METHODS OF CALCULATING THE HEAT TRANSFER BETWEEN NON-NEWTONIAN FLUID AND THE ENVIRONMENT

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1. INTRODUCTION

In previous papers some problems of non-Newtonian fluid flow in the channels were solved. The two classes of liquids were considered: Bingham fluid and the generalized shear. A special case of the latter is the power fluid. Heat transfer in channels with the environment is considered in two cases. In the first case the external environment is an infinite heat reservoir with a given temperature. In the second case, the role of the environment performs a channel in which the coolant moves. In the second case, the temperature of the coolant can not be given, it varies along the length of the channel. The equations of heat transfer contain convection terms and the terms of the thermal conductivity.

2. RECENT INVESTIGATIONS

As shown in previous studies heat transfer in a channel with non-Newtonian fluid occurs at small values of the Peclet number; despite the fact that Bingham and the generalized shear liquids have higher viscosity. The movement of fluid in the channel is considered inertial and also corresponds to small values of Peclet number. Thus, in the hydrodynamic aspect the non-Newtonian fluid and coolant are moving in different modes, where in the thermal aspect they are moving in the same mode.

3. RESULTS OF RESEARCH

The heat transfer scheme for Bingham fluid flow in the channel that is immersed in a thermal reservoir is shown on Fig. 1 a and b.

The heat transfer scheme of generalized shear fluid in the channel that is immersed in a thermal reservoir, is shown on Fig. 2 a, b. For the case of thermal interaction of non-Newtonian fluid in the channel with the coolant in another channel, the corresponding images are shown on Fig. 3, Fig. 4a and b, Fig. 5 a and b. From Fig. 3 follows that we consider the general case such that the coolant channel is pushing at an angle to the channel with non-Newtonian fluid.

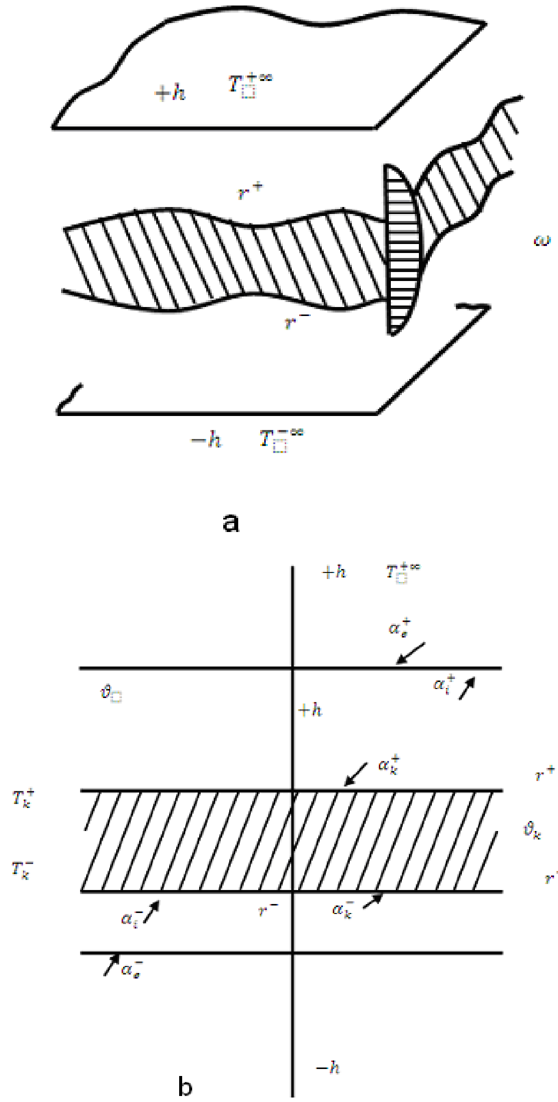

Figure 1. Bingham fluid: *a – view in dimensional space; b – scheme of flow and characteristics of heat transfer with heat reservoir.*

