ANALYSIS OF PRECESSIONAL GEAR GEOMETRIC PARAMETERS INFLUENCE ON THE TEETH HEIGHT AT THEIR PROCESSING BY PLASTIC DEFORMATION

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INTRODUCTION

One of the ways to enhance manufacturing processes of gears is to use volume processing by plastic deformation. Reduced consumption of energy, materials and workmanship by plastic deformation processing is influenced by correct determination of the dimensions of semi-finished goods. An important advantage of plastic deformation processing is significant improvement in the surface layer of components during the deformation process. The power required for plastic deformation is determined by the dimensions of the product, the nature of the material, temperature regime, the deformation speed and degree, geometry of the deformation roller, etc.

1. ANALYSIS OF PRECESSIONAL GEAR GEOMETRIC PARAMETERS INFLUENCE ON THE TEETH HEIGHT

Analysis of teeth profilograms obtained for different geometrical parameters (θ, β, δ and Z) [1] shows that their height (basic parameter determining the processing time and efficiency, also energetic parameters of the plastic deformation installation) is variable and depends on these parameters, and also on the outer conical radius R_{out} in a directly proportional form. The following is an analysis of the influence of nominated geometrical parameters on the tooth height [1, 2].

Dependence of the tooth height on the outer conical radius of the gear wheel in precessional gearing is demonstrated by the relation:

$$h_{teeth}=f\,(R_{out}).$$

Figure 1 shows the dependence of tooth height h on the outer conical radius of the toothed rim R_{out} for various values of the angle of nutation θ for the case when gearing multiplicity is $\varepsilon = 100\%$.

Influence of the number of teeth *Z*.

Analysis of profilograms [1] has demonstrated that teeth height is constant while the number of teeth varies. But increase of the number of teeth leads to unexpected reduction of the gearing angle α_w and, consequently, of the pressure angle in the tool-semi product contact, which is an important parameter for the justification of geometric parameters of the deformation device (increase of the pressure angle in contact leads to the reduction of the deformation useful force).

Influence of the angle of nutation *θ*.

The angle of nutation produces a major influence of the tooth profile height. Influence $h= f(\theta)$ is a directly proportional function (fig. 1).

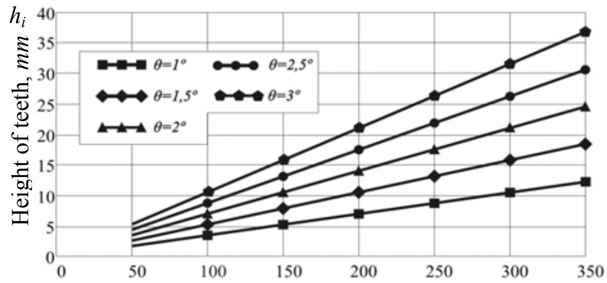


Figure 1. Tooth height h_i dependence of geometrical parameters of the multiplicity precessional gear $\varepsilon = 100\%$.

2. ASSESSMENT OF TEETH HEIGHT DEPENDING ON THE DEPTH FEED

The precessional tool, fixed onto the plastic deformation node, performs a spherical-spatial motion with a fixed point in the center *O* called the precession center. Continuous nature of plastic deformation, which would ensure high productivity of processing technology, requires a technological system with adaptive control. Thus the value of feed motion speed s_{cpi} should be correlated with the amount of crank-shaft rotation speed on which the node of plastic deformation with deformation precessional tools is installed [3, 4, 5].

Determination of some rational regimes for plastic deformation targets the choice of some appropriate values of work feed s_{cpi} (fig. 2) for working speed V_{Ei}. It is intended, therefore, obtaining the maximum processing productivity and minimum manufacturing costs. To achieve increased productivity by plastic deformation processing with toothing precessional tool of the precessional gearing, the processing feed s_{cpi} and, in fact, the number of passes is determined, taking into account the teeth size, the technological system stiffness and the effective power of the drive mechanism and characteristics of the material processed. Determination of the position of plastic

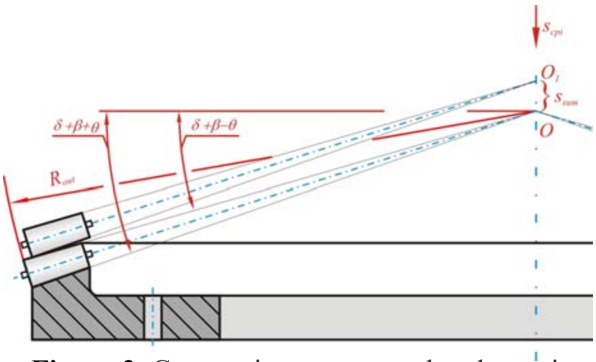


Figure 2. Geometric parameters that determine link roll forming semi-product.

deformation precessional tools, aiming at toothing execution throughout the tooth height and an appropriate adjustment of the technological system, involves the establishment of some components of the process. Determination of the whole process (fig.3) of plastic deformation roller in order to get the full height of the tooth requires calculation of tooth height by using the relationship:

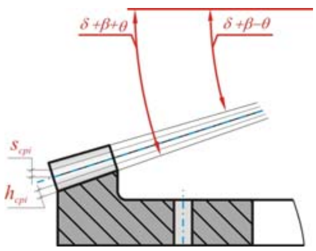


Figure 3. Dependence feed s_{cpi} of the height h_{cpi}.

$$h_{teeth} = R_{out} \cdot tg2\theta \qquad (1)$$

where: R_{out} is the outer conical radius of the semi-product, mm;
θ – angle of nutation, degrees.

