation operators $L^{\dagger}_{ex}(\vec{k}_{\parallel}), L_{ex}(\vec{k}_{\parallel})$ were introduced using the Hopfield coefficients and neglecting the antiresonant terms because the photon energies exceed the energy of the cavity mode. The BEC of the magnetoexciton–polariton takes place on the lower polariton branch at point $\vec{k}_{\parallel} = 0$ with the quantized value of the longitudinal component of the light wave vector, as in the point of the cavity mode.

The unitary coherent transformation of the obtained Hamiltonian leading to the breaking of its gauge symmetry was written as a Glauber-type coherent transformation using polariton operators $L^{\dagger}_0, L_0$ instead of the true Bose operators. It can be represented in a factorized form as a product of two unitary transformations acting separately on the magnetoexciton and photon subsystems. The first of them is similar to the Keldysh–Kozlov–Kapaev unitary transformation, whereas the second one is equivalent to the Bogoliubov canonical displacement transformation. It

was shown that the first transformation leads not only to the Bogoliubov u-v transformations of the electron and hole single-particle Fermi operators but also to the similar transformation of the two-particle integral operators. It becomes possible due to the extensive N-fold degeneracy of the lowest Landau levels (LLLs) in Landau gauge description, where N is proportional to the layer surface area S. In both cases, the u-v coefficients depend on the LLL filling factor, but in the last case, this dependence is doubled. The breaking of the gauge symmetry gives rise to the new mixed states expressed through the coherent superposition of the algebraic sum of the magnetoexciton creation and annihilation operators $(e^{i\alpha}\Psi_{ex}(-\vec{k}_{\|})+\Psi^{\dagger}_{ex}(\vec{k}_{\|})e^{-i\alpha})$ and density operator $\hat{D}(\vec{k}_{\|})$ representing the acoustical plasmon. In contrast, density operator $\hat{\rho}(\vec{Q})$ representing the optical plasmon does not take part in these superpositions.

1. Introduction

The present article is based on the background previous papers and monographs [1–16] as well on the recent contribution [17–26].

In [17], the Hamiltonian of the electron-radiation interaction in the second quantization representation for the case of 2D coplanar electron–hole (e–h) systems in a strong perpendicular magnetic field was derived. The s-type conduction band electrons with spin projections $s_z = \pm 1/2$ along the magnetic field direction and the heavy holes with total momentum projections $j_z = \pm 3/2$ in the p-type valence band were taken into account. The periodic parts of their Bloch wave functions are similar to $(x \pm iy)$ expressions with the orbital momentum projection $M_v = \pm 1$ on the same selected direction. The envelope parts of the Bloch wave functions have the forms of plane waves in the absence of a magnetic field. In its presence, they completely changed due to the Landau quantization event. In [17–26], the Landau quantization of the 2D electrons and holes is described in the Landau gauge and is characterized by the oscillator-type motion in one in-plane direction giving rise to discrete Landau levels enumerated by the quantum numbers n_e and n_h and by the free translation motion in another in-plane direction perpendicular to previous one. The one-dimensional (1D) plane waves describing this motion are marked by the 1D wave numbers p and q. In [18], the Landau quantization of the 2D electrons with non-parabolic dispersion law, pseudospin components and chirality terms were investigated. On this base, in [19], the influence of the Rashba spin–orbit coupling (RSOC) on the 2D magnetoexcitons was discussed. The spinor-type wave functions of the conduction and valence electrons in the presence of the RSOC have different numbers of Landau quantization levels for different spin projections. As was demonstrated in [18, 19, 22], the difference between these numbers is determined by the order of the chirality terms. Their origin is due to the influence of the external electric field applied to the layer parallel to the direction of the magnetic field. In [19], two lowest Landau levels (LLLs) of the conduction electron and four LLLs for the holes were used to calculate the matrix elements of the Coulomb interaction between the charged carriers as well as the matrix elements of the electron-radiation interaction. On these bases, the ionization potentials of the new magnetoexcitons and the probabilities of the quantum transitions from the ground state of the crystal to the magnetoexciton states were calculated. In the present description the number of the hole and magnetoexciton states will be enlarged and the formation of magnetopolaritons taking into account the RSOC will be described. A simpler version of magnetopolariton without taking into account the RSOC was described in [21] for the case of interband quantum transitions and in [23] for the case of intraband quantum transitions.

The paper is organized as follows. In section 2, the results concerning the Landau quantization of the 2D heavy holes, as well as of the electrons, in the conduction band taking into account the Rashba spin–orbit interaction were described. On this base, the Hamiltonians describing the electron-radiation interaction and of the Coulomb electron–electron interaction in the presence of the Rashba spin–orbit coupling were deduced in sections 3 and 4, respectively. Section 5 is focused on the description of the magnetoexcitons in the model of a Bose gas. In section 6, the breaking of the gauge symmetry of the obtained Hamiltonians is introduced and the mixed photon–magnetoexciton–acoustical plasmon states are discussed. Section 7 offers conclusions.

First of all, we will describe the Landau quantization of the 2D heavy holes following [19, 22].

2. Landau quantization of the 2D heavy holes

The full Landau-Rashba Hamiltonian for 2D heavy holes was discussed in [19] following formulas (13)–(20). It can be expressed through the Bose-type creation and annihilation operators $a^{\dagger}$, a acting on the Fock quantum states $\left|n\right\rangle = \frac{(a^{\dagger})^n}{\sqrt{n!}}\left|0\right\rangle$, where $\left|0\right\rangle$ is the vacuum state of the harmonic oscillator. The Hamiltonian has the form [22]

$$\hat{H}_h = \hbar\omega_{ch}\left\{\left[\left(a^{\dagger}a+\frac{1}{2}\right)+\delta\left(a^{\dagger}a+\frac{1}{2}\right)^2\right]\hat{I}+i\beta 2\sqrt{2}\begin{vmatrix}0 & (a^{\dagger})^3\\ -a^3 & 0\end{vmatrix}\right\},$$
$$\hat{I}=\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix} \tag{1}$$

with the denotations

$$\omega_{ch}=\frac{|e|H}{m_h c},\delta=\frac{|\delta_h E_z|\hbar^4}{l^4\hbar\omega_{ch}},\beta=\frac{\beta_h E_z}{l^3\hbar\omega_{ch}},l=\sqrt{\frac{\hbar c}{|e|H}}. \tag{2}$$

Parameter δ_h is not well known; therefore, different versions mentioned below were considered. The exact solutions of the Pauli-type Hamiltonian are described by formulas (21)–(31) of [19]. In more detail, they were described in [22] and have the spinor form

$$\hat{H}_h\begin{vmatrix}f_1\\ f_2\end{vmatrix}=E_h\begin{vmatrix}f_1\\ f_2\end{vmatrix}, f_1=\sum_{n=0}^{\infty}c_n\left|n\right\rangle, f_2=\sum_{n=0}^{\infty}d_n\left|n\right\rangle, \sum_{n=0}^{\infty}|c_n|^2+\sum_{n=0}^{\infty}|d_n|^2=1. \tag{3}$$

The first three solutions depend only on one quantum number m with values 0, 1, 2 as follows [6]:

$$E_h(m=0)=\hbar\omega_{ch}\left(\frac{1}{2}+\frac{\delta}{4}\right);\left|\Psi(m=0)\right\rangle=\begin{vmatrix}\left|0\right\rangle\\ 0\end{vmatrix},$$
$$E_h(m=1)=\hbar\omega_{ch}\left(\frac{3}{2}+\frac{9\delta}{4}\right);\left|\Psi(m=1)\right\rangle=\begin{vmatrix}\left|1\right\rangle\\ 0\end{vmatrix}, \tag{4}$$

$$E_h(m=2)=\hbar\omega_{ch}\left(\frac{5}{2}+\frac{25\delta}{4}\right);\left|\Psi(m=2)\right\rangle=\begin{vmatrix}\left|2\right\rangle\\0\end{vmatrix}.$$

All other solutions with $m\geq 3$ depend on two quantum numbers $(m-5/2)$ and $(m+1/2)$ and have the general expression

$$\varepsilon_h^{\pm}(m-\frac{5}{2};m+\frac{1}{2})=\frac{E_h^{\pm}(m-5/2;m+1/2)}{\hbar\omega_{ch}}=(m-1)+\frac{\delta}{8}\left[(2m+1)^2+(2m-5)^2\right]\pm$$
$$\pm(\left(\frac{3}{2}+\frac{\delta}{8}\left[(2m+1)^2-(2m-5)^2\right]\right)^2+8\beta^2 m(m-1)(m-2))^{1/2},m\geq 3. \quad (5)$$

The respective wave functions for $m=3$ and $m=4$ are

$$\left|\Psi_h^{\pm}(m=3)\right\rangle=\begin{vmatrix}c_3\left|3\right\rangle\\d_0\left|0\right\rangle\end{vmatrix}\text{and}\left|\Psi_h^{\pm}(m=4)\right\rangle=\begin{vmatrix}c_4\left|4\right\rangle\\d_1\left|1\right\rangle\end{vmatrix}. \quad (6)$$

They depend on coefficients c_m and d_{m-3}, which obey to the equations

$$c_m\left(m+\frac{1}{2}+\frac{\delta}{4}(2m+1)^2-\varepsilon_h\right)=-i\beta 2\sqrt{2}\sqrt{m(m-1)(m-2)}d_{m-3},$$
$$d_{m-3}\left(m-\frac{5}{2}+\frac{\delta}{4}(2m-5)^2-\varepsilon_h\right)=i\beta 2\sqrt{2}\sqrt{m(m-1)(m-2)}c_m, \quad (7)$$
$$|c_m|^2+|d_{m-3}|^2=1.$$

There are two different solutions $\varepsilon_h^{\pm}(m)$ at a given value of $m\geq 3$ and two different pairs of the coefficients $(c_m^{\pm},d_{m-3}^{\pm})$.

The dependences of parameters ω_{ch}, β, and δ on the electric and magnetic fields strengths may be represented for the GaAs-type quantum wells as follows $H=y\,\text{T}$; $E_z=x\,\text{kV/cm}$; $m_h=0.25m_0$; $\hbar\omega_{ch}=0.4y\,\text{meV}$; $\beta=1.062\cdot 10^{-2}x\sqrt{y}$; $\delta=10^{-4}Cxy$ with unknown parameter C, which will be varied in a larger interval of values. We cannot neglect the parameter C putting it equal to zero because, in this case, as was argued in [19] formula (10), the lower spinor branch of the heavy hole dispersion law

$$E_h^-(k_{\|})=\frac{\hbar^2\vec{k}_{\|}^2}{2m_h}-\left|\frac{\beta_h E_z}{2}\right|\left|\vec{k}_{\|}\right|^3$$

has an unlimited decreasing, deeply penetrating inside the semiconductor energy gap at great values of $\left|\vec{k}_{\|}\right|$. To avoid this unphysical situation, a positive quartic term $\left|\delta_h E_z\right|\vec{k}_{\|}^4$ was added in the starting Hamiltonian. The new dependences were compared with the drawings calculated in Fig. 2 of [19] in the case $E_z=10\,\text{kV/cm}$ and $C=10$. Four LLLs for heavy holes were selected in [19]. In addition to them, in [22], three other levels were studied as follows:

$$
\begin{aligned}
&E_h(R_1) = E_h^-(\frac{1}{2},\frac{7}{2});\\
&E_h(R_2) = E_h(m=0);\\
&E_h(R_3) = E_h^-(\frac{3}{2},\frac{9}{2});\\
&E_h(R_4) = E_h(m=1);\\
&E_h(R_5) = E_h^-(\frac{5}{2},\frac{11}{2});\\
&E_h(R_6) = E_h(m=2);\\
&E_h(R_7) = E_h^-(\frac{7}{2},\frac{13}{2}).
\end{aligned} \tag{8}
$$

Their dependences on the magnetic field strength were represented in Figs. 1 and 2 of [22] at different parameters x and C; they are reproduced below.

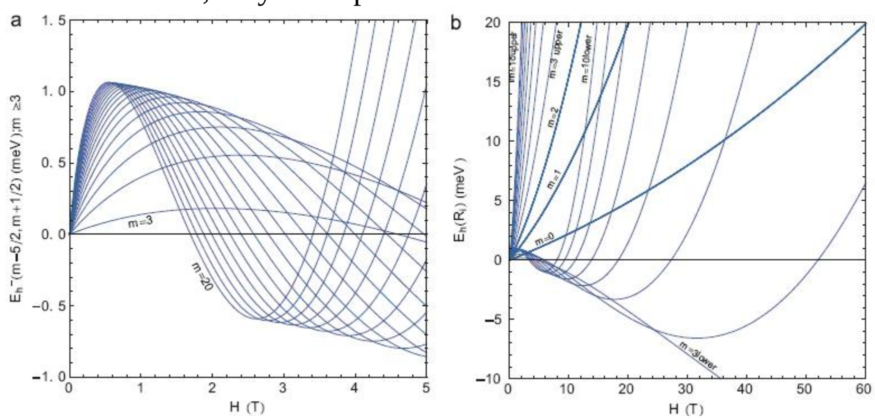

Fig. 1. (a) The lower branches of the heavy hole Landau quantization levels $E_h^-(m-5/2;m+1/2)$ for $m \geq 3$ at parameters $E_z = 10$ kV/cm and $C = 5.5$; (b) a general view of all the heavy hole Landau quantization levels with m=0,1,...,10 at the same parameters E_z and C. They are reproduced from Fig. 1 of [22].

The general view of the lower branches $E_h^-(m-\frac{5}{2},m+\frac{1}{2})$ of the heavy hole Landau quantization levels with $m \geq 3$ as a function of magnetic field strength is represented in Fig. 1a following formula (5). The upper branches exhibit a simpler monotonous behavior and are drawn together with some curves of the lower branches in Fig. 1b. All the lower branches in their initial parts have a linear increasing behavior up till they achieve the maximal values succeeded by the minimal values in the middle parts of their evolutions being transformed in the final quadratic increasing dependences. The values of the magnetic field strength corresponding to the minima and to the maxima decrease with increasing number m. These peculiarities can be compared with the case of Landau quantization of the 2D electron in the biased bilayer graphene described in [18]. The last-mentioned case is characterized by the initial dispersion law without parabolic part and by second order chirality terms. They both lead to dependences on magnetic field strength for the lower dispersion branches with sharp initial decreasing parts and minimal values succeeded by the quadratic increasing behavior. The differences between the initial dispersion laws and chirality terms in two cases of bilayer graphene and of heavy holes lead to different intersections and degeneracies of the Landau levels.

The spinor-type envelope wave functions of the heavy holes in the coordinate representation look as follows [22]:

$$\begin{aligned} &\left|\psi_h(\varepsilon_m,q;x,y)\right\rangle = \frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}\varphi_m(y+ql^2)\\ 0\end{vmatrix}, \\ &m=0,1,2. \\ &\left|\psi_h(\varepsilon_m^{\pm},q;x,y)\right\rangle = \frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}c_m^{\pm}\varphi_m(y+ql^2)\\ d_{m-3}^{\pm}\varphi_{m-3}(y+ql^2)\end{vmatrix}, \\ &m\geq 3 \end{aligned} \quad (9)$$

The valence electrons in comparison with the holes are characterized by the opposite signs of the spin projections, wave vector, and charge. The respective envelope wave functions can be obtained from the previous ones by the procedure

$$\left|\psi_v(\varepsilon,q;x,y)\right\rangle = i\hat{\sigma}_y\left|\psi_h(\varepsilon,-q;x,y)\right\rangle^* \quad (10)$$

where $\hat{\sigma}_y$ is the Pauli matrix. In coordinate representation, they are as follows:

$$\begin{aligned} &\left|\psi_v(\varepsilon_m,q;x,y)\right\rangle = \frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}0\\ -\varphi_m^*(y-ql^2)\end{vmatrix}, \\ &m=0,1,2. \\ &\left|\psi_v(\varepsilon_m^{\pm},q;x,y)\right\rangle = \frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}d_{m-3}^{\pm*}\varphi_{m-3}^*(y-ql^2)\\ -c_m^{\pm*}\varphi_m^*(y-ql^2)\end{vmatrix}, \\ &m\geq 3 \end{aligned} \quad (11)$$

To obtain the full valence electron Bloch wave functions, expressions (11) must be multiplied by the periodic parts. In the p-type valence band, they have the form $\frac{1}{\sqrt{2}}(U_{v,p,x,q}(x,y)\pm iU_{v,p,y,q}(x,y))$ and are characterized by the orbital momentum projections $M_v=\pm1$, respectively. The hole orbital projections $M_h=-M_v$ have opposite signs in comparison with the valence electron. The full Bloch wave functions of the valence electrons are now characterized by a supplementary quantum number M_v side by side with the previous ones ε_m, ${\varepsilon^{\pm}}_m$ and q as follows:

$$\begin{aligned} &\left|\psi_v(M_v,\varepsilon_m,q;x,y)\right\rangle = \frac{1}{\sqrt{2}}(u_{v,p,x,q}(\vec{r})\pm iU_{v,p,y,q}(\vec{r}))\frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}0\\ -\varphi_m^*(y-ql^2)\end{vmatrix}, \\ &m=0,1,2; M_v=\pm1, \\ &\left|\psi_v(M_v,\varepsilon_m^{\pm},q;x,y)\right\rangle = \frac{1}{\sqrt{2}}(u_{v,p,x,q}(\vec{r})\pm iU_{v,p,y,q}(\vec{r}))\frac{e^{iqx}}{\sqrt{L_x}}\begin{vmatrix}d_{m-3}^{\pm*}\varphi_{m-3}^*(y-ql^2)\\ -c_m^{\pm*}\varphi_m^*(y-ql^2)\end{vmatrix}, \\ &m\geq 3, M_v=\pm1. \end{aligned} \quad (12)$$

From this multitude of valence electron wave functions, the more important of them are characterized by the values of ε_m^- with $m=3$ and 4 as well as by ε_m with $m=0,1$. These four lowest hole energy levels being combined with two projections $M_h \pm 1$ form a set of eight lowest hole states, which will be taken into account below.

Now, for completeness, we will remember the main results obtained by Rashba [1] in the case of the electron conduction band. They are required to obtain a full description of the 2D e–h pair and of a 2D magnetoexciton under the condition of the Landau quantization under the influence of the RSOC.