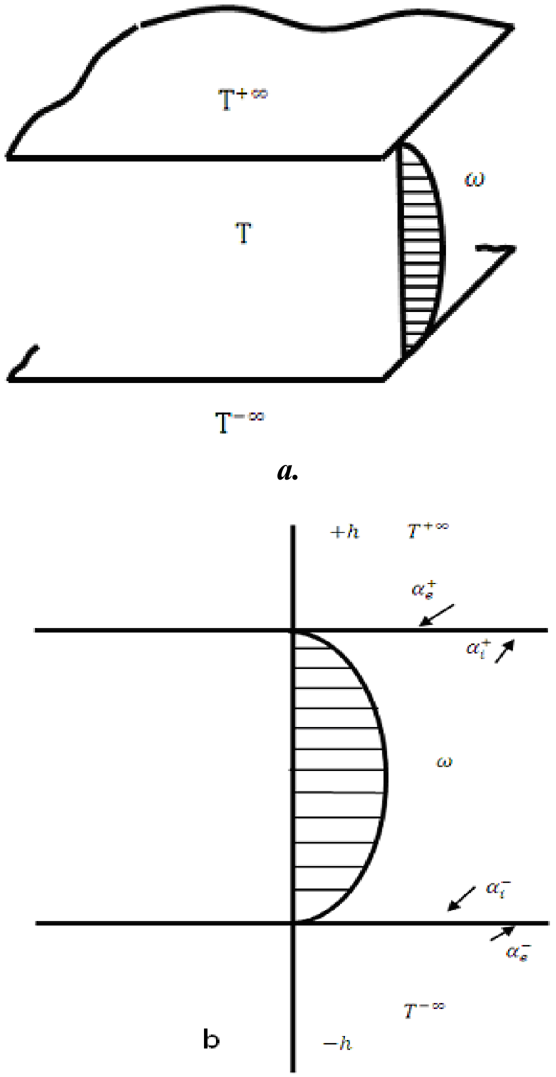

Figure 2. Generalized-shear fluid: *a – view in dimensional space; b – scheme of flow and characteristics of heat transfer with heat reservoir*

For this reason, two different interrelated longitudinal coordinate z and z_1 should be given. Then coordinate measured along the axis of the channel with non-Newtonian fluid is used as the longitudinal coordinate z, while the coordinate z_1 is attributed to wrapping channel with coolant. If the ascent angle of the wrapping channel is equal to zero, it means that both channels interact in forward flow or counter flow way. If the ascent angle of the wrapping channel is equal to π/2, then the two channels interact in cross flow way. When describing the heat exchange with generalized shear fluid the four heat-transfer coefficients (see Fig. 2b) for the thermal reservoir, and eight heat transfer coefficients for the wrapping channel with coolant (see Fig. 5b) should be used.

When describing the heat transfer with Bingham fluid the six heat-transfer coefficients for heat reservoir and ten heat transfer coefficients for the wrapping channel with coolant (see Fig. 1b and 4b) should be used. During the heat transfer in the core of Bingham fluid the heat is transported according to the heat conduction mechanism.

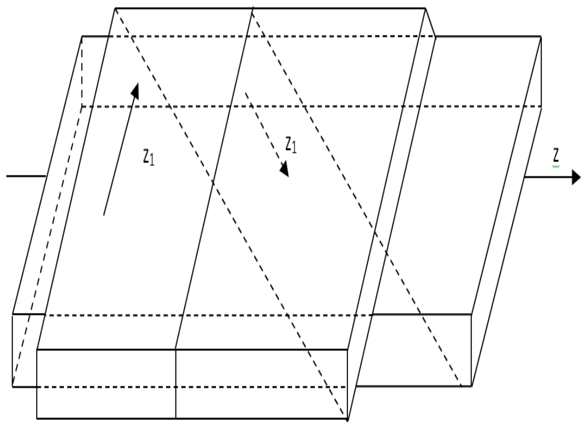

Figure 3. Schematic representation of relative disposition of direct and wrapping channel

For Bingham fluid heat transfer in a channel with a wall with a given temperature the following equations are performed:

$$\begin{aligned}
&\dot V^{+}\rho\, c_p \frac{dT^{+}}{dz} = \dot e^{+} s^{+} + \alpha_i^{+}(T_h^{+} - T^{+})p^{+} + \\
&+\alpha_k^{+}(T^{+} - T_k^{+})p_k^{+}, \\
&\dot V^{-}\rho\, c_p \frac{dT^{-}}{dz} = \dot e^{-} s^{-} + \alpha_i^{-}(T_h^{-} - T^{-})p^{-} + \\
&+\alpha_k^{-}(T^{-} - T_k^{-})p_k^{-}, \\
&\frac{\partial^2 T_k}{\partial y^2} + \frac{\partial^2 T_k}{\partial z^2} = 0\,;\; -\lambda_k \left.\frac{\partial T_k}{\partial y}\right|_{\Gamma^{\pm}} = \alpha_k^{\pm}(T_\Gamma - T^{\pm})\Big|_{\Gamma^{\pm}},
\end{aligned} \quad (1)$$

Where: $\dot V^{\pm}$ – the expenses of the fluid flow above and below the core flow, m^3/sec; ρ – density of the Bingham fluid, kg/m^3; c_p – heat capacity of the Bingham fluid, J/kg·deg; $T^{\pm}$ – the temperature of streaming portion above and below the core flow, deg; $\dot e^{\pm}$ – specific dissociations of energy of the fluid above and below the core flow, J/m^3; $s^{\pm}$ – cross-sectional areas of the fluid above and below the core flow, m^2; $T_h^{\pm}$ – temperatures of higher and lower walls, deg; $\alpha_i^{\pm}$ – the heat transfer coefficients on the walls at lines y = ±h, J/m2·sec·deg; $\alpha_k^{\pm}$ – heat transfer coefficients at the boundaries of the core flow on the lines y = $G^{\pm}$, J/m^2·sec·deg; $p^{\pm}$ – perimeters of channel

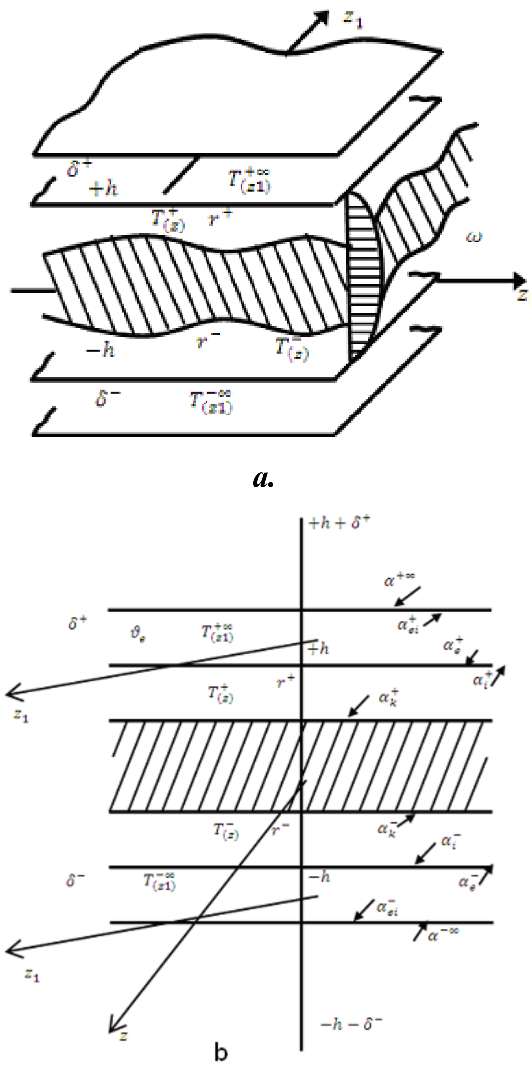

Figure 4. Bingham fluid*: a – view in dimensional section of direct and wrapping channel; b – scheme of flow and characteristics of heat transfer with heat reservoir.*

cross section walls, m; $p_k^{\pm}$ – perimeters of boundaries of the core flow, m;λ, λ_k – thermal conductivity coefficients of streaming portion and Bingham fluid core, respectively, J/m·sec·deg.