The LLL of the conduction electron in the presence of the RSOC was obtained in [1]:

$$\left|\Psi_{\mathrm{e}}\left(R_1, p; x_e, y_e\right)\right\rangle = \frac{\mathrm{e}^{ipx_{\mathrm{e}}}}{\sqrt{L_x}}\begin{vmatrix} a_0\varphi_0(y_{\mathrm{e}}). \\ b_1\varphi_1(y_{\mathrm{e}}) \end{vmatrix};$$

$$\varepsilon_{\mathrm{e}R_1} = 1 - \sqrt{\frac{1}{4} + 2\alpha^2}; |a_0|^2 + |b_1|^2 = 1 \tag{13}$$

$$|a_0|^2 = \frac{1}{1 + \dfrac{2\alpha^2}{\left[\dfrac{1}{2} + \sqrt{\dfrac{1}{4} + 2\alpha^2}\right]^2}}; |b_1|^2 = \frac{2\alpha^2 |a_0|^2}{\left[\dfrac{1}{2} + \sqrt{\dfrac{1}{4} + 2\alpha^2}\right]^2}.$$

The next electron level higher situated on the energy scale is characterized by the pure spin oriented state

$$\left|\Psi_{\mathrm{e}}\left(R_2, p; x_{\mathrm{e}}, y_{\mathrm{e}}\right)\right\rangle = \frac{\mathrm{e}^{ipx_{\mathrm{e}}}}{\sqrt{L_x}}\begin{vmatrix} 0 \\ \varphi_0(y_{\mathrm{e}}) \end{vmatrix}; \varepsilon_{\mathrm{e}R_2} = \frac{1}{2}. \tag{14}$$

Two LLLs for conduction electron are characterized by the values of $m_{\mathrm{e}} = 0.067 m_0$, $\hbar\omega_{\mathrm{ce}} = 1.49\ \mathrm{meV}\cdot y$ and parameter $\alpha = 8{\cdot}10^{-3} x / \sqrt{y}$. They are denoted as

$$E_{\mathrm{e}}(R_1) = \hbar\omega_{\mathrm{ce}}\left(1 - \sqrt{\frac{1}{4} + 2\alpha^2}\right),$$
$$E_{\mathrm{e}}(R_2) = \hbar\omega_{\mathrm{ce}}\frac{1}{2}. \tag{15}$$

The lowest Landau energy level for electron $E_{\mathrm{e}}(R_1)$ has a nonmonotonous anomalous dependence on magnetic field strength near the point $H = 0\,\mathrm{T}$. It is due to the singular dependence of the RSOC parameter $\alpha^2 = 6.4{\cdot}10^{-5} x^2 / y$, which, in the total energy level expression, is compensated for by factor $\hbar\omega_{\mathrm{ce}}$ of the cyclotron energy, where $\hbar\omega_{\mathrm{ce}} = 1.49\, y$ meV. As earlier, parameters x and y are related with electric field $E_z = x$ kV/cm and magnetic field $H = y$ T. The second electron Landau energy level has a simple linear dependence on H.

The full Bloch wave functions for conduction electrons including their s-type periodic parts look as follows:

$$\left|\psi_c(s, R_1, p; x, y)\right\rangle = U_{c,s,p}(\vec{r})\frac{e^{ipx}}{\sqrt{L_x}}\begin{vmatrix} a_0\varphi_0(y - pl^2) \\ b_1\varphi_1(y - pl^2) \end{vmatrix},$$
$$\left|\psi_c(s, R_2, p; x, y)\right\rangle = U_{c,s,p}(\vec{r})\frac{e^{ipx}}{\sqrt{L_x}}\begin{vmatrix} 0 \\ \varphi_0(y - pl^2) \end{vmatrix}. \tag{16}$$

Two lowest Rashba-type states for conduction electron will be combined with eight LLLs for heavy holes and with the corresponding states of the valence electrons. The e–h pair will be characterized by 16 states. Heaving the full set of the electron Bloch wave functions in conduction and in the valence bands one can construct the Hamiltonian describing in second quantization representation the Coulomb electron-electron interaction as well as the electron-radiation interaction. These tow tasks will be described in the next sections of our review paper. The results obtained earlier in [19, 22] taking into account only 8 e–h states will be supplemented below.

3. Electron-radiation interaction in the presence of the Rashba spin-orbit coupling

In [17, 21] the Hamiltonian of the electron-radiation interaction in the second quantization representation for the case of two-dimensional (2D) coplanar e–h system in a strong perpendicular magnetic field was discussed. The s-type conduction-band electrons with spin projections $s_z = \pm 1/2$ along the magnetic field direction and the heavy holes with the total momentum projections $j_z = \pm 3/2$ in the p-type valence band were taken into account. Their orbital Bloch wave functions are similar to $(x \pm iy)$ expressions with the orbital momentum projections $M = \pm 1$ on the same selected direction. The Landau quantization of the 2D electrons and holes was described in the Landau gauge with oscillator type motion in one in-plane direction characterized by the quantum numbers n_e and n_h and with the free translational motion described by the uni-dimensional(1D) wave numbers p and q in another in-plane direction perpendicular to the previous one. The electron and hole creation and annihilation operators $a^{+}_{s_z,n_e,p}$, $a_{s_z,n_e,p}$, and $b^{+}_{j_z,n_h,q}$, $b_{j_z,n_h,q}$ were introduced correspondingly. The Zeeman effect and the Rashba spin–orbit coupling in [17, 21] were not taken into account.

The electrons and holes have a free orbital motion on the surface of the layer with the area S and are completely confined in $\vec{a}_3$ direction. The degeneracy of their Landau levels equals $N = S/(2\pi l_0^2)$, where l_0 is the magnetic length. In contrast, the photons were supposed to move in any direction in the three-dimensional (3D) space with the wave vector $\vec{k}$ arbitrary oriented as regards the 2D layer as it is represented in the Fig.2 reproduced from [17]. There are three unit vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, the first two being in-plane oriented, whereas the third $\vec{a}_3$ is perpendicular to the layer. We will use the 3D and 2D wave vectors $\vec{k}$ and $\vec{k}_{\parallel}$ and will introduce circular polarization vectors $\vec{\sigma}_M$ for the valence electrons, heavy holes, and magnetoexcitons as follows:

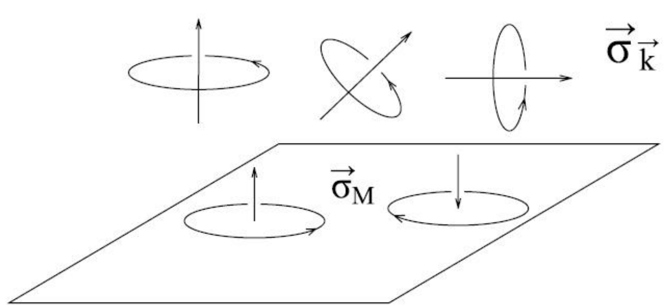

Fig. 2. Reciprocal orientations of circularly polarized vectors $\vec{\sigma}_{\vec{k}}$ and $\vec{\sigma}_M$ reproduced from [17].

$$\vec{k}=\vec{k}_{\parallel}+\vec{a}_3k_z;\vec{k}_{\parallel}=\vec{a}_1k_x+\vec{a}_2k_y;\vec{\sigma}_M=\frac{1}{\sqrt{2}}(\vec{a}_1\pm i\vec{a}_2);M=\pm1. \tag{17}$$

The photons are characterized by two linear vectors $\vec{e}_{k,j}$ or by two circular polarization vectors $\vec{\sigma}_{\vec{k}}^{\pm}$ obeying the transversality conditions:

$$\vec{\sigma}_{\vec{k}}^{\pm}=\frac{1}{\sqrt{2}}(\vec{e}_{\vec{k},1}\pm i\vec{e}_{\vec{k},2});(\vec{e}_{k,j}\cdot\vec{k})=0;\ j=1,2. \tag{18}$$

The photon creation and annihilation operators can be introduced in two different polarizations as follows:

$$\begin{aligned}&C_{\vec{k},\pm}=\frac{1}{\sqrt{2}}\left(C_{\vec{k},1}\pm iC_{\vec{k},2}\right);\left(C_{\vec{k},\pm}\right)^{\dagger}=\frac{1}{\sqrt{2}}\left(C_{\vec{k},1}^{\dagger}\mp iC_{\vec{k},2}^{\dagger}\right);\\&\sum_{j=1}^{2}\vec{e}_{\vec{k},j}C_{\vec{k},j}=C_{\vec{k},-}\vec{\sigma}_{\vec{k}}^{+}+C_{\vec{k},+}\vec{\sigma}_{\vec{k}}^{-};\\&\sum_{j=1}^{2}\vec{e}_{\vec{k},j}C_{\vec{k},j}^{\dagger}=\left(C_{\vec{k},-}\right)^{\dagger}\vec{\sigma}_{\vec{k}}^{-}+\left(C_{\vec{k},+}\right)^{\dagger}\vec{\sigma}_{\vec{k}}^{+}.\end{aligned} \tag{19}$$

The reciprocal orientations of circular polarizations $\vec{\sigma}_{\vec{k}}^{\pm}$ and $\vec{\sigma}_M$ will determine the values of scalar products $(\vec{\sigma}_{\vec{k}}^{\pm}\cdot\vec{\sigma}_M^{*})$. The electron-radiation interaction describing only the band-to-band quantum transitions with the participation of the e–h pairs in the presence of a strong perpendicular magnetic field was obtained in [17] and can be used as initial expression for obtaining the interaction of 2D magnetoexcitons with the electromagnetic field.

These results will be generalized below taking into account the Rashba spin–orbit coupling, which means the use of the spinor-type wave functions (12) and (16) instead of the scalar ones [17, 21]. The Hamiltonian looks as

$$\begin{aligned}&H_{e-rad}=\left(-\frac{e}{m_0}\right)\sum_{\vec{k}(\vec{k}_{\parallel},k_z)}\sqrt{\frac{2\pi\hbar}{V\omega_{\vec{k}}}}\sum_{i=1,2}\sum_{M_v=\pm1}\sum_{\varepsilon=\varepsilon_m,\varepsilon_m^-}\sum_{g,q}\{[C_{\vec{k},-}\vec{\sigma}_{\vec{k}}^{+}+C_{\vec{k},+}\vec{\sigma}_{\vec{k}}^{-}]\bullet\\&\bullet[\vec{P}(c,R_i,g;v,M_v,\varepsilon,q;\vec{k})a_{c,R_i,g}^{\dagger}a_{v,M_v,\varepsilon,q}+\vec{P}(v,M_v,\varepsilon,q;c,R_i,g;\vec{k})a_{v,M_v,\varepsilon,q}^{\dagger}a_{c,R_i,g}]+\\&+[(C_{\vec{k},-})^{\dagger}\vec{\sigma}_{\vec{k}}^{-}+(C_{\vec{k},+})^{\dagger}\vec{\sigma}_{\vec{k}}^{+}]\bullet[\vec{P}(c,R_i,g;v,M_v,\varepsilon,q;-\vec{k})\times\\&\times a_{c,R_i,g}^{\dagger}a_{v,M_v,\varepsilon,q}+\vec{P}(v,M_v,\varepsilon,q;c,R_i,g;-\vec{k})a_{v,M_v,\varepsilon,q}^{\dagger}a_{c,R_i,g}]\}\end{aligned} \tag{20}$$

The matrix elements will be discussed below. One of them has the form

$$\begin{aligned}&\vec{P}(c,R_1,g;v,M_v,\varepsilon^-{}_m,q;\vec{k})=\int d^2\vec{r}\left\langle\hat{\psi}_{c,R_i,g}(\vec{r})\right|e^{i\vec{k}\vec{r}}\hat{\vec{p}}\left|\hat{\psi}_{v,M_v,\varepsilon^-{}_m,q}(\vec{r})\right\rangle=\\&\frac{a_0^{*}\bar{d}_{m-3}^{*}}{\sqrt{2L_x}}\int d^2\vec{r}U_{c,s,g}^{*}(\vec{r})e^{-igx}\varphi_0^{*}(y-gl^2)e^{i\vec{k}\vec{r}}(\vec{a}_1\hat{p}_x+\vec{a}_2\hat{p}_y)(U_{v,p,x,q}(\vec{r})\pm iU_{v,p,y,q}(\vec{r}))e^{iqx}\varphi_{m-3}^{*}(y-pl^2)-\\&-\frac{b_1^{*}\bar{c}_m^{*}}{\sqrt{2L_x}}\int d^2\vec{r}U_{c,s,g}^{*}(\vec{r})e^{-igx}\varphi_1^{*}(y-gl^2)e^{i\vec{k}\vec{r}}(\vec{a}_1\hat{p}_x+\vec{a}_2\hat{p}_y)(U_{v,p,x,q}(\vec{r})\pm iU_{v,p,y,q}(\vec{r}))e^{iqx}\varphi_m^{*}(y-pl^2)\end{aligned} \tag{21}$$

One can represent the 2D coordinate vector $\vec{r}$ as a sum $\vec{r}=\vec{R}+\vec{\rho}$ of lattice point vector $\vec{R}$ and small vector $\vec{\rho}$ changing inside the unit lattice cell with lattice constant a_0 and volume $v_0=a_0^3$.

Any 2D semiconductor layer has at least minimal width a_0 and periodic parts $U_{nk}(\vec{r})$ are determined inside the elementary lattice cell. Periodic parts $U_{nk}(\vec{r})$ do not depend on $\vec{R}$ because $U_{nk}(\vec{R}+\vec{\rho})=U_{nk}(\vec{\rho})$. On the other hand, envelope functions $\varphi_n(\vec{r})$ describing the Landau quantization have a spread of the order of magnetic length l_0 which is much greater than a_0 ($l_0 >> a_0$). It means that they hardly depend on $\vec{\rho}$, i.e., $\varphi_n(\vec{R}+\vec{\rho}) \cong \varphi_n(\vec{R})$. The matrix elements (21) contains some functions that do not depend on $\vec{R}$ and other ones that do not depend on $\vec{\rho}$. Only the plane wave $e^{i\vec{k}\vec{r}} = e^{i\vec{k}\vec{R}+i\vec{k}\vec{\rho}}$ contains both of them. Derivative $\frac{\partial}{\partial \vec{r}}$ acts in the same manner on functions $U_{nk}(\vec{\rho})$ and $\varphi_n(\vec{R})$ because $\vec{R}$ and $\vec{\rho}$ are the components of $\vec{r}$. These properties suggested transforming the 2D integral on variable $\vec{r}$ in two separate integrals on variables $\vec{R}$ and $\vec{\rho}$ as follows:

$$\int d^2\vec{r} A(\vec{R})B(\vec{\rho}) = \sum_{\vec{R}} A(\vec{R})\int d^2\vec{\rho}B(\vec{\rho}) = \sum_{\vec{R}} A(\vec{R})a_0^2 \frac{1}{v_0}\int d^3\vec{\rho}B(\vec{\rho}) = \int d^2\vec{R}A(\vec{R})\frac{1}{v_0}\int_{v_0} d\vec{\rho}B(\vec{\rho}) \qquad (22)$$

Here the small value of a_0^2 is substituted by the infinitesimal differential value $d^2\vec{R}$ because $A(\vec{R})$ is a smooth function on $\vec{R}$. The integrals on te volume v_0 of the elementary lattice cell contain the quickly oscillating periodic parts $U_{c,s,g}(\vec{\rho})$ and $U_{v,p,i,q}(\vec{\rho})$ belonging to s-type conduction band and to p-type valence band. They have different parities and obey to selection rules

$$\begin{aligned} &\frac{1}{v_0}\int_{v_0} d\vec{\rho}U^*_{c,s,g}(\vec{\rho})e^{ik_y\rho_y}U_{v,p,i,q}(\vec{\rho}) = 0, \ \ i,j = x,y \\ &\frac{1}{v_0}\int_{v_0} d\vec{\rho}U^*_{c,s,g}(\vec{\rho})e^{ik_y\rho_y}\frac{\partial}{\partial\rho_i}U_{v,p,j,q}(\vec{\rho}) = 0, \ \ \text{if } i \neq j \end{aligned} \qquad (23)$$

The case $i = j$ is different from zero and gives rise to the expression

$$\frac{1}{v_0}\int_{v_0} d\vec{\rho}U^*_{c,s,g}(\vec{\rho})e^{ik_y\rho_y}\frac{\partial}{\partial\rho_i}U_{v,pi,g-k_x}(\vec{\rho}) = P_{cv}(\vec{k}_{\|}, g) \qquad (24)$$

The last integral in the zeroth approximation is of the allowed type in the definition of Elliott [26, 27] and can be considered as a constant $P_{cv}(\vec{k}_{\|}, g) \approx P_{cv}(0)$ which does not depend on wave vectors $\vec{k}_{\|}$ and g. Due to these selection rules, derivatives $\partial / \partial\vec{r}$ in expression (21) must be taken only from the periodic parts $U_{v,p,i,q}(\vec{\rho})$ because all other integrals vanish.

The integration on variable R_x engages only the plane wave functions and gives rise to the selection rule for the 1D wave numbers g, q, k_x as follows:

$$\frac{1}{L_x}\int e^{iR_x(q-g+k_x)} = \frac{2\pi}{L_x}\delta(q-g+k_x) = \delta_{kr}(q, g-k_x) \qquad (25)$$

The integral on variable R_y engages only Landau quantization functions $\varphi_n(R_y)$ and gives rise to the third selection rule concerning numbers n_e and n_h of the Landau levels for electrons and holes. It looks as

$$\int_{-\infty}^{\infty} dR_y \varphi_{n_e}^*(R_y - gl_0^2)\varphi_{n_h}^*(R_y - (g-k_x)l_0^2)e^{ik_yR_y} = e^{ik_ygl_0^2}e^{-i\frac{k_xk_y}{2}l_0^2}\tilde{\phi}(n_e,n_h;\vec{k}_\parallel), \quad (26)$$

where

$$\tilde{\phi}(n_e,n_h;\vec{k}_\parallel) = \int_{-\infty}^{\infty} dy \varphi_{n_e}(y - \frac{k_xl_0^2}{2})\varphi_{n_h}(y + \frac{k_xl_0^2}{2})e^{ik_yy},$$

$$e^{-i\frac{k_xk_y}{2}l_0^2}\tilde{\phi}(n_e,n_h;\vec{k}_\parallel) = \phi(n_e,n_h;\vec{k}_\parallel).$$

Here, we took into account that Landau quantization functions $\varphi_n(y)$ are real.

This selection rule coincides with formula (30) in the absence of the RSOC, and its interpretation remains the same. Once again one can underlain that, during the dipole-active band-to-band quantum transition, the numbers of the Landau levels in the initial starting band, as well as in the final arriving band, coincide, i.e., $n_e = n_h$. It is equally true both in the absence and in the presence of the RSOC.