If temperatures of the channel walls are known (which happens often), the heat transfer equations take the following form:

$$\dot{V}^{+}\rho\, c_p \frac{dT^{+}}{dz} = \dot{e}^{+} s^{+} + K^{+}(T^{+\infty} - T^{+})p^{+} + $$
$$+ K_k^{+}(T^{+} - T_k^{+})p_k^{+},$$
$$\dot{V}^{-}\rho\, c_p \frac{dT^{-}}{dz} = \dot{e}^{-} s^{-} + K^{-}(T^{-\infty} - T^{-})p^{-} + \quad (2)$$
$$+ K_k^{-}(T^{-} - T_k^{-})p_k^{-},$$

$$\frac{\partial^2 T_k}{\partial y^2} + \frac{\partial^2 T_k}{\partial z^2} = 0\,;$$
$$-\lambda_k \frac{\partial T_k}{\partial y}\bigg|_{\Gamma^{\pm}} = \alpha_k^{\pm}(T_{\Gamma} - T^{\pm})\Big|_{\Gamma^{\pm}},$$

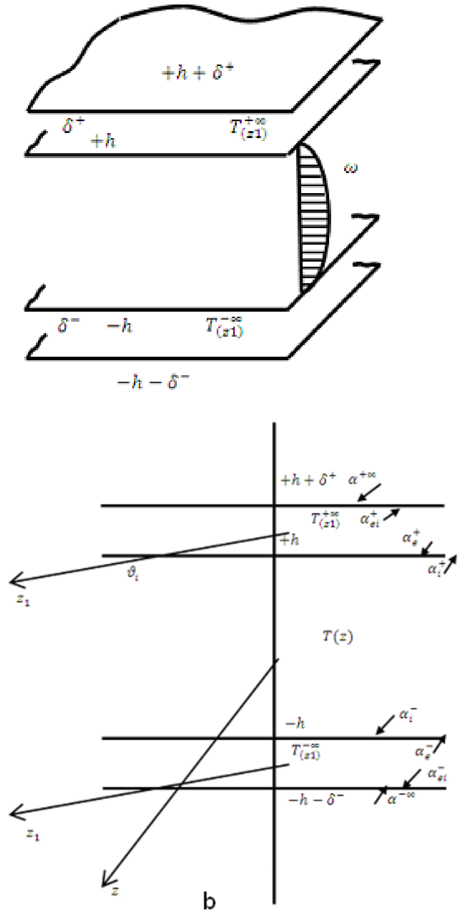

Figure 5. Generalized-shear fluid: *a – view in dimensional section of direct and wrapping channel; b – scheme of flow and characteristics of heat transfer with heat reservoir.*

Where: $T^{\pm\infty}$ – temperatures of heat reservoir above and below the channel, deg; $K^{\pm}$ – heat transfer coefficients between parts of the heat reservoir and the streaming portion of the Bingham fluid above and below the core flow, J/m sec deg; $K_k^{\pm}$ – heat transfer coefficients between the streaming portion and the core of Bingham fluid, J/m^2·sec·deg. The heat transfer coefficients are based on heat removal coefficients according to the usual rules.

The equation of heat transfer in a channel of generalized shear fluid with the specified wall temperatures has the following form:

$$\dot{V}\,\rho\,c_p\,\frac{dT}{dz}=\dot{e}s+\alpha_i^+(T_h^+-T)p^+ + \\ +\alpha_i^+(T_h^- -T)p^-, \qquad (3)$$

Where the meaning of all symbols is the same as in (1) and (2); except for the absence of difference between higher and lower flows in relation to the core flow.

The equation of heat transfer in a channel with a thermal reservoir can be represented as follows:

$$\dot{V}\,\rho\,c_p\,\frac{dT}{dz}=\dot{e}s+K^+(T^{+\infty}-T)p^+ + \\ +K^-(T^{-\infty}-T)p^-. \qquad (4)$$

The record of the equations of heat transfer with the given wall temperatures for the flow of Bingham fluid in a rectangular channel should consider the partition of channel cross-section. Moreover, the number of heat transfer coefficients is doubled. In the longitudinal and transverse flow the partition for the longitudinal part of the flow and partitioning of the transverse part of the flow may not coincide.

The equations of heat transfer for the flow of Bingham fluid in a rectangular channel with the given wall temperatures have the following form:

$$\dot{V}_y^+\,\rho\,c_p\,\frac{dT_y^+}{dz}=\dot{e}_y^+s_y^+ +\alpha_{iy}^+(T_{hy}^+-T_y^+)p_y^+ + \\ +\alpha_{ky}^+(T_y^+-T_{k\Gamma}^+)p_{ky}^+,$$

$$\dot{V}_y^-\,\rho\,c_p\,\frac{dT_y^-}{dz}=\dot{e}_y^-s_y^- +\alpha_{iy}^-(T_{hy}^--T_y^-)p_y^- + \\ +\alpha_{ky}^-(T_y^--T_{k\Gamma}^-)p_{ky}^-,$$

$$\dot{V}_x^+\,\rho\,c_p\,\frac{dT_x^+}{dz}=\dot{e}_x^+s_x^+ +\alpha_{ix}^+(T_{hx}^+-T_x^+)p_x^+ + \\ +\alpha_{kx}^+(T_x^+-T_{k\Gamma}^+)p_{kx}^+, \qquad (5)$$

$$\dot{V}_x^-\,\rho\,c_p\,\frac{dT_x^-}{dz}=\dot{e}_x^-s_x^- +\alpha_{ix}^-(T_{hx}^--T_x^-)p_x^- + \\ +\alpha_{kx}^-(T_x^--T_{k\Gamma}^-)p_{kx}^-,$$

$$\frac{\partial^2 T_k}{\partial x^2}+\frac{\partial^2 T_k}{\partial z^2}=0\,;\ -\lambda_k\left.\frac{\partial T_k}{\partial y}\right|_{\Gamma_y^\pm}=\alpha_{ky}^\pm(T_{ky}^\pm-T_y^\pm)\Big|_{\Gamma_y^\pm},$$

$$\frac{\partial^2 T_k}{\partial y^2}+\frac{\partial^2 T_k}{\partial z^2}=0\,;\ -\lambda_k\left.\frac{\partial T_k}{\partial x}\right|_{\Gamma_x^\pm}=\alpha_{kx}^\pm(T_{kx}^\pm-T_x^\pm)\Big|_{\Gamma_x^\pm}$$

The equations of heat transfer for the flow of Bingham fluid in a heat reservoir with the given wall temperatures have the following form:

$$\dot{V}_y^+\,\rho\,c_p\,\frac{dT_y^+}{dz}=\dot{e}_y^+s_y^+ +K_y^+(T_y^{+\infty}-T_y^+)p_y^+ + \\ +K_{ky}^+(T_y^+-T_{ky}^+)p_{ky}^+,$$

$$\dot{V}_y^-\,\rho\,c_p\,\frac{dT_y^-}{dz}=\dot{e}_y^-s_y^- +K_y^-(T_y^{-\infty}-T_y^-)p_y^- + \\ +K_{ky}^-(T_y^--T_{ky}^-)p_{ky}^-,$$

$$\dot{V}_x^+\,\rho\,c_p\,\frac{dT_x^+}{dz}=\dot{e}_x^+s_x^+ +K_x^+(T_x^{+\infty}-T_x^+)p_x^+ + \\ +K_{kx}^+(T_x^+-T_{kx}^+)p_{kx}^+, \qquad (6)$$

$$\dot{V}_x^-\,\rho\,c_p\,\frac{dT_x^-}{dz}=\dot{e}_x^-s_x^- +K_x^-(T_x^{-\infty}-T_x^-)p_x^- + \\ +K_{kx}^-(T_x^--T_{kx}^-)p_{kx}^-,$$

$$\frac{\partial^2 T_k}{\partial x^2}+\frac{\partial^2 T_k}{\partial z^2}=0\,;$$

$$-\lambda_k\left.\frac{\partial T_k}{\partial y}\right|_{\Gamma_y^\pm}=\alpha_{ky}^\pm(T_{ky}^\pm-T_y^\pm)\Big|_{\Gamma_y^\pm},$$

$$\frac{\partial^2 T_k}{\partial y^2}+\frac{\partial^2 T_k}{\partial z^2}=0\,;$$

$$-\lambda_k\left.\frac{\partial T_k}{\partial x}\right|_{\Gamma_x^\pm}=\alpha_{kx}^\pm(T_{kx}^\pm-T_x^\pm)\Big|_{\Gamma_x^\pm}.$$