Three separate integrations on $\vec{\rho}, R_x$ and R_y taking into account selection rules (23), (25), and (26) lead to the expression

$$\begin{aligned}&\vec{P}(c,R_1,g;v,M_v,\varepsilon_m^-,q;\vec{k}_\parallel) = \delta_{kr}(q,g-k_x)\vec{\sigma}_{M_v}P_{cv}(0)e^{ik_ygl_0^2} \times \\ &\times e^{-i\frac{k_xk_y}{2}l_0^2}[a_0^*d_{m-3}^{-*}\tilde{\phi}(0,m-3;\vec{k}_\parallel) - b_1^*c_m^{-*}\tilde{\phi}(1,m;\vec{k}_\parallel)], \\ &m \geq 3\end{aligned} \quad (27)$$

Here, the vectors of circular polarizations $\vec{\sigma}_{M_v}$ describing the valence electron states were introduced following formula (17). One can introduce also the vectors of the heavy hole circular polarizations $\vec{\sigma}_{M_h}$ in the form

$$\begin{aligned}&\vec{\sigma}_{M_v} = \frac{1}{\sqrt{2}}(\vec{a}_1 \pm i\vec{a}_2), \quad M_v = \pm 1, \\ &\vec{\sigma}_{M_h} = \vec{\sigma}_{M_v}^* = \frac{1}{\sqrt{2}}(\vec{a}_1 \mp i\vec{a}_2), \quad M_h = \mp 1\end{aligned} \quad (28)$$

The magnetoexciton states are characterized by quantum numbers M_h, R_i, ε and by wave vectors $\vec{k}_\parallel$.

The general expressions for the matrix elements are as follows:

$$\begin{aligned}&\vec{P}(c,R_i,g;v,M_v,\varepsilon,q;\vec{k}_\parallel) = \delta_{kr}(q,g-k_x)\vec{\sigma}_{M_v}P_{cv}(0)e^{ik_ygl_0^2}T(cR_i,\varepsilon;\vec{k}_\parallel)e^{-i\frac{k_xk_y}{2}l_0^2}, \\ &i=1,2,\ \mathrm{M}_v = \pm 1,\ \varepsilon = \varepsilon_m,\ \text{with } m = 0,1,2,\ \text{and } \varepsilon = \varepsilon_m^-,\ \text{with } m \geq 3\end{aligned} \quad (29)$$

Coefficients $T(cR_i,\varepsilon;\vec{k}_\parallel)$ have the forms

$$\begin{aligned}
&T(R_1,\varepsilon_m^-;\vec{k}_\parallel)=[a_0^* d_{m-3}^{-*}\tilde{\phi}(0,m-3;\vec{k}_\parallel)-b_1^* c_m^{-*}\tilde{\phi}(1,m;\vec{k}_\parallel)],\\
&m\ge 3,\\
&T(R_1,\varepsilon_m;\vec{k}_\parallel)=[-b_1^*\tilde{\phi}(1,m;\vec{k}_\parallel)],\\
&m=0,1,2,\\
&T(R_2,\varepsilon_m^-;\vec{k}_\parallel)=[-c_m^{-*}\tilde{\phi}(0,m;\vec{k}_\parallel)],\\
&m\ge 3,\\
&T(R_2,\varepsilon_m;\vec{k})=[-\tilde{\phi}(0,m;\vec{k}_\parallel)],\\
&m=0,1,2
\end{aligned} \tag{30}$$

The other matrix elements can be calculated in a similar way. They are

$$\begin{aligned}
&\vec{P}(v,M_v,\varepsilon,q;c,R_i,g;-\vec{k}_\parallel)=\vec{P}^*(c,R_i,g;v,M_v,\varepsilon,q;\vec{k}_\parallel)=\\
&=\delta_{kr}(q,g-k_x)\vec{\sigma}^*_{M_v}P^*_{cv}(0)e^{-ik_y g l_0^2}e^{i\frac{k_x k_y}{2}l_0^2}T^*(R_i,\varepsilon;\vec{k}_\parallel),\\
&\vec{P}(c,R_i,g;v,M_v,\varepsilon,q;-\vec{k}_\parallel)=\delta_{kr}(q,g+k_x)\vec{\sigma}_{M_v}P_{cv}(0)e^{-ik_y g l_0^2}e^{-i\frac{k_x k_y}{2}l_0^2}T(R_i,\varepsilon;-\vec{k}_\parallel),\\
&\vec{P}(v,M_v,\varepsilon,q;c,R_i,g;\vec{k}_\parallel)=\delta_{kr}(q,g+k_x)\vec{\sigma}^*_{M_v}P^*_{cv}(0)e^{ik_y g l_0^2}e^{i\frac{k_x k_y}{2}l_0^2}T^*(R_i,\varepsilon;-\vec{k}_\parallel)
\end{aligned} \tag{31}$$

They permit calculating the electron operator parts in Hamiltonian (20) and expressing them in terms of the magnetoexciton creation and annihilation operators determined as follows:

$$\hat{\psi}^\dagger_{ex}(\vec{k}_\parallel,M_h,R_i,\varepsilon)=\frac{1}{\sqrt{N}}\sum_t e^{ik_y t l_0^2}a^\dagger_{R_i,\frac{k_x}{2}+t}b^\dagger_{M_h,\varepsilon,\frac{k_x}{2}-t} \tag{32}$$

Here, he electron and hole creation and annihilation operator were introduced

$$\begin{aligned}
&a_{R_i,g}=a_{c,R_i,g},\\
&a_{v,M_v,\varepsilon,q}=b^\dagger_{-M_h,\varepsilon,-q}
\end{aligned} \tag{33}$$

Here, we have supposed that the Coulomb electron–hole interaction leading to the formation of the magnetoexciton is greater than the magnetoexciton–photon interaction leading to the formation of the magnetopolariton. It means that the ionization potential of the magnetoexciton I_l is greater than the Rabi energy $\hbar|\omega_R|\approx\frac{|e|}{m_0 l_0}|P_{cv}(0)|\sqrt{\frac{\hbar}{L_z\omega_{\vec{k}}}}$. It was determined in [21].

The existence of the phase factors of the type $e^{\pm ik_y g l_0^2}$ in expressions (29) and (31) similar with that appearing in the definitions of the magnetoexciton creation operators permits obtaining the expressions

$$\begin{aligned}
&\sum_{q,g} \vec{P}(c,R_i,g;v,M_v,\varepsilon,q;\vec{k}_\parallel)a^\dagger_{c,R_i,g}a_{v,M_v,\varepsilon,q} = \\
&= \vec{\sigma}^*_{M_h}P_{cv}(0)T(R_i,\varepsilon;\vec{k}_\parallel)\sqrt{N}\hat{\psi}^\dagger_{ex}(\vec{k}_\parallel,M_h,R_i,\varepsilon), \\
&\sum_{q,g} \vec{P}(v,M_v,\varepsilon,q;c,R_i,g;-\vec{k}_\parallel)a^\dagger_{v,M_v,\varepsilon,q}a_{c,R_i,g} = \\
&= \vec{\sigma}_{M_h}P^*_{cv}(0)T^*(R_i,\varepsilon;\vec{k}_\parallel)\sqrt{N}\hat{\psi}_{ex}(\vec{k}_\parallel,M_h,R_i,\varepsilon), \\
&\sum_{q,g} \vec{P}(c,R_i,g;v,M_v,\varepsilon,q;-\vec{k}_\parallel)a^\dagger_{c,R_i,g}a_{v,M_v,\varepsilon,q} = \\
&= \vec{\sigma}^*_{M_h}P_{cv}(0)T(R_i,\varepsilon;-\vec{k}_\parallel)\sqrt{N}\hat{\psi}^\dagger_{ex}(-\vec{k}_\parallel,M_h,R_i,\varepsilon), \\
&\sum_{q,g} \vec{P}(v,M_v,\varepsilon,q;c,R_i,g;\vec{k}_\parallel)a^\dagger_{v,M_v,\varepsilon,q}a_{c,R_i,g} = \\
&= \vec{\sigma}_{M_h}P^*_{cv}(0)T^*(R_i,\varepsilon;-\vec{k}_\parallel)\sqrt{N}\hat{\psi}_{ex}(-\vec{k}_\parallel,M_h,R_i,\varepsilon),
\end{aligned} \tag{34}$$

In [21], the Hamiltonian of the electron-radiation interaction was deduced in the absence of the RSOC. In its presence, the mentioned Hamiltonian also can be expressed in a compact form in terms of the photon and magnetoexciton creation and annihilation operators. As earlier, we introduced the values $N = S/2\pi l_0^2$, $V = SL_z$, where L_z is the size of the 3D space in direction perpendicular to the layer. In the case of microcavity L_z equals cavity length L_c. The electron-radiation interaction has the form

$$\begin{aligned}
&\hat{H}_{e-rad} = \left(-\frac{e}{m_0 l_0}\right)\sum_{\vec{k}(\vec{k}_\parallel,k_z)}\sum_{M_h=\pm1}\sum_{i=1,2}\sum_{\varepsilon=\varepsilon_m,\varepsilon_m^-}\sqrt{\frac{\hbar}{L_z\omega_{\vec{k}}}}\times \\
&\times\{P_{cv}(0)T\left(R_i,\varepsilon,\vec{k}_\parallel\right)[C_{\vec{k},-}\left(\vec{\sigma}^+_{\vec{k}}\cdot\vec{\sigma}^*_{M_h}\right)+C_{\vec{k},+}\left(\vec{\sigma}^-_{\vec{k}}\cdot\vec{\sigma}^*_{M_h}\right)]\hat{\Psi}^\dagger_{ex}(\vec{k}_\parallel,M_h,R_i,\varepsilon)+ \\
&+P^*_{cv}(0)T^*\left(R_i,\varepsilon,\vec{k}_\parallel\right)[\left(C_{\vec{k},-}\right)^\dagger\left(\vec{\sigma}^-_{\vec{k}}\cdot\vec{\sigma}_{M_h}\right)+\left(C_{\vec{k},+}\right)^\dagger\left(\vec{\sigma}^+_{\vec{k}}\cdot\vec{\sigma}_{M_h}\right)]\hat{\Psi}_{ex}(\vec{k}_\parallel,M_h,R_i,\varepsilon)+ \\
&+P_{cv}(0)T\left(R_i,\varepsilon,-\vec{k}_\parallel\right)[\left(C_{\vec{k},-}\right)^\dagger\left(\vec{\sigma}^-_{\vec{k}}\cdot\vec{\sigma}^*_{M_h}\right)+\left(C_{\vec{k},+}\right)^\dagger\left(\vec{\sigma}^+_{\vec{k}}\cdot\vec{\sigma}^*_{M_h}\right)]\hat{\Psi}^\dagger_{ex}(-\vec{k}_\parallel,M_h,R_i,\varepsilon)+ \\
&+P^*_{cv}(0)T^*\left(R_i,\varepsilon,-\vec{k}_\parallel\right)[C_{\vec{k},-}\left(\vec{\sigma}^+_{\vec{k}}\cdot\vec{\sigma}_{M_h}\right)+C_{\vec{k},+}\left(\vec{\sigma}^-_{\vec{k}}\cdot\vec{\sigma}_{M_h}\right)]\hat{\Psi}_{ex}(-\vec{k}_\parallel,M_h,R_i,\varepsilon)\}
\end{aligned} \tag{35}$$

This expression is similar to the Hamiltonian in the absence of the RSOC. Now the Coulomb interaction between charged carriers in the presence of the RSOC will be investigated.

4. The Coulomb interaction in the 2D electron-hole system under the influence of the Rashba spin-orbit coupling

The Coulomb interaction in the 2D e–h system taking into account the Rashba spin−orbit coupling was discussed in [19, 22]. Below, we will remind these results including all valence electron states (12). In the present description, the multicomponent electron field contains a larger variety of the valence band states than in [19, 22]. For the very beginning, the properties of the density operator of electron field $\hat{\rho}(\vec{r})$ and of its Fourier components $\hat{\rho}(\vec{Q})$ will be discussed. To this end, the Fermi-type creation and annihilation operators of the electron on different states were introduced. They are denoted as $a^{\dagger}_{R_i,g}, a_{R_i,g}$ for the conduction band Rashba-type states (16) $\left|\psi_c(R_i,g;r)\right\rangle$, as $a^{\dagger}_{M_v,\varepsilon_m,g}, a_{M_v,\varepsilon_m,g}$ for the spinor valence band states (12) $\left|\psi_v(M_v,\varepsilon_m,g;r)\right\rangle$ and as $a^{\dagger}_{M_v,\varepsilon_m^-,g}, a_{M_v,\varepsilon_m^-,g}$ for other spinor valence band states (12) $\left|\psi_v(M_v,\varepsilon_m^-,g;r)\right\rangle$. These spinor-type functions have a form of a column with two components corresponding to two spin projections on the direction of the magnetic field. The conjugate functions $\left\langle\psi_c(R_i,g;r)\right|$, $\left\langle\psi_c(R_i,g;r)\right|$ and $\left\langle\psi_v(M_v,\varepsilon_m^-,g;r)\right|$ have a form of a row with two components conjugate to the components of the columns. With the aid of the electron creation and annihilation operators and the spinor-type wave functions, creation and annihilation operators $\hat{\Psi}^{\dagger}(r)$ and $\hat{\Psi}(r)$ of the multi-component electron field can be written as

$$\begin{aligned}
\hat{\Psi}(r) &= \sum_{i=1,2}\sum_{g} \left|\psi_c(R_i,g;r)\right\rangle a_{R_i,g} + \sum_{M_v}\sum_{\varepsilon_m}\sum_{g} \left|\psi_v(M_v,\varepsilon_m,g;r)\right\rangle a_{M_v,\varepsilon_m,g} + \\
&+ \sum_{M_v}\sum_{\varepsilon_m^-}\sum_{g} \left|\psi_v(M_v,\varepsilon_m^-,g;r)\right\rangle a_{M_v,\varepsilon_m^-,g}, \\
\hat{\Psi}^{\dagger}(r) &= \sum_{i=1,2}\sum_{g} \left\langle\psi_c(R_i,g;r)\right| a^{\dagger}_{R_i,g} + \sum_{M_v}\sum_{\varepsilon_m}\sum_{g} \left\langle\psi_v(M_v,\varepsilon_m,g;r)\right| a^{\dagger}_{M_v,\varepsilon_m,g} + \\
&+ \sum_{M_v}\sum_{\varepsilon_m^-}\sum_{g} \left\langle\psi_v(M_v,\varepsilon_m^-,g;r)\right| a^{\dagger}_{M_v,\varepsilon_m^-,g}
\end{aligned} \tag{36}$$

The density operator of electron field $\hat{\rho}(\vec{r})$ and its Fourier components $\hat{\rho}(\vec{Q})$ are determined by the expressions

$$\begin{aligned}
\hat{\rho}(\vec{r}) &= \hat{\Psi}^{\dagger}(r)\hat{\Psi}(r), \\
\hat{\rho}(\vec{Q}) &= \int d^2\vec{r}\,\hat{\rho}(\vec{r})e^{i\vec{Q}\vec{r}}
\end{aligned} \tag{37}$$

The density operator looks as

$$\hat{\rho}(\vec{r}) = \sum_{i,j=1,2}\sum_{g,q} a^{\dagger}_{R_j,q}a_{R_i,g}\left\langle\psi_c(R_j,q;r)\,|\,\psi_c(R_i,g;r)\right\rangle +$$
$$+\sum_{M_v,M'_v}\sum_{\varepsilon_m,\varepsilon_{m'}}\sum_{g,q} a^{\dagger}_{M'_v,\varepsilon_{m'},q}a_{M_v,\varepsilon_m,g}\left\langle\psi_v(M'_v,\varepsilon_{m'},q;r)\middle|\psi_v(M_v,\varepsilon_m,g;r)\right\rangle +$$
$$+\sum_{M_v,M'_v}\sum_{\varepsilon^-_m,\varepsilon^-_{m'}}\sum_{g,q} a^{\dagger}_{M'_v,\varepsilon^-_{m'},q}a_{M_v,\varepsilon^-_m,g}\left\langle\psi_v(M'_v,\varepsilon^-_{m'},q;r)\middle|\psi_v(M_v,\varepsilon^-_m,g;r)\right\rangle +$$
$$+\sum_{M_v,M'_v}\sum_{\varepsilon_{m'},\varepsilon^-_m}\sum_{g,q} a^{\dagger}_{M'_v,\varepsilon_{m'},q}a_{M_v,\varepsilon^-_m,g}\left\langle\psi_v(M'_v,\varepsilon_{m'},q;r)\middle|\psi_v(M_v,\varepsilon^-_m,g;r)\right\rangle +$$
$$+\sum_{M_v,M'_v}\sum_{\varepsilon^-_{m'},\varepsilon_m}\sum_{g,q} a^{\dagger}_{M'_v,\varepsilon^-_{m'},q}a_{M_v,\varepsilon_m,g}\left\langle\psi_v(M'_v,\varepsilon^-_{m'},q;r)\middle|\psi_v(M_v,\varepsilon_m,g;r)\right\rangle +$$
$$+\sum_{i=1,2}\sum_{M_v}\sum_{\varepsilon_m}\sum_{g,q} a^{\dagger}_{R_i,q}a_{M_v,\varepsilon_m,g}\left\langle\psi_c(R_i,q;r)\,|\,\psi_v(M_v,\varepsilon_m,g;r)\right\rangle +$$
$$+\sum_{i=1,2}\sum_{M_v}\sum_{\varepsilon^-_m}\sum_{g,q} a^{\dagger}_{R_i,q}a_{M_v,\varepsilon^-_m,g}\left\langle\psi_c(R_i,q;r)\,|\,\psi_v(M_v,\varepsilon^-_m,g;r)\right\rangle +$$
$$+\sum_{i=1,2}\sum_{M_v}\sum_{\varepsilon_m}\sum_{g,q} a^{\dagger}_{M_v,\varepsilon_m,q}a_{R_i,g}\left\langle\psi_v(M_v,\varepsilon_m,q;r)\,|\,\psi_c(R_i,g;r)\right\rangle +$$
$$+\sum_{i=1,2}\sum_{M_v}\sum_{\varepsilon^-_m}\sum_{g,q} a^{\dagger}_{M_v,\varepsilon^-_m,q}a_{R_i,g}\left\langle\psi_v(M_v,\varepsilon^-_m,q;r)\,|\,\psi_c(R_i,g;r)\right\rangle \tag{38}$$