The heat transfer of generalized shear fluid in a rectangular channel with the given wall temperatures is represented by the following equations:

$$\dot{V}_y^+\,\rho\,c_p\,\frac{dT_y^+}{dz}=\dot{e}_y^+s_y^+ +\alpha_{iy}^+(T_h^+-T_y^+)p_y^+,$$

$$\dot{V}_y^-\,\rho\,c_p\,\frac{dT_y^-}{dz}=\dot{e}_y^-s_y^- +\alpha_{iy}^-(T_h^--T_y^-)p_y^-,$$

$$\dot{V}_x^+\,\rho\,c_p\,\frac{dT_x^+}{dz}=\dot{e}_x^+s_x^+ +\alpha_{ix}^+(T_a^+-T_x^+)p_x^+, \qquad (7)$$

$$\dot{V}_x^-\,\rho\,c_p\,\frac{dT_x^-}{dz}=\dot{e}_x^-s_x^- +\alpha_{ix}^-(T_a^--T_x^-)p_x^-.$$

Equations (7) in case of submerging into the a heat reservoir have the following form:

$$\dot{V}_y^+ \rho\, c_p \frac{dT_y^+}{dz} = \dot{e}_y^+ s_y^+ + K_y^+ (T_y^{+\infty} - T_y^+) p_y^+ ,$$
$$\dot{V}_y^- \rho\, c_p \frac{dT_y^-}{dz} = \dot{e}_y^- s_y^- + K_y^- (T_y^{-\infty} - T_y^-) p_y^- ,$$
$$\dot{V}_x^+ \rho\, c_p \frac{dT_x^+}{dz} = \dot{e}_x^+ s_x^+ + K_x^+ (T_x^{+\infty} - T_x^+) p_x^+ , \qquad (8)$$
$$\dot{V}_x^- \rho\, c_p \frac{dT_x^-}{dz} = \dot{e}_x^- s_x^- + K_x^- (T_x^{-\infty} - T_x^-) p_x^- .$$

Equations (2), (4), (6), (8), in case where a flat or rectangular channel is covered with wrapping channel, in which a liquid coolant flows, should be supplemented by equations for heat reservoir temperature variations. In order not to rewrite all the equations anew, we can consider only the equations of heat transfer for the thermal reservoir. These equations are as follows:

$$\dot{V}_e^+ \rho_e\, c_{pe} \frac{dT_e^+}{dz_e} = K^+ (T_e^+ - T^+) p^+ +$$
$$+ K_e^+ (T_e^+ - T_{ee}^+) p_{ee}^+ ,$$
(9)
$$\dot{V}_e^- \rho_e\, c_{pe} \frac{dT_e^-}{dz_e} = K^- (T_e^- - T^-) p^- +$$
$$+ K_e^- (T_e^- - T_{ee}^-) p_{ee}^- .$$

For flat channel with Bingham fluid. In these equations $T_e^{\pm}$ shows the temperature of the channel with coolant, deg; index “ee” mean the environment external in relation to the channel with coolant. Temperatures $T_e^{\pm}$ in case of heat reservoir with fixed temperatures gain the values of $T^{\pm\infty}$. The the rectangular channel the number of (9) equations is doubled; and all the values that are contained in these equations should be supplemented with indexes x and y. For the generalized shear fluid the same should be done. Variables z and z_e as well as the relation between them will be discussed later. If a channel with a very viscous liquid is surrounded by a wrapping channel, then both for the Bingham fluid and for the generalized shear fluid the single $T_e(z_1)$ temperature will be instead of the individual temperatures $T_e^{\pm}$. But this single temperature will be taken in the different points along the z_1 coordinate. The detailed construction will be performed below.