Fourier components $\hat{\rho}(\vec{Q})$ of the density operator determine the Coulomb interaction between the electrons. They will be calculated below taking into account spinor-type wave functions (12) and (16). For example, the first term in expressions (38) looks as

$$\hat{\rho}_{c-c}(R_1;R_1;\vec{Q}) = \sum_{q,g} a^{\dagger}_{R_1,q}a_{R_1,g}\left\langle\psi_c(R_1,q;\vec{r})\middle|\psi_c(R_1,g;\vec{r})\right\rangle =$$
$$= \sum_{q,g} a^{\dagger}_{R_1,q}a_{R_1,g}\int d^2\vec{r}\,U^*_{c,s,q}(\vec{r})U_{c,s,g}(\vec{r})\frac{e^{i(g+Q_x-q)x}}{L_x}\times \tag{39}$$
$$\times[|a_0|^2\varphi^*_0(y-ql_0^2)\varphi_0(y-gl_0^2)+|b_1|^2\varphi^*_1(y-ql_0^2)\varphi_1(y-gl_0^2)],$$
$$\vec{r}=\vec{R}+\vec{\rho}$$

Following formula (22), it is necessary to separate the integration of the quickly varying periodic parts on volume v_0 of the elementary lattice cell and the integration of the slowly varying envelope parts on lattice point vectors $\vec{R}$ as follows:

$$\frac{1}{v_0}\int_{v_0} d\rho\,U^*_{c,s,g+Q_x}(\rho)U_{c,s,g}(\rho)e^{iQ_y\rho_y} = 1+O(\vec{Q}),$$
$$\frac{1}{L_x}\int e^{i(g-q+Q_x)R_x}dR_x = \delta_{kr}(q,g+Q_x), \tag{40}$$
$$\tilde{\phi}(n,m;\vec{Q}) = \int dR_y\varphi^*_n\left(R_y-\frac{Q_xl_0^2}{2}\right)\varphi_m\left(R_y+\frac{Q_xl_0^2}{2}\right)e^{iQ_yR_y} = \tilde{\phi}^*(m,n;-\vec{Q})$$

Here $O(\vec{Q})$ is an infinitesimal value much smaller than unity, tending to zero in the limit $Q\to 0$. It will be neglected in all calculations below. The calculation gives rise to the final form

$$\hat{\rho}_{c-c}(R_1;R_1;\vec{Q}) = [|a_0|^2\tilde{\phi}(0,0;\vec{Q}) + |b_1|^2\tilde{\phi}(1,1;\vec{Q})]\hat{\rho}(R_1;R_1;\vec{Q}) = \tilde{S}(R_1;R_1;\vec{Q})\hat{\rho}(R_1;R_1;\vec{Q}),$$
$$\hat{\rho}(R_1;R_1;\vec{Q}) = \sum_t e^{iQ_y t l_0^2} a^{\dagger}_{R_1,t+\frac{Q_x}{2}} a_{R_1,t-\frac{Q_x}{2}}, \quad (41)$$
$$\tilde{S}(R_1;R_1;\vec{Q}) = [|a_0|^2\tilde{\phi}(0,0;\vec{Q}) + |b_1|^2\tilde{\phi}(1,1;\vec{Q})]$$

Expression (41) looks as a product of one numeral factor $\tilde{S}(R_1;R_1;\vec{Q})$, which concerns the concrete electron spinor state and another operator type factor of the general form

$$\hat{\rho}(\xi,\eta;\vec{Q}) = \sum_t e^{iQ_y t l_0^2} a^{\dagger}_{\xi,t+\frac{Q_x}{2}} a_{\eta,t-\frac{Q_x}{2}} = \hat{\rho}^{\dagger}(\eta,\xi;-\vec{Q}) \quad (42)$$

It will be met in all expressions listed below, but with different meanings of ξ and η, as follows:

$$\hat{\rho}_{c-c}(R_2;R_2;\vec{Q}) = \tilde{S}(R_2;R_2;\vec{Q})\hat{\rho}(R_2;R_2;\vec{Q}),$$
$$\tilde{S}(R_2;R_2;\vec{Q}) = \tilde{\phi}(0,0;\vec{Q}),$$
$$\hat{\rho}_{c-c}(R_1;R_2;\vec{Q}) = \tilde{S}(R_1;R_2;\vec{Q})\hat{\rho}(R_1;R_2;\vec{Q}),$$
$$\tilde{S}(R_1;R_2;\vec{Q}) = b_1^*\tilde{\phi}(1,0;\vec{Q}), \quad (43)$$
$$\hat{\rho}_{c-c}(R_2;R_1;\vec{Q}) = \tilde{S}(R_2;R_1;\vec{Q})\hat{\rho}(R_2;R_1;\vec{Q}) = \rho^{\dagger}_{c-c}(R_1;R_2;-\vec{Q}),$$
$$\tilde{S}(R_2;R_1;\vec{Q}) = b_1\tilde{\phi}(0,1;\vec{Q})$$

One of the valence electron density fluctuation operator looks as

$$\hat{\rho}_{v-v}(M_v,\varepsilon_m^-,M_v',\varepsilon_{m'}^-;\vec{Q}) = \sum_{q,g} a^{\dagger}_{M_v,\varepsilon_m^-,q} a_{M_v',\varepsilon_{m'}^-,g} \int d^2\vec{r} e^{i\vec{Q}\vec{r}} \left\langle \psi_v(M_v,\varepsilon_m^-,q;\vec{r}) \middle| \psi_v(M_v',\varepsilon_{m'}^-,g;\vec{r}) \right\rangle =$$
$$= \tilde{S}(M_v,\varepsilon_m^-;M_v',\varepsilon_{m'}^-;\vec{Q})\hat{\rho}(M_v,\varepsilon_m^-;M_v,\varepsilon_{m'}^-\vec{Q}), \quad (44)$$
$$\tilde{S}(M_v,\varepsilon_m^-;M_v',\varepsilon_{m'}^-;\vec{Q}) = \delta_{M_v,M_v'}[d_{m-3}^- d_{m'-3}^{-*}\tilde{\phi}(m-3,m'-3;\vec{Q}) + c_m^- c_{m'}^{-*}\tilde{\phi}(m,m';\vec{Q})],$$
$$m,m' \geq 3$$

Here, we have taken into account the following property of the valence band periodic parts

$$\frac{1}{v_0}\int_{v_0} d\vec{\rho} U^*_{v,p,i,g+Q_x}(\rho) U_{v,p,j,g}(\rho) e^{iQ_y\rho_y} = \delta_{ij} + O(\vec{Q}),$$
$$i,j = x,y \quad (45)$$

They lead to Kronecker symbol δ_{M_v,M'_v} in expression (50) and in the next ones concerning the valence band as follows:

$$\begin{aligned}
&\hat{\rho}_{v-v}(M_v,\varepsilon_m;M'_v,\varepsilon_{m'};\vec{Q}) = \tilde{S}(M_v,\varepsilon_m;M'_v,\varepsilon_{m'};\vec{Q})\hat{\rho}(M_v,\varepsilon_m;M_v,\varepsilon_{m'};\vec{Q}),\\
&\tilde{S}(M_v,\varepsilon_m;M'_v,\varepsilon_{m'};\vec{Q}) = \delta_{M_v,M'_v}\tilde{\phi}(m,m';\vec{Q}),\\
&m,m' = 0,1,2,\\
&\hat{\rho}_{v-v}(M'_v,\varepsilon_{m'};M_v,\varepsilon^-_m;\vec{Q}) = \tilde{S}(M'_v,\varepsilon_{m'};M_v,\varepsilon^-_m;\vec{Q})\hat{\rho}(M_v,\varepsilon_{m'};M_v,\varepsilon^-_m;\vec{Q}),\\
&\tilde{S}(M'_v,\varepsilon_{m'};M_v,\varepsilon^-_m;\vec{Q}) = \delta_{M_v,M'_v}c^{-*}_m\tilde{\phi}(m',m;\vec{Q}),\\
&m' = 0,1,2,\ m\geq 3,\\
&\hat{\rho}_{v-v}(M_v,\varepsilon^-_m;M'_v,\varepsilon_{m'};\vec{Q}) = \tilde{S}(M_v,\varepsilon^-_m;M'_v,\varepsilon_{m'};\vec{Q})\hat{\rho}(M_v,\varepsilon^-_m;M_v,\varepsilon_{m'};\vec{Q}),\\
&\tilde{S}(M_v,\varepsilon^-_m;M'_v,\varepsilon_{m'};\vec{Q}) = \delta_{M_v,M'_v}c^-_m\tilde{\phi}(m,m';\vec{Q}),\\
&m\geq 3,\ m' = 0,1,2
\end{aligned} \tag{46}$$

As usual, they obey to the equalities

$$\begin{aligned}
&\hat{\rho}^\dagger_{v-v}(\xi;\eta;\vec{Q}) = \hat{\rho}_{v-v}(\eta;\xi;-\vec{Q}),\\
&\hat{\rho}(\xi;\eta;\vec{Q}) = \hat{\rho}^\dagger(\eta;\xi;-\vec{Q}),\\
&\tilde{\phi}(n;m;\vec{Q}) = \tilde{\phi}^*(m;n;-\vec{Q}),\\
&\tilde{S}(\xi;\eta;\vec{Q}) = \tilde{S}^*(\eta;\xi;-\vec{Q})
\end{aligned} \tag{47}$$

Up till now, we have dealt with intraband density operators $\hat{\rho}_{c-c}(\xi;\eta;\vec{Q})$ and $\hat{\rho}_{v-v}(\xi;\eta;\vec{Q})$. Interband density operators $\hat{\rho}_{c-v}(\xi;\eta;\vec{Q})$ and $\hat{\rho}_{v-c}(\xi;\eta;\vec{Q})$ depend on the interband exchange electron densities of the type $U^*_{c,s,g+Q_x}(\rho)\frac{1}{\sqrt{2}}(U_{v,p,x,g}(\rho)\pm iU_{v,p,y,g}(\rho))$ and its complex conjugate value. They contain the quickly oscillating periodic parts with different parities and the orthogonality integral on the elementary lattice cell has an infinitesimal value

$$\frac{1}{v_0}\int_{v_0} d\rho U^*_{c,s,g+Q_x}(\rho)e^{iQ_y\rho_y}\frac{1}{\sqrt{2}}(U_{v,p,x,g}(\rho)\pm iU_{v,p,y,g}(\rho)) = O(\vec{Q}) \tag{48}$$

This integral is different from zero if one takes into account, for example, term $iQ_y\rho_y$ appearing in the series expansion of function $e^{iQ_y\rho_y}$. It gives rise to interband dipole momentum $\vec{d}_{cv}$ with the component

$$\begin{aligned}
&d_{cv,y} = \frac{e}{v_0}\int_{v_0} d\rho U^*_{c,s,g}(\rho)\rho_y U_{v,p,y,g}(\rho),\\
&O(\vec{Q}) \approx Q_y d_{cv,y}
\end{aligned} \tag{49}$$

The Coulomb interaction depending on interband exchange electron densities $\hat{\rho}_{cv}(\vec{Q})\hat{\rho}_{vc}(-\vec{Q})$ has a form of the dipole–dipole interaction instead of the charge–charge interaction, which takes place only in the intraband cases. It is known as a long-range Coulomb interaction and gives rise to the longitudinal-transverse splitting of the three-fold degenerate levels of the dipole-active excitons in the cubic crystals [28, 29]. These effects with the participation of the 2D magnetoexcitons have not been investigated up till now, to the best of our knowledge, and remain outside the present review article.

Density operator $\hat{\rho}(\vec{Q})$ in the frame of electron spinor states (12) and (16) looks as

$$\begin{aligned}\hat{\rho}(\vec{Q}) &= \sum_{i,j} \hat{\rho}_{c-c}(R_i;R_j;\vec{Q}) + \sum_{M_v,M_v'}\sum_{\varepsilon_m,\varepsilon_{m'}} \hat{\rho}_{v-v}(M_v,\varepsilon_m;M_v',\varepsilon_{m'};\vec{Q}) + \\ &+ \sum_{M_v,M_v'}\sum_{\varepsilon_m^-,\varepsilon_{m'}^-} \hat{\rho}_{v-v}(M_v,\varepsilon_m^-;M_v',\varepsilon_{m'}^-;\vec{Q}) + \sum_{M_v,M_v'}\sum_{\varepsilon_m^-,\varepsilon_{m'}} \hat{\rho}_{v-v}(M_v',\varepsilon_{m'};M_v,\varepsilon_m^-;\vec{Q}) + \\ &+ \sum_{M_v,M_v'}\sum_{\varepsilon_m^-,\varepsilon_{m'}} \hat{\rho}_{v-v}(M_v,\varepsilon_m^-;M_v',\varepsilon_{m'};\vec{Q}) + \sum_i\sum_{M_v}\sum_{\varepsilon_m} \hat{\rho}_{c-v}(R_i;M_v,\varepsilon_m;\vec{Q}) + \\ &+ \sum_i\sum_{M_v}\sum_{\varepsilon_m^-} \hat{\rho}_{c-v}(R_i;M_v,\varepsilon_m^-;\vec{Q}) + \sum_i\sum_{M_v}\sum_{\varepsilon_m} \hat{\rho}_{v-c}(M_v,\varepsilon_m;R_i;\vec{Q}) + \\ &+ \sum_i\sum_{M_v}\sum_{\varepsilon_m^-} \hat{\rho}_{v-c}(M_v,\varepsilon_m^-;R_i;\vec{Q})\end{aligned} \tag{50}$$

The first five terms of this expression depend on the intraband electron densities and determine the charge–charge Coulomb interaction. The last four terms depend on the interband electron densities and lead to the dipole–dipole long-range Coulomb interaction.

The strength of the Coulomb interaction is determined by coefficients a_n, b_n, c_n, d_n of the spinor-type wave function (12) and (16) as well as by the normalization and orthogonality-type integrals $\tilde{\phi}(n,m,\vec{Q})$. They have the properties:

$$\begin{aligned}&\tilde{\phi}(n,m,\vec{Q}) = e^{-\frac{Q^2l^2}{4}} A_{n,m}(\vec{Q}) \\ &A_{n,m}(0) = \delta_{n,m}\end{aligned} \tag{51}$$

Diagonal coefficients $A_{n,n}(\vec{Q})$ with $n = 0,1,3$ will be calculated below. The nondiagonal coefficients with $n \neq m$ in the limit $Q \to 0$ are proportional to vector components Q_i in a degree of $|n-m|$. They can be neglected in the zeroth order approximation together with other corrections denoted as $O(\vec{Q})$. It essentially diminishes the number of the actual components of density operator $\hat{\rho}(\vec{Q})$.

In the zeroth order approximation, neglecting the corrections of the order $O(\vec{Q})$, we will deal only with diagonal terms that permit the simplified denotations

$$\begin{aligned}&\hat{\rho}(\xi;\xi;\vec{Q}) = \hat{\rho}(\xi;\vec{Q}), \\ &\tilde{S}(\xi;\xi;\vec{Q}) = \tilde{S}(\xi;\vec{Q}) = e^{-\frac{Q^2l_0^2}{4}} S(\xi;\vec{Q})\end{aligned} \tag{52}$$

The concrete values of coefficients $S(\xi;\vec{Q})$ are

$$\begin{aligned}&S(R_1;\vec{Q}) = [|a_0|^2 A_{0,0}(\vec{Q}) + |b_1|^2 A_{1,1}(\vec{Q})], \\ &S(R_2;\vec{Q}) = A_{0,0}(\vec{Q}), \\ &S(\varepsilon_m;\vec{Q}) = A_{m,m}(\vec{Q}), m = 0,1,2, \\ &S(\varepsilon_m^-;\vec{Q}) = [|d_{m-3}^-|^2 A_{m-3,m-3}(\vec{Q}) + |c_m^-|^2 A_{m,m}(\vec{Q})], \\ &m \geq 3\end{aligned} \tag{53}$$

The calculated values of $A_{m,m}(\vec{Q})$ equal

$$A_{0,0}(\vec{Q})=1,\ A_{1,1}(\vec{Q})=\left(1-\frac{Q^2l_0^2}{2}\right),$$
$$A_{3,3}(\vec{Q})=1-\frac{3}{2}Q^2l_0^2+\frac{3}{8}Q^4l_0^4-\frac{1}{48}Q^6l_0^6 \tag{54}$$

The diagonal part of density operator $\hat{\rho}(\vec{Q})$ looks as

$$\hat{\rho}(\vec{Q})=e^{-\frac{Q^2l_0^2}{4}}\{\sum_i S(R_i;\vec{Q})\hat{\rho}(R_i;\vec{Q})+\sum_{M_v}\sum_{\varepsilon_m} S(M_v,\varepsilon_m;\vec{Q})\hat{\rho}(M_v,\varepsilon_m;\vec{Q})+$$
$$+\sum_{M_v}\sum_{\varepsilon_m^-} S(M_v,\varepsilon_m^-;\vec{Q})\hat{\rho}(M_v,\varepsilon_m^-;\vec{Q})\} \tag{55}$$

It contains two separate contributions from the conduction and valence bands. The latter contribution in turn can be represented as due to the electrons of the full filled valence band extracting the contribution of the holes created in its frame. To show it, one can introduce the hole creation and annihilation operators as follows:

$$b^{\dagger}_{M_h,\varepsilon,q}=a_{v,-M_v,\varepsilon,-q},$$
$$b_{M_h,\varepsilon,q}=a^{\dagger}_{v,-M_v,\varepsilon,-q},$$
$$\varepsilon=\varepsilon_m, m=0,1,2,$$
$$\varepsilon=\varepsilon_m^-, m\geq 3 \tag{56}$$

This leads to the relation

$$\hat{\rho}_v(-M_v,\varepsilon,\vec{Q})=N\delta_{kr}(\vec{Q},0)-\hat{\rho}_h(M_h,\varepsilon,\vec{Q}),$$
$$N=\frac{S}{2\pi l_0^2} \tag{57}$$

where hole density operator $\rho_h(M_h,\varepsilon,\vec{Q})$ looks as

$$\hat{\rho}_h(M_h,\varepsilon,\vec{Q})=\sum_t e^{-iQ_ytl_0^2}b^{\dagger}_{M_h,\varepsilon,t+\frac{Q_x}{2}}b_{M_h,\varepsilon,t-\frac{Q_x}{2}} \tag{58}$$

The constant part $N\delta_{kr}(\vec{Q},0)$ in (57) created by electron of the full filled valence band is compensated by the influence of the positive electric charges of the background nuclei. In the jelly model of the system, their presence is taken into account excluding the point $\vec{Q}=0$ from the Hamiltonian of the Coulomb interaction [29]. Taking into account the fully neutral system of the bare electrons and the positive jelly background, we will operate only with the conduction band electrons and with the holes in the valence band. In this electron–hole description, density operator $\hat{\rho}(\vec{Q})$ becomes equal to

$$\hat{\rho}(\vec{Q})=\hat{\rho}_e(\vec{Q})-\hat{\rho}_h(\vec{Q}),$$
$$\vec{Q}\neq 0 \tag{59}$$

where

$$\hat{\rho}_h(\vec{Q}) = \sum_{M_h,\varepsilon_m} \tilde{S}(\varepsilon_m;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m;\vec{Q}) + \sum_{M_h,\varepsilon_m^-} \tilde{S}(\varepsilon_m^-;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q}) =$$

$$= e^{-\frac{Q^2 l_0^2}{4}}\{\sum_{M_h,\varepsilon_m} S(\varepsilon_m;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m;\vec{Q}) + \sum_{M_h,\varepsilon_m^-} S(\varepsilon_m^-;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q})\}, \quad (60)$$

$$\hat{\rho}_e(\vec{Q}) = \hat{\rho}_c(\vec{Q}) = \sum_{i=1,2} \tilde{S}(R_i;\vec{Q})\hat{\rho}_e(R_i;\vec{Q}) = e^{-\frac{Q^2 l_0^2}{4}} \sum_{i=1,2} S(R_i;\vec{Q})\hat{\rho}_e(R_i;\vec{Q})$$

The Hamiltonian of the Coulomb interaction of the initial bare electrons can be expressed in terms of the electron field and density operators (36) and (50) as follows:

$$\begin{aligned} H_{Coul} &= \frac{1}{2}\int d\vec{1}\int d\vec{2}\hat{\Psi}^\dagger(1)\hat{\Psi}^\dagger(2)V(1-2)\hat{\Psi}(2)\hat{\Psi}(1) = \\ &= \frac{1}{2}\sum_{\vec{Q}} V(\vec{Q})\int d\vec{1}\int d\vec{2}e^{i\vec{Q}(\vec{1}-\vec{2})}\hat{\Psi}^\dagger(1)\hat{\rho}(2)\hat{\Psi}(1) = \\ &= \frac{1}{2}\sum_{\vec{Q}} V(\vec{Q})\int\int d\vec{r}e^{i\vec{Q}\vec{r}}\hat{\Psi}^\dagger(\vec{r})\hat{\rho}(-\vec{Q})\hat{\Psi}(\vec{r}), \\ V(\vec{Q}) &= \frac{2\pi e^2}{\varepsilon_0 S|\vec{Q}|} \end{aligned} \quad (61)$$

$V(\vec{Q})$ is the Fourier transform of the Coulomb interaction of the electrons situated on the surface of the 2D layer with area S and dielectric constant ε_0 of the medium. The expression $\hat{\Psi}^\dagger(\vec{r})\hat{\rho}(-\vec{Q})\hat{\Psi}(\vec{r})$ contains density operator $\hat{\rho}(-\vec{Q})$ intercalated between field operators $\hat{\Psi}^\dagger(\vec{r})$ and $\hat{\Psi}(\vec{r})$. Operator $\hat{\Psi}(\vec{r})$ cannot be transposed over operator $\hat{\rho}(-\vec{Q})$ because they do not commute, but its nonoperator part expressed through the spinor-type wave function can be transposed forming together with the conjugate wave function of field operator $\hat{\Psi}^\dagger(\vec{r})$ a scalar.

After the integration on coordinate $\vec{r}$, the quadratic intercalated density operators will appear in the form

$$\begin{aligned} K(\xi;\eta;x;\eta;\vec{Q}) &= \sum_t e^{iQ_y t l_0^2} a^\dagger_{\xi,t+\frac{Q_x}{2}}\hat{\rho}(x;y;-\vec{Q})a_{\eta,t-\frac{Q_x}{2}} = \\ &= \sum_t\sum_s e^{iQ_\eta t l_0^2}e^{-iQ_y s l_0^2}a^\dagger_{\xi,t+\frac{Q_x}{2}}a^\dagger_{x,s-\frac{Q_x}{2}}a_{y,s+\frac{Q_x}{2}}a_{\eta,t-\frac{Q_x}{2}} = \\ &= \hat{\rho}(\xi;\eta;\vec{Q})\hat{\rho}(x;y;-\vec{Q}) - \delta_{\eta,x}\hat{\rho}(\xi;y;0) \end{aligned} \quad (62)$$

The same relations remain in the electron–hole description.

The commutation relations between the density operators are the following:

$$\hat{\rho}(\xi;\eta;\vec{Q}) = \sum_t e^{iQ_y t l_0^2} a^\dagger_{\xi,t+\frac{Q_x}{2}} a_{\eta,t-\frac{Q_x}{2}},$$

$$\hat{\rho}(x;y;\vec{P}) = \sum_t e^{iP_y t l_0^2} a^\dagger_{x,t+\frac{P_x}{2}} a_{y,t-\frac{P_x}{2}},$$

$$[\hat{\rho}(\xi;\eta;\vec{Q}),\hat{\rho}(x;y;\vec{P})] = \delta_{x,\eta}\hat{\rho}(\xi;y;\vec{P}+\vec{Q})e^{\frac{i[\vec{P}\times\vec{Q}]_z l_0^2}{2}} - \delta_{\xi,y}\hat{\rho}(x;\eta;\vec{P}+\vec{Q})e^{\frac{-i[\vec{P}\times\vec{Q}]_z l_0^2}{2}} = \tag{63}$$

$$= \cos\left(\frac{[\vec{P}\times\vec{Q}]_z l_0^2}{2}\right)[\delta_{x,\eta}\hat{\rho}(\xi;y;\vec{P}+\vec{Q}) - \delta_{\xi,y}\hat{\rho}(x;\eta;\vec{P}+\vec{Q})] +$$

$$+ i\sin\left(\frac{[\vec{P}\times\vec{Q}]_z l_0^2}{2}\right)[\delta_{x,\eta}\hat{\rho}(\xi;y;\vec{P}+\vec{Q}) + \delta_{\xi,y}\hat{\rho}(x;\eta;\vec{P}+\vec{Q})]$$

Factor $e^{-\frac{Q^2 l_0^2}{4}}$ arising from the product of the density operators $\hat{\rho}(\vec{Q})$ and $\hat{\rho}(-\vec{Q})$ being multiplied by coefficient $V(\vec{Q})$ gives rise to coefficient $W(\vec{Q})$ describing the effective Coulomb interaction under the conditions of the Landau quantization

$$W(\vec{Q}) = V(\vec{Q})e^{-\frac{Q^2 l_0^2}{2}} \tag{64}$$

Excluding the intercalations, the Hamiltonian of the Coulomb interaction in the presence of the Landau quantization and Rashba spin–orbit coupling has the form:

$$H_{Coul} = \frac{1}{2}\sum_{\vec{Q}} W(\vec{Q})\{\sum_{i,j} S(R_i;\vec{Q})S(R_j;-\vec{Q})[\hat{\rho}(R_i;\vec{Q})\hat{\rho}(R_j;-\vec{Q}) - \delta_{i,j}\hat{\rho}(R_i;0)] +$$

$$+ \sum_{M_v,M'_v}\sum_{\varepsilon_m,\varepsilon_{m'}} S(M_v,\varepsilon_m;\vec{Q})S(M'_v,\varepsilon_{m'};-\vec{Q})[\hat{\rho}(M_v,\varepsilon_m;\vec{Q})\hat{\rho}(M'_v,\varepsilon_{m'};-\vec{Q}) - \delta_{M_v,M'_v}\delta_{m,m'}\hat{\rho}(M_v,\varepsilon_m;0)] +$$

$$+ \sum_{M_v,M'_v}\sum_{\varepsilon^-_m,\varepsilon^-_{m'}} S(M_v,\varepsilon^-_m;\vec{Q})S(M'_v,\varepsilon^-_{m'};-\vec{Q})[\hat{\rho}(M_v,\varepsilon^-_m;\vec{Q})\hat{\rho}(M'_v,\varepsilon^-_{m'};-\vec{Q}) - \delta_{M_v,M'_v}\delta_{m,m'}\hat{\rho}(M_v,\varepsilon^-_m;0)] +$$

$$+ \sum_{i=1,2}\sum_{M_v,\varepsilon_m} S(R_i;\vec{Q})S(M_v,\varepsilon_m;-\vec{Q})\hat{\rho}(R_i;\vec{Q})\hat{\rho}(M_v,\varepsilon_m;-\vec{Q}) +$$

$$+ \sum_{i=1,2}\sum_{M_v,\varepsilon^-_m} S(R_i;\vec{Q})S(M_v,\varepsilon^-_m;-\vec{Q})\hat{\rho}(R_i;\vec{Q})\hat{\rho}(M_v,\varepsilon^-_m;-\vec{Q}) +$$

$$+ \sum_{i=1,2}\sum_{M_v,\varepsilon_m} S(M_v,\varepsilon_m;\vec{Q})S(R_i;-\vec{Q})\hat{\rho}(M_v,\varepsilon_m;\vec{Q})\hat{\rho}(R_i;-\vec{Q}) +$$

$$+ \sum_{i=1,2}\sum_{M_v,\varepsilon^-_m} S(M_v,\varepsilon^-_m;\vec{Q})S(R_i;-\vec{Q})\hat{\rho}(M_v,\varepsilon^-_m;\vec{Q})\hat{\rho}(R_i;-\vec{Q}) +$$

$$+ \sum_{M_v,M'_v}\sum_{\varepsilon_m,\varepsilon^-_{m'}} S(M_v,\varepsilon_m;\vec{Q})S(M'_v,\varepsilon^-_{m'};-\vec{Q})\hat{\rho}(M_v,\varepsilon_m;\vec{Q})\hat{\rho}(M'_v,\varepsilon^-_{m'};-\vec{Q}) +$$

$$+ \sum_{M_v,M'_v}\sum_{\varepsilon^-_m,\varepsilon_{m'}} S(M_v,\varepsilon^-_m;\vec{Q})S(M'_v,\varepsilon_{m'};-\vec{Q})\hat{\rho}(M_v,\varepsilon^-_m;\vec{Q})\hat{\rho}(M'_v,\varepsilon_{m'};-\vec{Q})\} \tag{65}$$

The Hamiltonian of the Coulomb interaction in the electron–hole representation looks as

$$H_{Coul} = \frac{1}{2}\sum_{\vec{Q}} W(\vec{Q})\{\sum_{i,j} S(R_i;\vec{Q})S(R_j;-\vec{Q})[\hat{\rho}_e(R_i;\vec{Q})\hat{\rho}_e(R_j;-\vec{Q}) - \delta_{i,j}\hat{\rho}_e(R_i;0)] +$$
$$+\sum_{M_h,M'_h}\sum_{\varepsilon_m,\varepsilon_{m'}} S(M_h,\varepsilon_m;\vec{Q})S(M'_h,\varepsilon_{m'};-\vec{Q})[\hat{\rho}_h(M_h,\varepsilon_m;\vec{Q})\hat{\rho}_h(M'_h,\varepsilon_{m'};-\vec{Q}) - \delta_{M_h,M'_h}\delta_{m,m'}\hat{\rho}_h(M_h,\varepsilon_m;0)] +$$
$$+\sum_{M_h,M'_h}\sum_{\varepsilon^-_m,\varepsilon^-_{m'}} S(M_h,\varepsilon^-_m;\vec{Q})S(M'_h,\varepsilon^-_{m'};-\vec{Q})[\hat{\rho}_h(M_h,\varepsilon^-_m;\vec{Q})\hat{\rho}_h(M'_h,\varepsilon^-_{m'};-\vec{Q}) - \delta_{M_h,M'_h}\delta_{m,m'}\hat{\rho}_h(M_h,\varepsilon^-_m;0)] +$$
$$+\sum_{M_h,M'_h}\sum_{\varepsilon_m,\varepsilon^-_{m'}} S(M_h,\varepsilon_m;\vec{Q})S(M'_h,\varepsilon^-_{m'};-\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m;\vec{Q})\hat{\rho}_h(M'_h,\varepsilon^-_{m'};-\vec{Q}) +$$
$$+\sum_{M_h,M'_h}\sum_{\varepsilon^-_m,\varepsilon_{m'}} S(M_h,\varepsilon^-_m;\vec{Q})S(M'_h,\varepsilon_{m'};-\vec{Q})\hat{\rho}_h(M_h,\varepsilon^-_m;\vec{Q})\hat{\rho}_h(M'_h,\varepsilon_{m'};-\vec{Q}) -$$
$$-\sum_{i}\sum_{M_h,\varepsilon_m} S(R_i;\vec{Q})S(M_h,\varepsilon_m;-\vec{Q})\hat{\rho}_e(R_i;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m;-\vec{Q}) -$$
$$-\sum_{i}\sum_{M_h,\varepsilon^-_m} S(R_i;\vec{Q})S(M_h,\varepsilon^-_m;-\vec{Q})\hat{\rho}_e(R_i;\vec{Q})\hat{\rho}_h(M_h,\varepsilon^-_m;-\vec{Q}) -$$
$$-\sum_{i}\sum_{M_h,\varepsilon_m} S(M_h,\varepsilon_m;\vec{Q})S(R_i;-\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m;\vec{Q})\hat{\rho}_e(R_i;-\vec{Q}) -$$
$$-\sum_{i}\sum_{M_h,\varepsilon^-_m} S(M_h,\varepsilon^-_m;\vec{Q})S(R_i;-\vec{Q})\hat{\rho}_h(M_h,\varepsilon^-_m;\vec{Q})\hat{\rho}_e(R_i;-\vec{Q})\} \quad (66)$$

In the concrete variant named as F_1, where the electrons are in state R_1, whereas the holes are in state ε^-_3 with a given value of M_h, Hamiltonian (66) looks as

$$H_{Coul}(R_1;\varepsilon^-_3) = \frac{1}{2}\sum_{\vec{Q}} W(\vec{Q})\{(|a_0|^2 + |b_1|^2\ A_{1,1}(\vec{Q}))^2[\hat{\rho}_e(R_1;\vec{Q})\hat{\rho}_e(R_1;-\vec{Q}) - \hat{\rho}_e(R_1;0)] +$$
$$+(|d^-_0|^2 + |c^-_3|^2\ A_{3,3}(\vec{Q}))^2[\hat{\rho}_h(M_h,\varepsilon^-_3;\vec{Q})\hat{\rho}_h(M_h,\varepsilon^-_3;-\vec{Q}) - \hat{\rho}_h(M_h,\varepsilon^-_3;0)] - \quad (67)$$
$$-2(|a_0|^2 + |b_1|^2\ A_{1,1}(\vec{Q}))(|d^-_0|^2 + |c^-_3|^2\ A_{3,3}(\vec{Q}))\hat{\rho}_e(R_1;\vec{Q})\hat{\rho}_h(M_h,\varepsilon^-_3;-\vec{Q})\}$$

In the absence of the RSOC, we have $a_0 = d^-_0 = 1$ and $b_1 = c^-_3 = 0$. In the variant $F_1 = (R_1,\varepsilon^-_3)$ described by Hamiltonian (67), the 2D magnetoexciton can be described by the wave function

$$\left|\Psi_{ex}(F_1,\vec{K})\right\rangle = \frac{1}{\sqrt{N}}\sum_t e^{iK_y t l_0^2} a^\dagger_{R_1,\frac{K_x}{2}+t}\, b^\dagger_{M_h,\varepsilon^-_3,\frac{K_x}{2}-t}\left|0\right\rangle \quad (68)$$

where $\left|0\right\rangle$ is the vacuum state determined by the equalities

$$a_{\xi,t}\left|0\right\rangle = b_{\eta,t}\left|0\right\rangle = 0 \quad (69)$$

In [22], other seven combinations of the electron and hole states were considered as follows:

$$F_2 = (R_2,\varepsilon^-_3), F_3 = (R_1,\varepsilon_0), F_4 = (R_2,\varepsilon_0), F_5 = (R_1,\varepsilon^-_4),$$
$$F_6 = (R_2,\varepsilon^-_4), F_7 = (R_1,\varepsilon_1), F_8 = (R_2,\varepsilon_1) \quad (70)$$

In all these cases, the exciton creation energies were calculated using the formulas

$$E_{ex}(F_n,\vec{k}) = E_{cv}(F_n) - I_{ex}(F_n,\vec{k})$$
$$E_{cv}(F_n) - E_g = E_e(\xi) + E_h(\eta), \quad (71)$$
$$F_n = (\xi,\eta)$$

Here, E_g is the semiconductor energy gap in the absence of a magnetic field. $I_{ex}(F_n,\vec{k})$ is the ionization potential of the magnetoexciton moving with wave vector $\vec{k}$.