The form of equations (1) ÷ (9) shows that the longitudinal temperature field can be set by the following sequence of actions: calculate the values of specific dissipations, build line borders of rectangle partition in the section of the channel, calculate the subdomains areas, calculate integrals of the specific dissipation of subdomains; calculate the heat transfer coefficients on the solid walls and boundaries (for Bingham fluid), construct the heat transfer coefficients, and solve the problem of heat conduction in the core of Bingham fluid.

All these questions are consistently described below, but before examining them it is important to derive the equations of heat transfer for an arbitrarily oriented heat transfer direction in relation to the main channel.

This situation is realized during the heat transfer between the direct channel and the channel that is wrapping the direct channel with an arbitrary ascent angle of its axis’s helix line. The longitudinal coordinates z and z_e of the wrapping channel and the main channel do not coincide. The relationship between these coordinates is established through the ascent angle φ and has the following form:

$$z_e = z\sqrt{1 + tg^2\varphi} . \qquad (10)$$

If the specific cross-section is selected in the direct channel, then the opposite sides of this section will correspond to the different cross-sections of the wrapping channel, which are arranged one after another and fill some gap along z_e coordinate. For each set of sections of the wrapping channel which are related to the one side of the direct channel’s cross-section, the average (in terms of its location) section can be specified. This average section is related to some definite coordinate z_e. If this procedure is repeated for all sides of the main channel’s cross-section, then the rectangular cross-section of the direct channel will be surrounded by the cross-section of the channel with temperature T_e taken at four different points along the coordinate z_e. Arguments z_{ei} for temperature T_e take the following values:

$$z_{e1} = \xi\, z ;\; z_{e2} = \xi\, z + \left(\frac{a+h}{2} + \delta\right)\sqrt{1-\xi^2} ;$$
$$z_{e3} = \xi\, z + \left(\frac{2a+2h}{2} + 2\delta\right)\sqrt{1-\xi^2} ;$$
$$z_{e4} = \xi\, z + \left(\frac{3a+3h}{2} + 3\delta\right)\sqrt{1-\xi^2} ; \qquad (11)$$
$$\xi \equiv 1 + tg^2\varphi ,$$

where: a, h, δ – dimensions of cross-section rectangle of the direct channel and width of wrapping channel, respectively, m. The heat transfer between generalized-shear fluid and coolant in flat channel can be taken as an example of heat transfer equations. This simplest case demonstrates all the basic features of the cross-precise heat transfer. In this case, the equations have a complex form:

$$\dot{V}\rho\, c_p \frac{dT}{dz} = \dot{e}s + K_1(T_1^\infty - T)a + K_2(T_2^\infty - T)h + $$
$$+ K_3(T_3^\infty - T)a + K_4(T_4^\infty - T)h,$$

(1)

$$\dot{V}_e \rho_e\, c_{pe} \frac{dT_y^+}{dz} = -K_1(T_1^\infty - T)a - K_2(T_2^\infty - T)h -$$
$$- K_3(T_3^\infty - T)a - K_4(T_4^\infty - T)h -$$
$$- K_{e1}(T_1^\infty - T_{1ee})(a+2\delta) - K_{e2}(T_2^\infty - T_{2ee})(h+2\delta)$$
$$- K_{e3}(T_3^\infty - T_{3ee})(a+2\delta) - K_{4e}(T_4^\infty - T_{4ee})(h+2\delta),$$

$$K_i \equiv K_i(z_{ei})\,;\; K_{ie} = K_{ie}(z_{ei})\,,\; T_i^\infty = T_i^\infty(z_{ei})\,,$$

where: T_{iee} – temperatures of the environment which surrounds the wrapping channel, deg. The second equation in (12) retains its form for both generalized-shear fluid and Bingham fluid in direct channel. Addition of equations (1) ÷ (9) to this equation.

4. CONCLUSIONS

It follows that the heat transfer equation for a system of "*direct channel plus wrapping channel*" is a system of differential equations in finite differences. And this is their main difference from the corresponding equations for the cases of fixed temperatures on the walls of direct channel and the cases of direct channel immersion in the heat reservoir with fixed temperature.

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