The Hamiltonian of the Coulomb electron-electron interaction in the case of e–h pairs with the electrons in the degenerate state (S_e,R_1) and the holes in the degenerate state $(S_h,M_h,\varepsilon_m^-)$ has the form

$$\begin{aligned}
&H_{Coul}(R_1,\varepsilon_m^-)= \\
&=\frac{1}{2}\sum_{\vec{Q}}\Big\{W_{e-e}(R_1;\vec{Q})\Big[\hat{\rho}_e(R_1;\vec{Q})\,\hat{\rho}_e(R_1;-\vec{Q})-\hat{N}_e(R_1)\Big]+ \\
&+W_{h-h}(\varepsilon_m^-;\vec{Q})\Big[\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m^-;-\vec{Q})-\hat{N}_h(M_h,\varepsilon_m^-)\Big]- \\
&-2W_{e-h}(R_1,\varepsilon_m^-;\vec{Q})\hat{\rho}_e(R_1;\vec{Q})\hat{\rho}_h(M_h,\varepsilon_m^-;-\vec{Q})\Big\}
\end{aligned} \tag{72}$$

Once again, the electron and hole density operators are recalled:

$$\begin{aligned}
&\hat{\rho}_e(R_1;\vec{Q})=\sum_t e^{iQ_y t l_0^2} a^{\dagger}_{R_1,t+\frac{Q_x}{2}} a_{R_1,t-\frac{Q_x}{2}}, \\
&\hat{\rho}_h(M_h,\varepsilon_m^-;\vec{Q})=\sum_t e^{-iQ_y t l_0^2} b^{\dagger}_{M_h,\varepsilon_m^-,t+\frac{Q_x}{2}} b_{M_h,\varepsilon_m^-,t-\frac{Q_x}{2}}, \\
&\hat{N}_e(R_1)=\hat{\rho}_e(R_1;0), \quad \hat{N}_h(M_h,\varepsilon_m^-)=\hat{\rho}_h(M_h,\varepsilon_m^-;0)
\end{aligned} \tag{73}$$

Coefficients $W_{i-j}(\vec{Q})$ in (72) are

$$\begin{aligned}
&W_{e-e}(R_1;\vec{Q})=W(\vec{Q})\Big(|a_0|^2 A_{0,0}(\vec{Q})+|b_1|^2 A_{1,1}(\vec{Q})\Big)^2, \\
&W_{h-h}(\varepsilon_m^-;\vec{Q})=W(\vec{Q})\Big(|d_{m-3}^-|^2 A_{m-3,m-3}(\vec{Q})+|c_m^-|^2 A_{m,m}(\vec{Q})\Big)^2, \quad m\geq 3, \\
&W_{e-h}(R_1,\varepsilon_m^-;\vec{Q})=W(\vec{Q})\Big(|a_0|^2 A_{0,0}(\vec{Q})+|b_1|^2 A_{1,1}(\vec{Q})\Big)\times \\
&\times\Big(|d_{m-3}^-|^2 A_{m-3,m-3}(\vec{Q})+|c_m^-|^2 A_{m,m}(\vec{Q})\Big), \; m\geq 3
\end{aligned} \tag{74}$$

The normalization conditions take place

$$\begin{aligned}
&|a_0|^2+|b_1|^2=1, \\
&|d_{m-3}^-|^2+|c_m^-|^2=1, \\
&m\geq 3
\end{aligned} \tag{75}$$

In the actual case $m=3$ we obtain

$$W_{e-e}(R_1;\vec{Q}) = W(\vec{Q})\left(1-\frac{|b_1|^2}{2}Q^2 l_0^2\right)^2,$$

$$W_{h-h}(\varepsilon_3^-;\vec{Q}) = W(\vec{Q})\left(1-\frac{|c_3^-|^2}{2}\left(3Q^2 l_0^2-\frac{3}{4}Q^4 l_0^4+\frac{1}{24}Q^6 l_0^6\right)\right)^2, \tag{76}$$

$$W_{e-h}(R_1,\varepsilon_3^-;\vec{Q}) = W(\vec{Q})\left(1-\frac{|b_1|^2}{2}Q^2 l_0^2\right)\left(1-\frac{|c_3^-|^2}{2}\left(3Q^2 l_0^2-\frac{3}{4}Q^4 l_0^4+\frac{1}{24}Q^6 l_0^6\right)\right),$$

$$W(\vec{Q}) = e^{-\frac{Q^2 l_0^2}{2}}V(\vec{Q}),\quad V(\vec{Q}) = \frac{2\pi e^2}{\varepsilon_0 S|\vec{Q}|}$$

The terms proportional to $\hat{N}_e(R_1)$ and $\hat{N}_h(M_h,\varepsilon_m^-)$ in (72) have coefficients $I_e(R_1)$ and $I_h(\varepsilon_m^-)$, which describe the Coulomb self-actions of the electrons and holes, are listed below together with the binding energy of the electron and the hole forming the magnetoexciton. The last value is determined by the diagonal matrix element of Hamiltonian (72) calculated with wave function (68) as follows:

$$\left\langle \Psi_{ex}(F_1,\vec{k})\left| H_{Coul}\right| \Psi_{ex}(F_1,\vec{k})\right\rangle = -I_l(F_1)+E(F_1,\vec{k}),$$

$$I_l(F_1) = I_l(R_1;\varepsilon_m^-) = \sum_{\vec{Q}} W_{e-h}(R_1;\varepsilon_m^-;\vec{Q}),$$

$$E(F_1,\vec{k}) = E(R_1;\varepsilon_m^-;\vec{k}) = 2\sum_{\vec{Q}} W_{e-h}(R_1;\varepsilon_m^-;\vec{Q})\sin^2\left(\frac{[\vec{k}\times\vec{Q}]_z l_0^2}{2}\right), \tag{77}$$

$$\lim_{k\to\infty} E(R_1;\varepsilon_m^-;\vec{k}) = I_l(R_1;\varepsilon_m^-),$$

$$I_e(R_1) = \frac{1}{2}\sum_{\vec{Q}} W_{e-e}(R_1;\vec{Q}),\ I_h(\varepsilon_m^-) = \frac{1}{2}\sum_{\vec{Q}} W_{h-h}(\varepsilon_m^-;\vec{Q}),$$

$$I_S(R_1;\varepsilon_m^-) = I_e(R_1)+I_h(\varepsilon_m^-)$$

The binding energy of the magnetoexciton and its ionization potential, which has the opposite sign as compared with the binding energy, tend to zero when wave vector $\vec{k}$ tends to infinity and the magnetoexciton is transformed into a free e–h pair:

$$H_{mex,1} = \left(\frac{E_g^0}{2}+E_e(R_1)-I_e(R_1)-\mu_e\right)\hat{N}_e(R_1)+ \\ +\left(\frac{E_g^0}{2}+E_h(M_h,\varepsilon_m^-)-I_h(\varepsilon_m^-)-\mu_h\right)\hat{N}_h(M_h,\varepsilon_m^-) \tag{78}$$

Here, semiconductor energy band gap E_g^0 in the absence of the Landau quantization was introduced.

Now, instead of electron and hole density operators $\hat{\rho}_e(\vec{Q})$ and $\hat{\rho}_h(\vec{Q})$, we will introduce the density operators of the optical plasmon denoted as $\hat{\rho}(\vec{Q})$ and the acoustical plasmon denoted as $\hat{D}(\vec{Q})$ following the relations

$$\begin{aligned}
&\hat{\rho}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(\vec{Q}),\\
&\hat{D}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(\vec{Q}),\\
&\hat{\rho}_e(\vec{Q}) = \frac{\hat{\rho}(\vec{Q}) + \hat{D}(\vec{Q})}{2},\\
&\hat{\rho}_h(\vec{Q}) = \frac{\hat{D}(\vec{Q}) - \hat{\rho}(\vec{Q})}{2}
\end{aligned} \tag{79}$$

Here, for simplicity, many indices that label the electron, hole, and plasmon density operators are omitted. But they must be kept in mind and may be restored in concrete cases.

In the plasmon representation, Hamiltonian $H_{mex,1}$ (78) looks as

$$\begin{aligned}
H_{mex,1} &= \left(E_{mex}(R_1; M_h, \varepsilon_m^-) - \mu_{mex}\right)\frac{\hat{D}(0)}{2} +\\
&+\left(G_{e-h}(R_1; M_h, \varepsilon_m^-) - \mu_e + \mu_h\right)\frac{\hat{\rho}(0)}{2}
\end{aligned} \tag{80}$$

Here, the sums and differences of the Landau quantization level energies, the Coulomb self-interaction terms, and the chemical potentials are defined as follows:

$$\begin{aligned}
&E_{mex}(R_1; M_h, \varepsilon_m^-) = E_g(R_1; M_h, \varepsilon_m^-) - I_l(R_1; \varepsilon_m^-),\\
&E_g(R_1; M_h, \varepsilon_m^-) = E_g^0 + E_e(R_1) + E_h(M_h, \varepsilon_m^-),\\
&\mu_{ex} = \mu_e + \mu_h,\\
&G_{e-h}(R_1; M_h, \varepsilon_m^-) = E_e(R_1) - E_h(M_h, \varepsilon_m^-) - I_e(R_1) + I_h(\varepsilon_m^-)
\end{aligned} \tag{81}$$

The remaining part $H_{mex,2}$ of Hamiltonian (72), after the excluding of the linear terms, is quadratic in the plasmon density operators. It has the form

$$\begin{aligned}
H_{mex,2} &= \frac{1}{2}\sum_{\vec{Q}}\Big\{W_{0-0}(\vec{Q})\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) + W_{a-a}(\vec{Q})\hat{D}(\vec{Q})\hat{D}(-\vec{Q}) +\\
&+W_{0-a}(\vec{Q})\left(\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q}) + \hat{D}(\vec{Q})\hat{\rho}(-\vec{Q})\right)\Big\}
\end{aligned} \tag{82}$$

The new coefficients are expressed in terms of the former ones by the formulas

$$\begin{aligned}
&W_{0-0}(\vec{Q}) = \frac{1}{4}\left(W_{e-e}(\vec{Q}) + W_{h-h}(\vec{Q}) + 2W_{e-h}(\vec{Q})\right),\\
&W_{a-a}(\vec{Q}) = \frac{1}{4}\left(W_{e-e}(\vec{Q}) + W_{h-h}(\vec{Q}) - 2W_{e-h}(\vec{Q})\right),\\
&W_{0-a}(\vec{Q}) = \frac{1}{4}\left(W_{e-e}(\vec{Q}) - W_{h-h}(\vec{Q})\right)
\end{aligned} \tag{83}$$

In the case of the e–h pairs of the type $(R_1; \varepsilon_m^-)$ they take the form

$$W_{0-0}(R_1;\varepsilon_m^-;\vec{Q})=\frac{1}{4}\Big(W_{e-e}(R_1;\vec{Q})+W_{h-h}(\varepsilon_m^-;\vec{Q})+2W_{e-h}(R_1;\varepsilon_m^-;\vec{Q})\Big)=$$
$$=\frac{W(\vec{Q})}{4}\Big(|a_0|^2A_{0,0}(\vec{Q})+|b_1|^2A_{1,1}(\vec{Q})+|d_{m-3}^-|^2A_{m-3,m-3}(\vec{Q})+|c_m^-|^2A_{m,m}(\vec{Q})\Big)^2,$$
$$W_{a-a}(R_1;\varepsilon_m^-;\vec{Q})=\frac{1}{4}\Big(W_{e-e}(R_1;\vec{Q})+W_{h-h}(\varepsilon_m^-;\vec{Q})-2W_{e-h}(R_1;\varepsilon_m^-;\vec{Q})\Big)=$$
$$=\frac{W(\vec{Q})}{4}\Big(|a_0|^2A_{0,0}(\vec{Q})+|b_1|^2A_{1,1}(\vec{Q})-|d_{m-3}^-|^2A_{m-3,m-3}(\vec{Q})-|c_m^-|^2A_{m,m}(\vec{Q})\Big)^2, \quad (84)$$
$$W_{0-a}(R_1;\varepsilon_m^-;\vec{Q})=\frac{1}{4}\Big(W_{e-e}(R_1;\vec{Q})-W_{h-h}(\varepsilon_m^-;\vec{Q})\Big)=$$
$$=\frac{W(\vec{Q})}{4}\Big[\Big(|a_0|^2A_{0,0}(\vec{Q})+|b_1|^2A_{1,1}(\vec{Q})\Big)^2-\Big(|d_{m-3}^-|^2A_{m-3,m-3}(\vec{Q})+|c_m^-|^2A_{m,m}(\vec{Q})\Big)^2\Big]$$

In a special case $m=3$ we have

$$W_{0-0}(R_1;\varepsilon_m^-;\vec{Q})=W(\vec{Q})\left(1-\frac{|b_1|^2}{4}Q^2l_0^2-\frac{|c_m^-|^2}{4}\left(3Q^2l_0^2-\frac{3}{4}Q^4l_0^4+\frac{1}{24}Q^6l_0^6\right)\right)^2,$$
$$W_{a-a}(R_1;\varepsilon_m^-;\vec{Q})=W(\vec{Q})\left(-\frac{|b_1|^2}{4}Q^2l_0^2+\frac{|c_m^-|^2}{4}\left(3Q^2l_0^2-\frac{3}{4}Q^4l_0^4+\frac{1}{24}Q^6l_0^6\right)\right)^2, \quad (85)$$
$$W_{0-a}(R_1;\varepsilon_m^-;\vec{Q})=W(\vec{Q})\left[\left(1-\frac{|b_1|^2}{4}Q^2l_0^2-\frac{|c_m^-|^2}{4}\left(3Q^2l_0^2-\frac{3}{4}Q^4l_0^4+\frac{1}{24}Q^6l_0^6\right)\right)\times\right.$$
$$\left.\times\left(-\frac{|b_1|^2}{4}Q^2l_0^2+\frac{|c_m^-|^2}{4}\left(3Q^2l_0^2-\frac{3}{4}Q^4l_0^4+\frac{1}{24}Q^6l_0^6\right)\right)\right]$$

Side by side with the magnetoexciton subsystem, the photon subsystem does exist. In our case, it is composed of photons with a given circular polarization, for example, $\vec{\sigma}_{\vec{k}}^+$. Their wave vectors $\vec{k}=\vec{a}_3\frac{\pi}{L_c}+\vec{k}_\parallel$ have the same quantized longitudinal component equal to $\frac{\pi}{L_c}$, where L_c is the resonator length and arbitrary values of the in-plane 2D vectors $\vec{k}_\parallel$. The photon energies are $\hbar\omega_{\vec{k}}=\frac{\hbar c}{n_0}\sqrt{\frac{\pi^2}{L_c^2}+\vec{k}_\parallel^2}$, where n_0 is the refractive index of the microcavity. The full number of the photons captured into the resonator is determined by their chemical potential μ_{ph}.

The zeroth order Hamiltonian of the photons in the microcavity looks as

$$H_{0,ph}=\sum_{\vec{k}_\parallel}\left(\hbar\omega_{\vec{k}}-\mu_{ph}\right)c_{\vec{k},\sigma}^\dagger c_{\vec{k},\sigma} \quad (86)$$

where $c_{\vec{k},\sigma}^\dagger, c_{\vec{k},\sigma}$ are the creation and annihilation photon operators and σ denotes a definite circular polarization. Only the case $\sigma=-$ will be considered. It must be supplemented by the

Hamiltonian of the magnetoexciton–photon interaction deduced above in a more general case. In the case of dipole-active band-to-band quantum transition with the combination of the e–h states (R_1, ε_3^-) we have

$$H_{mex-ph} = \sum_{\vec{k}_{\|}} \Big[\varphi(\vec{k}_{\|}; R_1; \varepsilon_3^-) \left(\vec{\sigma}_{\vec{k}}^{+} \bullet \vec{\sigma}_{M_h}^{*} \right) c_{\vec{k},-} \hat{\Psi}_{ex}^{\dagger}(\vec{k}_{\|}) + \\ + \varphi^{*}(\vec{k}_{\|}; R_1; \varepsilon_3^-) \left(\vec{\sigma}_{\vec{k}}^{-} \bullet \vec{\sigma}_{M_h} \right) c_{\vec{k},-}^{\dagger} \hat{\Psi}_{ex}(\vec{k}_{\|}) \Big] \tag{87}$$

The interaction coefficient is as follows:

$$\varphi(\vec{k}_{\|}; R_1; \varepsilon_3^-) = \left(-\frac{e}{m_0 l_0} \right) \sqrt{\frac{\hbar}{L_c \omega_{\vec{k}}}} P_{cv}(0) T(\vec{k}_{\|}; R_1; \varepsilon_3^-), \\ T(\vec{k}_{\|}; R_1; \varepsilon_3^-) = a_0^{*} d_0^{-*} \tilde{\phi}(0,0; \vec{k}_{\|}) - b_1^{*} c_3^{-*} \tilde{\phi}(1,3; \vec{k}_{\|}) \tag{88}$$

The magnetoexciton creation and annihilation operators were written in a shortened form in (87) because there are too many indices in its full description as follows:

$$\hat{\Psi}_{ex}^{\dagger}(\vec{Q}) = \hat{\Psi}_{ex}^{\dagger}(\vec{Q}; R_1; M_h, \varepsilon_3^-) = \frac{1}{\sqrt{N}} \sum_t e^{iQ_y t l_0^2} a_{R_1, t+\frac{Q_x}{2}}^{\dagger} b_{M_h, \varepsilon_3^-, -t+\frac{Q_x}{2}}^{\dagger} \tag{89}$$

The full Hamiltonian of the magnetoexciton–photon system for a more actual combination (R_1, ε_3^-) may be written

$$H = H_{mex,1} + H_{0,ph} + H_{mex-ph} + H_{mex,2} \tag{90}$$

Its remarkable peculiarity is the presence only of the two-particle integral plasmon and magnetoexciton operators, rather than of the single-particle electron and hole Fermi operators. It permits considerably simplifying the deduction of their equations of motion. For this reason, the commutation relations between the full set of four two-particle integral operators $\hat{\rho}(\vec{Q}), \hat{D}(\vec{Q}), \hat{\Psi}_{ex}^{\dagger}(\vec{Q})$ and $\hat{\Psi}_{ex}(\vec{Q})$ are needed. They are listed below

$$\left[\hat{\rho}(\vec{Q}), \hat{\rho}(\vec{P}) \right] = \left[\hat{D}(\vec{Q}), \hat{D}(\vec{P}) \right] = 2i \sin\left(Z(\vec{P}, \vec{Q}) \right) \hat{\rho}\left(\vec{Q} + \vec{P} \right), \\
\left[\hat{\rho}(\vec{Q}), \hat{D}(\vec{P}) \right] = 2i \sin\left(Z(\vec{P}, \vec{Q}) \right) \hat{D}\left(\vec{P} + \vec{Q} \right), \\
\left[\hat{\Psi}_{ex}(\vec{P}), \hat{\Psi}_{ex}^{\dagger}(\vec{Q}) \right] = \delta_{kr}(\vec{P}, \vec{Q}) - \frac{1}{N} [i \sin\left(Z(\vec{Q}, \vec{P}) \right) \hat{\rho}\left(\vec{Q} - \vec{P} \right) + \cos\left(Z(\vec{Q}, \vec{P}) \right) \hat{D}\left(\vec{Q} - \vec{P} \right)], \\
Z(\vec{P}, \vec{Q}) = -Z(\vec{Q}, \vec{P}) = \frac{[\vec{P} \times \vec{Q}]_z l_0^2}{2} = -Z(\vec{P}, -\vec{Q}) = Z(-\vec{Q}, \vec{P}), \\
\left[\hat{\Psi}_{ex}(\vec{P}), \hat{\Psi}_{ex}^{\dagger}(\vec{P}) \right] = 1 - \frac{1}{N} \hat{D}(0), \\
\left[\hat{\rho}(\vec{Q}), \hat{\Psi}_{ex}^{\dagger}(\vec{P}) \right] = 2i \sin\left(Z(\vec{P}, \vec{Q}) \right) \hat{\Psi}_{ex}^{\dagger}(\vec{P} + \vec{Q}), \\
\left[\hat{\rho}(\vec{Q}), \hat{\Psi}_{ex}(\vec{P}) \right] = -2i \sin\left(Z(\vec{P}, \vec{Q}) \right) \hat{\Psi}_{ex}(\vec{P} - \vec{Q}), \\
\left[\hat{D}(\vec{Q}), \hat{\Psi}_{ex}^{\dagger}(\vec{P}) \right] = 2 \cos\left(Z(\vec{P}, \vec{Q}) \right) \hat{\Psi}_{ex}^{\dagger}(\vec{P} + \vec{Q}), \\
\left[\hat{D}(\vec{Q}), \hat{\Psi}_{ex}(\vec{P}) \right] = -2 \cos\left(Z(\vec{P}, \vec{Q}) \right) \hat{\Psi}_{ex}(\vec{P} - \vec{Q}) \tag{91}$$

5. The magnetoexcitons in the Bose-gas model description

The Hamiltonian describing the 2D e–h pairs with electrons and holes situated on the given Landau quantization levels and interacting between themselves through the Coulomb forces was represented as a sum: $H_{mex,1} + H_{mex,2}$. It is expressed in terms of plasmon density operators $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$. It is useful to represent it in the model of weakly interacting Bose gas. To this end, the wave functions describing the free single magnetoexcitons $\left|\psi_{mex}(\vec{P})\right\rangle$ as well as the pairs of the free magnetoexcitons with wave vectors $\vec{P}$ and $\vec{R}$ $\left|\psi_{mex}(\vec{P}),\psi_{mex}(\vec{R})\right\rangle$ were introduced:

$$\begin{aligned}
&\left|\psi_{ex}(\vec{P})\right\rangle = \hat{\Psi}^{\dagger}_{ex}(\vec{P})\left|0\right\rangle, \\
&\left\langle\psi_{ex}(\vec{P})\right| = \left\langle 0\right|\hat{\Psi}_{ex}(\vec{P}), \\
&\left|\psi_{ex}(\vec{P}),\psi_{ex}(\vec{R})\right\rangle = \hat{\Psi}^{\dagger}_{ex}(\vec{P})\hat{\Psi}^{\dagger}_{ex}(\vec{R})\left|0\right\rangle, \\
&\left\langle\psi_{ex}(\vec{P}),\psi_{ex}(\vec{R})\right| = \left\langle 0\right|\hat{\Psi}_{ex}(\vec{R})\hat{\Psi}_{ex}(\vec{P})
\end{aligned} \tag{92}$$

where $\left|0\right\rangle$ is the vacuum state of the semiconductor. They were used to calculate the matrix elements

$$\begin{aligned}
&E_{mex}(\vec{P}) = \left\langle\psi_{ex}(\vec{P})\right| H_{mex,1} + H_{mex,2} \left|\psi_{ex}(\vec{P})\right\rangle, \\
&W(\vec{P}_1,\vec{R}_1;\vec{P}_2,\vec{R}_2) = \left\langle\psi_{ex}(\vec{P}_1)\psi_{ex}(\vec{R}_1)\right| H_{mex,1} + H_{mex,2} \left|\psi_{ex}(\vec{P}_2)\psi_{ex}(\vec{R}_2)\right\rangle
\end{aligned} \tag{93}$$

With these matrix elements and with the magnetoexciton creation and annihilation operators, the new Hamiltonian in the model of weakly interacting Bose gas can be constructed. It looks as

$$\begin{aligned}
&\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}, \\
&\hat{H}_0 = \sum_{\vec{P}} E_{mex}(\vec{P})\hat{\Psi}^{\dagger}_{ex}(\vec{P})\hat{\Psi}_{ex}(\vec{P}), \\
&\hat{H}_{\text{int}} = \sum_{\vec{P}_1,\vec{R}_1,\vec{P}_2,\vec{R}_2} W(\vec{P}_1,\vec{R}_1;\vec{P}_2,\vec{R}_2)\hat{\Psi}^{\dagger}_{ex}(\vec{P}_1)\hat{\Psi}^{\dagger}_{ex}(\vec{R}_1)\hat{\Psi}_{ex}(\vec{R}_2)\hat{\Psi}_{ex}(\vec{P}_2), \\
&\vec{P}_1 + \vec{R}_1 = \vec{P}_2 + \vec{R}_2
\end{aligned} \tag{94}$$

Recall that the magnetoexciton creation and annihilation operators in turn are constructed from electron and hole creation and annihilation Fermi-type operators $a^{\dagger}_p, a_p, b^{\dagger}_p, b_p$ as follows:

$$\hat{\Psi}^{\dagger}_{ex}(\vec{P}) = \frac{1}{\sqrt{N}}\sum_t e^{iP_y t l_0^2} a^{\dagger}_{t+\frac{P_x}{2}} b^{\dagger}_{-t+\frac{P_x}{2}} \tag{95}$$

and their composition in all calculations is taken into account. Some of them are demonstrated below using commutation relations (91):

$$
\begin{aligned}
&a_p|0\rangle = b_p|0\rangle = 0,\\
&\hat{\rho}(\vec{Q})|0\rangle = \hat{D}(\vec{Q})|0\rangle = \hat{\Psi}_{ex}(\vec{Q})|0\rangle = 0,\\
&\hat{D}(0)\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle = 2\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle,\\
&\hat{\rho}(0)\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle = 0,\\
&\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle = 4\sin^2(Z(\vec{P},\vec{Q}))\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle,\\
&\hat{D}(\vec{Q})\hat{D}(-\vec{Q})\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle = 4\cos^2(Z(\vec{P},\vec{Q}))\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle,\\
&(\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q}) + \hat{D}(\vec{Q})\hat{\rho}(-\vec{Q}))\hat{\Psi}^{\dagger}_{ex}(\vec{P})|0\rangle = 0
\end{aligned}
\tag{96}
$$

In the present model, the main role is played by the magnetoexciton creation and annihilation operators, rather than by the plasmon density operators.

Magnetoexciton creation energy $E_{mex}(\vec{P})$ from Hamiltonian H_0 consists of three parts:

$$
\begin{aligned}
&E_{mex}(\vec{P}) = E_{mex}(R_1; M_h, \varepsilon_m^-; \vec{P}) = E_g(R_1; M_h, \varepsilon_m^-) - I_S(R_1; \varepsilon_m^-) +\\
&+2\sum_{\vec{Q}} W_{0-0}(R_1; \varepsilon_m^-; \vec{Q})\sin^2(Z(\vec{P},\vec{Q})) + 2\sum_{\vec{Q}} W_{a-a}(R_1; \varepsilon_m^-; \vec{Q})\cos^2(Z(\vec{P},\vec{Q})) =\\
&= E_g(M_h, \varepsilon_m^-; R_1) - I_l(R_1; \varepsilon_m^-) + E(R_1; \varepsilon_m^-; \vec{P})
\end{aligned}
\tag{97}
$$

The first component $E_g(S_e, R_1; S_h, M_h, \varepsilon_m^-)$ plays the role of the band gap, whereas difference $I_l(\varepsilon_m^-; R_1) - E(R_1; \varepsilon_m^-; \vec{P})$ determines the resulting ionization potential of the moving magnetoexciton with wave vector $\vec{P}$. In the limiting case $\vec{P} \to \infty$, when $\lim_{\vec{P}\to\infty} E(R_1; \varepsilon_m^-; \vec{P}) = I_l(R_1; \varepsilon_m^-)$, the resulting ionization potential vanishes and the e–h pair becomes unbound. Nevertheless, the presence of positive term $E(R_1; \varepsilon_m^-; \vec{P})$ in formula (97) plays the role of the kinetic energy of the magnetoexciton at least in the region of the small values of wave vector $\vec{P}$, where this term can be represented in a quadratic form $\frac{\hbar^2 P^2}{2M(B)}$ with effective mass $M(B)$ depending on magnetic field strength *B*. Zeroth-order Hamiltonian H_0 (94), together with the similar Hamiltonian for the cavity photons and with the Hamiltonian describing the magnetoexciton–photon interaction, gives rise to quadratic Hamiltonian H_2 forming the base of the polariton conception. It looks as

$$
\begin{aligned}
H_2 = &\sum_{\vec{k}_{\|}} E_{mex}(\vec{k}_{\|})\hat{\Psi}^{\dagger}_{ex}(\vec{k}_{\|})\hat{\Psi}_{ex}(\vec{k}_{\|}) + \sum_{\vec{k}_{\|}} \hbar\omega_{\vec{k}} c^{\dagger}_{\vec{k},-} c_{\vec{k},-} +\\
&+\sum_{\vec{k}_{\|}} \left[\varphi(\vec{k}_{\|})(\vec{\sigma}^{+}_{\vec{k}} \bullet \vec{\sigma}^{*}_{M_h}) c_{\vec{k},-} \hat{\Psi}^{\dagger}_{ex}(\vec{k}_{\|}) + \varphi^{*}(\vec{k}_{\|})(\vec{\sigma}^{-}_{\vec{k}} \bullet \vec{\sigma}_{M_h}) c^{\dagger}_{\vec{k},-} \hat{\Psi}_{ex}(\vec{k}_{\|}) \right]
\end{aligned}
\tag{98}
$$

In this expression, the chemical potentials of the magnetoexcitons and the photons are omitted until the single-particle polariton formation is investigated. They will be restored when the collective properties of the polaritons will be discussed. The diagonalization of quadratic form (98) is achieved introducing the polariton creation and annihilation operators $\hat{L}^{\dagger}_{\vec{k}_{\|}}, \hat{L}_{\vec{k}_{\|}}$ in the form of a linear superposition

$$\hat{L}_{\vec{k}_{\|}} = x(\vec{k}_{\|})\Psi_{ex}(\vec{k}_{\|}) + y(\vec{k}_{\|})c_{\vec{k},-} \tag{99}$$

It is a simplified form without the antiresonant terms because they are not introduced in the starting Hamiltonian H_2 (98). Quantities $x(\vec{k}_{\|})$ and $y(\vec{k}_{\|})$ are known as Hopfield coefficients [28, 29]. In the case where the scalar product of two circular polarized vectors equals 1, the energy spectrum of two polariton branches looks as

$$\hbar\omega(\vec{k}_{\|}) = \frac{E_{mex}(\vec{k}_{\|}) + \hbar\omega_{\vec{k}}}{2} \pm \frac{1}{2}\sqrt{(E_{mex}(\vec{k}_{\|}) - \hbar\omega_{\vec{k}})^2 + 4|\varphi(\vec{k}_{\|})|^2} \tag{100}$$

The Rabi frequency for the e–h pair in the states (R_1, ε_3^-) is as follows:

$$|\omega_R| = \left|\frac{\varphi(0)}{\hbar}\right| = \frac{e}{m_0 l_0}\sqrt{\frac{1}{L_c \hbar\omega_k}}\left|P_{cv}(0)a_0 d_0\right|\left|\vec{\sigma}_{\vec{k}}^- \bullet \vec{\sigma}_{M_h}\right| \tag{101}$$

In the absence of the RSOI, coefficients $a_0 = d_0 = 1$ and expression (101) coincides with formula (12) in [21].

The Hopfield coefficients obey to the normalization condition and are equal to

$$\begin{aligned}
&\left|x(\vec{k}_{\|})\right|^2 = \frac{\left(\hbar\omega_p(\vec{k}_{\|}) - \hbar\omega_{\vec{k}}\right)^2}{\left(\hbar\omega_p(\vec{k}_{\|}) - \hbar\omega_{\vec{k}}\right)^2 + |\varphi(\vec{k}_{\|})(\vec{\sigma}_{\vec{k}}^- \bullet \vec{\sigma}_{M_h})|^2}, \\
&\left|y(\vec{k}_{\|})\right|^2 = \frac{|\varphi(\vec{k}_{\|})(\vec{\sigma}_{\vec{k}}^- \bullet \vec{\sigma}_{M_h})|^2}{\left(\hbar\omega_p(\vec{k}_{\|}) - \hbar\omega_{\vec{k}}\right)^2 + |\varphi(\vec{k}_{\|})(\vec{\sigma}_{\vec{k}}^- \bullet \vec{\sigma}_{M_h})|^2}, \\
&\left|x(\vec{k}_{\|})\right|^2 + \left|y(\vec{k}_{\|})\right|^2 = 1, \\
&\varphi(\vec{k}_{\|})(\vec{\sigma}_{\vec{k}}^+ \bullet \vec{\sigma}_{M_h}^*) = |\varphi(\vec{k}_{\|})(\vec{\sigma}_{\vec{k}}^- \bullet \vec{\sigma}_{M_h})|\, e^{i\gamma(\vec{k}_{\|})}, \\
&|(\vec{\sigma}_{\vec{k}}^- \bullet \vec{\sigma}_{M_h})| = \left|(\vec{\sigma}_{\vec{k}}^+ \bullet \vec{\sigma}_{M_h}^*)\right|, \\
&x(\vec{k}_{\|}) = \left|x(\vec{k}_{\|})\right| e^{i\alpha(\vec{k}_{\|})},\ y(\vec{k}_{\|}) = \left|y(\vec{k}_{\|})\right| e^{i\beta(\vec{k}_{\|})}, \\
&\alpha(\vec{k}_{\|}) - \beta(\vec{k}_{\|}) + \gamma(\vec{k}_{\|}) = 0
\end{aligned} \tag{102}$$

The last equality results from the fact that polariton energy spectrum $\hbar\omega_p(\vec{k}_{\|})$, the magnetoexciton and cavity photon bare energies are real entities. This relation will be used below at point $\vec{k}_{\|} = 0$ where these phases will be simply denoted as α, β and γ.

Now the breaking of the gauge symmetry of the 2D magnetoexciton–photon system leading to the BEC of the magnetopolaritons on the lower polariton branch at point $\vec{k}_{\|} = 0$ will be discussed.

6. Breaking of the gauge symmetry and the mixed photon–magnetoexciton–acoustical plasmon states

A method to introduce the coherent macroscopic polariton states in a system of 2D e–h pairs and photons captured in the microcavity was proposed in [30, 31]. It was assumed that the e–h pairs were excited on the quantum well embedded into the microcavity and interacted with the photons captured in the resonator giving rise to the 2D Wannier–Mott excitons and polariton formation. As was shown in [30], the proposed method is equivalent to the u-v Bogoliubov transformation for the electron and hole Fermi operators and to Bogoliubov displacement transformation for the photon Bose operators. This method will be now applied to the case of 2D magnetoexcitons and photons in microcavity with the aim to investigate the BEC of magnetopolaritons in the state with $\vec{k}_{\|}=0$ on the lower polariton branch. The unitary transformation proposed in [30] looks as

$$D(\sqrt{N_p})=\exp\left(\sqrt{N_p}\left(L_0^{\dagger}-L_0\right)\right) \tag{103}$$

where N_p is a macroscopic number of the condensed polaritons at point $\vec{k}_{\|}=0$ of the lower polariton branch. The cavity photon with $\vec{k}_{\|}=0$ has a quantized longitudinal projection of its wave vector $\vec{k}$ equal to π / L_c. Only the photons with a given circular polarization are considered. In this case, we have

$$\begin{aligned} &\hat{L}_0=x(0)\Psi_{ex}(0)+y(0)c_{\frac{\pi}{L_c},-}, \\ &x(0)=|x(0)|e^{i\alpha}, \\ &y(0)=|y(0)|e^{i\beta} \end{aligned} \tag{104}$$

and the starting unitary transformation can be factorized in two independent unitary transformations acting separately in two subsystems of magnetoexcitons and of the photons as follows:

$$\begin{aligned} &D(\sqrt{N_p})=D_{ex}(\sqrt{N_p}|x(0)|)D_{ph}(\sqrt{N_p}|y(0)|), \\ &D_{ex}(\sqrt{N_p}|x(0)|)=\exp\left[\sqrt{N_p}|x(0)|\left(e^{-i\alpha}\hat{\Psi}_{ex}^{\dagger}(0)-e^{i\alpha}\hat{\Psi}_{ex}(0)\right)\right], \\ &D_{ph}(\sqrt{N_p}|y(0)|)=\exp\left[\sqrt{N_p}|y(0)|\left(e^{-i\beta}c_{\frac{\pi}{L_c},-}^{\dagger}-e^{i\beta}c_{\frac{\pi}{L_c},-}\right)\right] \end{aligned} \tag{105}$$

Taking into account the expressions for the magnetoexciton operators

$$\begin{aligned} &\hat{\Psi}_{ex}^{\dagger}(0)=\frac{1}{\sqrt{N}}\sum_t a_t^{\dagger}b_{-t}^{\dagger}, \\ &\hat{\Psi}_{ex}(0)=\frac{1}{\sqrt{N}}\sum_t b_{-t}a_t \end{aligned} \tag{106}$$

one can transcribe operator $D_{ex}(\sqrt{N_p}|x(0)|)$ in the form $D_{ex}(\sqrt{N_p}|x(0)|)=e^{z}=\prod_t e^{z_t}$, where

$$\begin{aligned} &z=\sqrt{N_p}|x(0)|\left(e^{-i\alpha}\hat{\Psi}_{ex}^{\dagger}(0)-e^{i\alpha}\hat{\Psi}_{ex}(0)\right)=\sum_t z_t, \\ &z_t=\nu_p|x(0)|\left(e^{-i\alpha}a_t^{\dagger}b_{-t}^{\dagger}-e^{i\alpha}b_{-t}a_t\right) \end{aligned} \tag{107}$$

The unitary transformations of the Fermi operator are

$$\begin{aligned}
&D_{ex}(\sqrt{N_p}\,|x(0)|)a_t D_{ex}^{-1}(\sqrt{N_p}\,|x(0)|) = e^{z_t} a_t e^{-z_t} = \alpha_t = \\
&= a_t\cos(v_p|x(0)|) - b_{-t}^{\dagger} e^{-i\alpha}\sin(v_p|x(0)|), \\
&D_{ex}(\sqrt{N_p}\,|x(0)|)b_{-t} D_{ex}^{-1}(\sqrt{N_p}\,|x(0)|) = e^{z_t} b_{-t} e^{-z_t} = \beta_{-t} = \\
&= b_{-t}\cos(v_p|x(0)|) + a_t^{\dagger} e^{-i\alpha}\sin(v_p|x(0)|)
\end{aligned} \tag{108}$$

Here, the filling factor of the Bose-Einstein condensate was introduced

$$\frac{N_p}{N} = v_p^2 \tag{109}$$

Side by side with unitary transformations (108) for the single-particle Fermi operators, one can also obtain the transformations for the two-particle integral operators. They were obtained using commutation relations (91) and look as follows

$$\begin{aligned}
&e^{\hat{z}}\frac{\hat{D}(\vec{Q})}{\sqrt{N}}e^{-\hat{z}} = \frac{\hat{D}(\vec{Q})}{\sqrt{N}}\cos(2v_p|x(0)|) - \hat{\theta}(\vec{Q})\sin(2v_p|x(0)|), \\
&e^{\hat{z}}\hat{\theta}(\vec{Q})e^{-\hat{z}} = \hat{\theta}(\vec{Q})\cos(2v_p|x(0)|) + \frac{\hat{D}(\vec{Q})}{\sqrt{N}}\sin(2v_p|x(0)|), \\
&\hat{\theta}(\vec{Q}) = e^{-i\alpha}\hat{\Psi}_{ex}^{\dagger}(\vec{Q}) + e^{i\alpha}\hat{\Psi}_{ex}(-\vec{Q}),\ e^{\hat{z}}\hat{\rho}(\vec{Q})e^{-\hat{z}} = \hat{\rho}(\vec{Q}), \\
&e^{z}e^{-i\alpha}\Psi_{ex}^{\dagger}(\vec{Q})e^{-z} = e^{-i\alpha}\Psi_{ex}^{\dagger}(\vec{Q}) + \frac{1}{2}\sin\left(2v_p\,|\,x(0)\,|\right)\frac{D(\vec{Q})}{\sqrt{N}} + \frac{1}{2}[\cos\left(2v_p\,|\,x(0)\,|\right) - 1]\theta(\vec{Q}), \\
&e^{z}e^{i\alpha}\Psi_{ex}(-\vec{Q})e^{-z} = e^{i\alpha}\Psi_{ex}(-\vec{Q}) + \frac{1}{2}\sin\left(2v_p\,|\,x(0)\,|\right)\frac{D(\vec{Q})}{\sqrt{N}} + \frac{1}{2}[\cos\left(2v_p\,|\,x(0)\,|\right) - 1]\theta(\vec{Q})
\end{aligned} \tag{110}$$

As one can see, the superposition of the magnetoexciton creation and annihilation operators in the form $\theta(\vec{Q})$ forms a coherent mixed state with acoustical plasmon density operator $\frac{\hat{D}(\vec{Q})}{\sqrt{N}}$. These mixed magnetoexciton–plasmon states were discussed in [32–34].

The full Hamiltonian of the magnetoexciton-photon system consists of four parts as follows:

$$\hat{H} = \hat{H}_{mex,1} + \hat{H}_{0,ph} + \hat{H}_{mex,2} + \hat{H}_{mex-ph} \tag{111}$$

It will be subjected to unitary gauge transformation (105), which means calculation of the following unitary transformations:

$$D_{ex}(\sqrt{N_p}\,|\,x(0)\,|)(\hat{H}_{mex,1} + \hat{H}_{mex,2})D_{ex}^{-1}(\sqrt{N_p}\,|\,x(0)\,|), \qquad D_{ph}(\sqrt{N_p}\,|\,y(0)\,|)\hat{H}_{0,ph}D_{ph}^{-1}(\sqrt{N_p}\,|\,y(0)\,|),$$

$$D_{ex}(\sqrt{N_p}\,|\,x(0)\,|)D_{ph}(\sqrt{N_p}\,|\,y(0)\,|)\hat{H}_{mex-ph}D_{ph}^{-1}(\sqrt{N_p}\,|\,y(0)\,|)D_{ex}^{-1}(\sqrt{N_p}\,|\,x(0)\,|)$$

The first of them is

$$\begin{aligned}
&\hat{\hat{H}}_{mex,1} = D_{ex}(\sqrt{N_p}\,|x(0)|)\hat{H}_{mex,1}D_{ex}^{-1}(\sqrt{N_p}\,|x(0)|) = \\
&= \left(E_{mex}(R_1;M_h,\varepsilon_m^-) - \mu_{ex}\right)\cos(2v_p\,|x(0)|)\frac{\hat{D}(0)}{2} + \\
&+(G_{e-h}(R_1;M_h,\varepsilon_m^-) - \mu_e + \mu_h)\frac{\hat{\rho}(0)}{2} - \\
&-\frac{\sqrt{N}}{2}\sin(2v_p\,|x(0)|)\left(E_{mex}(R_1;M_h,\varepsilon_m^-) - \mu_{ex}\right)\hat{\theta}(0), \\
&\mu_{ex} = \mu_e + \mu_h
\end{aligned} \tag{112}$$

The second one looks as

$$\begin{aligned}
&\hat{\hat{H}}_{mex,2} = D_{ex}(\sqrt{N_p}\,|x(0)|)\hat{H}_{mex,2}D_{ex}^{-1}(\sqrt{N_p}\,|x(0)|) = \\
&= \frac{1}{2}\sum_{\vec{Q}}\Big\{W_{0-0}(\vec{Q})\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) + W_{a-a}(\vec{Q})\Big[\cos^2(2v_p\,|x(0)|)\hat{D}(\vec{Q})\hat{D}(-\vec{Q}) + \\
&+\sin^2(2v_p\,|x(0)|)N\theta(\vec{Q})\theta(-\vec{Q}) - \\
&-\cos(2v_p\,|x(0)|)\sin(2v_p\,|x(0)|)\sqrt{N}\,(\hat{D}(\vec{Q})\theta(-\vec{Q}) + \theta(\vec{Q})\hat{D}(-\vec{Q}))\Big] + \\
&+W_{0-a}(\vec{Q})\Big[\cos(2v_p\,|x(0)|)\,(\hat{\rho}(\vec{Q})\hat{D}(-\vec{Q}) + \hat{D}(\vec{Q})\hat{\rho}(-\vec{Q})) - \\
&-\sin(2v_p\,|x(0)|)\sqrt{N}\,(\hat{\rho}(\vec{Q})\theta(-\vec{Q}) + \theta(\vec{Q})\hat{\rho}(-\vec{Q}))\Big]\Big\}
\end{aligned} \tag{113}$$

The third transformation concerns the captured photons

$$\begin{aligned}
&\hat{\hat{H}}_{0,ph} = D_{ph}(\sqrt{N_p}\,|y(0)|)\hat{H}_{0,ph}D_{ph}^{-1}(\sqrt{N_p}\,|y(0)|) = \\
&= \left(\hbar\omega_{\frac{\pi}{L_c}} - \mu_{ph}\right)N_p\,|y(0)|^2 + \sum_{\vec{k}_\parallel}\left(\hbar\omega_{\vec{k}} - \mu_{ph}\right)c_{\vec{k},-}^{\dagger}c_{\vec{k},-} - \\
&-\sqrt{N_p}\,|y(0)|\left(\hbar\omega_{\frac{\pi}{L_c}} - \mu_{ph}\right)\left(e^{-i\beta}c_{\frac{\pi}{L_c},-}^{\dagger} + e^{i\beta}c_{\frac{\pi}{L_c},-}\right), \\
&\vec{k} = \frac{\pi}{L_c}\vec{a}_3 + \vec{k}_\parallel,\ \vec{k}_\parallel = \vec{a}_1k_x + \vec{a}_2k_y
\end{aligned} \tag{114}$$

The last transformation involves the magnetoexciton and photons operators as follows:

$$\begin{aligned}
&\hat{\hat{H}}_{mex-ph} = D(\sqrt{N_p})\hat{H}_{mex-ph}D^{-1}(\sqrt{N_p}) = \\
&D_{ex}(\sqrt{N_p}\,|x(0)|)\{\hat{H}_{mex-ph} - \sqrt{N_p}\,|y(0)|[\varphi(0)e^{-i\beta}(\vec{\sigma}^{+}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}^{*}_{M_h})\hat{\Psi}^{\dagger}_{ex}(0) + \\
&+\varphi^{*}(0)e^{i\beta}(\vec{\sigma}^{-}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}_{M_h})\hat{\Psi}_{ex}(0)]\}D^{-1}_{ex}(\sqrt{N_p}\,|x(0)|) = \\
&= -\sqrt{N_p}\,|y(0)|\{\hat{\Psi}^{\dagger}_{ex}(0)[\frac{1}{2}(\cos(2v_p\,|x(0)|)+1)\varphi(0)e^{-i\beta}(\vec{\sigma}^{+}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}^{*}_{M_h}) + \\
&+\frac{1}{2}(\cos(2v_p\,|x(0)|)-1)\varphi^{*}(0)e^{i\beta-2i\alpha}(\vec{\sigma}^{-}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}_{M_h})] + \\
&+\hat{\Psi}_{ex}(0)[\frac{1}{2}(\cos(2v_p\,|x(0)|)+1)\varphi^{*}(0)e^{i\beta}(\vec{\sigma}^{-}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}_{M_h}) + \\
&+\frac{1}{2}(\cos(2v_p\,|x(0)|)-1)\varphi(0)e^{-i\beta+2i\alpha}(\vec{\sigma}^{+}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}^{*}_{M_h})] + \\
&+\frac{\hat{D}(0)}{2\sqrt{N}}\sin(2v_p\,|x(0)|)[\varphi(0)e^{-i\beta+i\alpha}(\vec{\sigma}^{+}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}^{*}_{M_h}) + \varphi^{*}(0)e^{i\beta-i\alpha}(\vec{\sigma}^{-}_{\frac{\pi}{L_c}}\bullet\vec{\sigma}_{M_h})]\} + \\
&+\frac{1}{2}(\cos(2v_p\,|x(0)|)+1)\sum_{\vec{k}_{\parallel}}[\varphi(\vec{k}_{\parallel})(\vec{\sigma}^{+}_{\vec{k}}\bullet\vec{\sigma}^{*}_{M_h})c_{\vec{k},-}\Psi^{\dagger}_{ex}(\vec{k}_{\parallel}) + \varphi^{*}(\vec{k}_{\parallel})(\vec{\sigma}^{-}_{\vec{k}}\bullet\vec{\sigma}_{M_h})c^{\dagger}_{\vec{k},-}\Psi_{ex}(\vec{k}_{\parallel})] + \\
&+\frac{1}{2}(\cos(2v_p\,|x(0)|)-1)\sum_{\vec{k}_{\parallel}}[\varphi(\vec{k}_{\parallel})e^{2i\alpha}(\vec{\sigma}^{+}_{\vec{k}}\bullet\vec{\sigma}^{*}_{M_h})c_{\vec{k},-}\Psi_{ex}(-\vec{k}_{\parallel}) + \qquad (115) \\
&+\varphi^{*}(\vec{k}_{\parallel})e^{-2i\alpha}(\vec{\sigma}^{-}_{\vec{k}}\bullet\vec{\sigma}_{M_h})c^{\dagger}_{\vec{k},-}\Psi^{\dagger}_{ex}(-\vec{k}_{\parallel})] + \\
&+\frac{1}{2}\sin(2v_p\,|x(0)|)\sum_{\vec{k}_{\parallel}}[\varphi(\vec{k}_{\parallel})e^{i\alpha}(\vec{\sigma}^{+}_{\vec{k}}\bullet\vec{\sigma}^{*}_{M_h})c_{\vec{k},-}\frac{\hat{D}(\vec{k}_{\parallel})}{\sqrt{N}} + \varphi^{*}(\vec{k}_{\parallel})e^{-i\alpha}(\vec{\sigma}^{-}_{\vec{k}}\bullet\vec{\sigma}_{M_h})c^{\dagger}_{\vec{k},-}\frac{\hat{D}^{\dagger}(\vec{k}_{\parallel})}{\sqrt{N}}]
\end{aligned}$$

Taking into account relation (102) between phases α,β and γ and definition (110) of operator $\theta(0)$, one can represent the transformed Hamiltonian with the broken gauge symmetry in the form

$$\hat{\hat{H}} = N_p |y(0)|^2 \left(\hbar\omega_{\frac{\pi}{L_c}} - \mu_{ph} \right) - \sqrt{N_p} |y(0)| \left(\hbar\omega_{\frac{\pi}{L_c}} - \mu_{ph} \right) \left(e^{-i\beta} c^{\dagger}_{\frac{\pi}{L_c},-} + e^{i\beta} c_{\frac{\pi}{L_c},-} \right) -$$
$$-\hat{\theta}(0)\sqrt{N_p} \left[\frac{1}{2} \left(E_{mex}(R_1; M_h, \varepsilon_m^-) - \mu_{ex} \right) \sin(2v_p |x(0)|) + \right.$$
$$\left. + v_p |y(0)| \left| \varphi(0) \left(\vec{\sigma}^-_{\frac{\pi}{L_c}} \bullet \vec{\sigma}_{M_h} \right) \right| \cos(2v_p |x(0)|) \right] +$$
$$+ \frac{\hat{D}(0)}{2} \left[E_{mex}(R_1; M_h, \varepsilon_m^-) - \mu_{ex} - 2v_p \sin(2v_p |x(0)|) |y(0)| \left| \varphi(0) \left(\vec{\sigma}^-_{\frac{\pi}{L_c}} \bullet \vec{\sigma}_{M_h} \right) \right| \right] +$$
$$+ \left(G_{e-h}(R_1; M_h, \varepsilon_m^-) - \mu_e + \mu_h \right) \frac{\hat{\rho}(0)}{2} + \sum_{\vec{k}_{\parallel}} \left(\hbar\omega_{\vec{k}} - \mu_{ph} \right) c^{\dagger}_{\vec{k},-} c_{\vec{k},-} +$$
$$+ \frac{1}{2} \sum_{\vec{Q}} \left\{ W_{0-0}(\vec{Q}) \hat{\rho}(\vec{Q}) \hat{\rho}(-\vec{Q}) + W_{a-a}(\vec{Q}) \left[\cos^2(2v_p |x(0)|) \hat{D}(\vec{Q}) \hat{D}(-\vec{Q}) + \right. \right.$$
$$+ \sin^2(2v_p |x(0)|) N \hat{\theta}(\vec{Q}) \hat{\theta}(-\vec{Q}) -$$
$$\left. - \frac{1}{2} \sin(4v_p |x(0)|) \sqrt{N} \left(\hat{D}(\vec{Q}) \hat{\theta}(-\vec{Q}) + \hat{\theta}(\vec{Q}) \hat{D}(-\vec{Q}) \right) \right] +$$
$$+ W_{0-a}(\vec{Q}) \left[\cos(2v_p |x(0)|) \left(\hat{\rho}(\vec{Q}) \hat{D}(-\vec{Q}) + \hat{D}(\vec{Q}) \hat{\rho}(-\vec{Q}) \right) - \right.$$
$$\left. \left. - \sin(2v_p |x(0)|) \sqrt{N} \left(\hat{\rho}(\vec{Q}) \hat{\theta}(-\vec{Q}) + \hat{\theta}(\vec{Q}) \hat{\rho}(-\vec{Q}) \right) \right] \right\} +$$
$$+ \frac{1}{2} (\cos(2v_p |x(0)|) + 1) \sum_{\vec{k}_{\parallel}} [\varphi(\vec{k}_{\parallel}) (\vec{\sigma}^+_{\vec{k}} \bullet \vec{\sigma}^*_{M_h}) c_{\vec{k},-} \Psi^{\dagger}_{ex}(\vec{k}_{\parallel}) + \varphi^*(\vec{k}_{\parallel}) (\vec{\sigma}^-_{\vec{k}} \bullet \vec{\sigma}_{M_h}) c^{\dagger}_{\vec{k},-} \Psi_{ex}(\vec{k}_{\parallel})] + \tag{116}$$
$$+ \frac{1}{2} (\cos(2v_p |x(0)|) - 1) \sum_{\vec{k}_{\parallel}} [\varphi(\vec{k}_{\parallel}) e^{2i\alpha} (\vec{\sigma}^+_{\vec{k}} \bullet \vec{\sigma}^*_{M_h}) c_{\vec{k},-} \Psi_{ex}(-\vec{k}_{\parallel}) +$$
$$+ \varphi^*(\vec{k}_{\parallel}) e^{-2i\alpha} (\vec{\sigma}^-_{\vec{k}} \bullet \vec{\sigma}_{M_h}) c^{\dagger}_{\vec{k},-} \Psi^{\dagger}_{ex}(-\vec{k}_{\parallel})] +$$
$$+ \frac{1}{2} \sin(2v_p |x(0)|) \sum_{\vec{k}_{\parallel}} [\varphi(\vec{k}_{\parallel}) e^{i\alpha} (\vec{\sigma}^+_{\vec{k}} \bullet \vec{\sigma}^*_{M_h}) c_{\vec{k},-} \frac{\hat{D}(\vec{k}_{\parallel})}{\sqrt{N}} + \varphi^*(\vec{k}_{\parallel}) e^{-i\alpha} (\vec{\sigma}^-_{\vec{k}} \bullet \vec{\sigma}_{M_h}) c^{\dagger}_{\vec{k},-} \frac{\hat{D}^{\dagger}(\vec{k}_{\parallel})}{\sqrt{N}}]$$

Looking at this expression, one may conclude that, side by side with the u-v-type transformation (110) of magnetoexciton superposition-type operator $\hat{\theta}(\vec{Q})$ and acoustical plasmon density operator $\hat{D}(\vec{Q})/\sqrt{N}$, another mixed state of the acoustical plasmon–photon type appeared under the influence of the magnetoexciton–polariton BEC. In addition to them, there are anti-resonant-type terms in the magnetoexciton-photon interaction, even if they were not included in initial Hamiltonian (87). The obtained results permit determining chemical potentials μ_{ex} and μ_{ph} and investigating the energy spectrum of the collective elementary excitations.

7. Conclusions

The influence of the RSOC on the properties of the 2D magnetoexcitons was described taking into account the results concerning the Landau quantization of the 2D electrons and holes with nonparabolic dispersion laws, pseudospin components and chirality terms [18, 19, 22]. The main attention was paid to the study of operators $\hat{\rho}(\vec{Q})$ and $\hat{D}(\vec{Q})$ that, together with magnetoexciton creation and annihilation operators $\hat{\Psi}^{\dagger}_{ex}(\vec{k}_{\|})$ and $\hat{\Psi}_{ex}(\vec{k}_{\|})$, form a set of four two-particle integral operators. It was shown that the Hamiltonians of the electron-radiation and Coulomb electron–electron interactions can be expressed in terms of these four integral two-particle operators. The unitary transformation breaking the gauge symmetry of the deduced Hamiltonian and the BEC of the magnetoexciton–polaritons were introduced in the frame of the Keldysh–Kozlov–Kopaev method using the polariton creation and annihilation operators. They were expressed in terms of the same magnetoexciton and photon operators using the Hopfield coefficients in a simplified form without the anti-resonance terms because the energies of the participant quasiparticles are finite situated near the energy of the cavity mode. The unitary transformation is factorized as a product of two unitary transformations acting independently in two magnetoexciton and photon subsystems. It was realized that the BEC of magnetoexciton polaritons supplementary gives rise to the acoustical plasmon–photon interaction and to a new type plasmon–polariton formation. The antiresonance terms of the magnetoexciton–photon interaction also appeared even if they were neglected in the starting Hamiltonian. The mixed magnetoexciton–acoustical plasmon states in the absence of the RSOC were investigated in [32–34]. The obtained final transformed Hamiltonian will be used to study the collective elementary excitations.

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