The growth of tooth height depending on the vertical feed of the plastic deformation node, where the plastic deformation rollers are fixed, is determined by the relation:

$$\Delta h_i = s_{cpi} \cdot cos(\delta + \beta + \theta) \qquad (2)$$

where: δ is the conical axoid angle, degrees;
s_{cpi} – plastic deformation node feed for a precessional cycle, mm;
β – taper angle of the rollers, degrees.

Knowing the teeth height we can determine the process value of the plastic deformation node, on which the plastic deformation rollers are fixed:

$$s_{sum} = \frac{h_{teeth}}{cos(\delta + \beta + \theta)} \qquad (3)$$

Generally, big feed values will be used for high power of the drive motors of the applied technological system, taking into account the material machinability of the semi-product by plastic deformation. Also, bigger feed values will be used for smaller working speeds on the tooth height. Number of precessional cycles and depth feed s_{cpi} are chosen depending on the type of material that is subjected to plastic deformation and on the gearing geometric parameters that define the tooth form and profile.

3. DETERMINATION OF TOOTH SURFACE IN NORMAL SECTION FOR THE GEARING MULTIPLICITY *ε=100%*

In order to determine the tooth profile in normal section in precessional transmission it is necessary to analyze the principled diagram (fig. 4). Points E_1 and E_2 have coordinates corresponding to minimum values of tooth profile curve on the ball. Plane P_1 is drawn perpendicular to the surface of a triangle OE_1E_2 via points E_1 and E_2. The coordinates of contact point E "tool - semi product" is determined by the relations [4]. Tooth profile

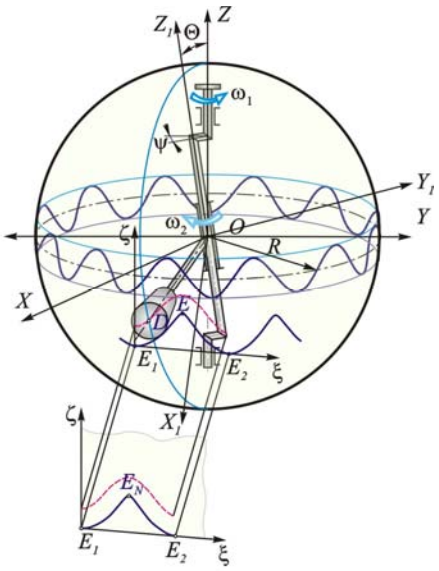


Figure 4. Determination of tooth profile in normal section.

from the sphere projected on the plane is identified by determining the coordinates of points of intersection E_N with this plane of the straight lines family that pass through the center of precession O, and the corresponding points of tooth profile on the sphere [1].

$$X_{1E} = k_2 Z_{1E} + d_2;$$
$$Y_{1E} = k_1 Z_{1E} - d_1; \qquad (4)$$
$$Z_{1E} = \frac{k_1 d_1 - k_2 d_2 - \sqrt{(k_1 d_1 - k_2 d_2)^2 + (k_1^2 + k_2^2 + 1)(R_D^2 - d_1^2 - d_2^2)}}{k_1^2 + k_2^2 + 1},$$

where:

$$k_1 = \frac{X_{1D}(X_{1D}\dot{X}_{1D} + Y_{1D}\dot{Y}_{1D}) + Z_{1D}^2 \dot{X}_{1D}}{Z_{1D}(X_{1D}\dot{Y}_{1D} - \dot{X}_{1D}Y_{1D})};\ k_2 = -\frac{R_D^2 \cos\beta + Z_{1D}}{X_{1D}};$$
$$d_1 = \frac{R_D^2 \cos\beta \cdot \dot{X}_{1D}}{X_{1D}\dot{Y}_{1D} - \dot{X}_{1D}Y_{1D}};\ d_2 = \frac{R_D^2 \cos\beta + d_1 Y_{1D}}{X_{1D}}.$$

where: β is the taper angle of plastic deformation roller.

Multiplicity of gearing is the most important characteristic that determines the bearing capacity of the transmission, kinematic precision, mass and gauges, gears' material requirements, etc. Multiplicity of teeth gearing can be evaluated analytically by analyzing graphs of functions $\zeta_1 = \zeta_1(\xi)$ and $\zeta = \zeta(\xi)$ (fig. 5) for the case $\varepsilon = 100\%$, which represents the trajectory projection of the satellite roller' center D motion and properly the central wheel tooth profile on the plane P_1 [1]. Each value of the function $\zeta_1 = \zeta_1(\xi)$ corresponds to a function value $\zeta = \zeta(\xi)$, defined as the contact point of the teeth involved. In other words, the position of any contact point on the graph $\zeta = \zeta(\xi)$ is determined by the precession phase, i.e. the precession angle ψ. For example, point D on the function graph $\zeta_1 = \zeta_1(\xi)$ corresponds to point E on the graph $\zeta = \zeta(\xi)$. The initial conditions for obtaining $\zeta = \zeta(\xi)$ function provides the following interaction of teeth: roller with the center located in point D contacts the central wheel tooth profile in point E.

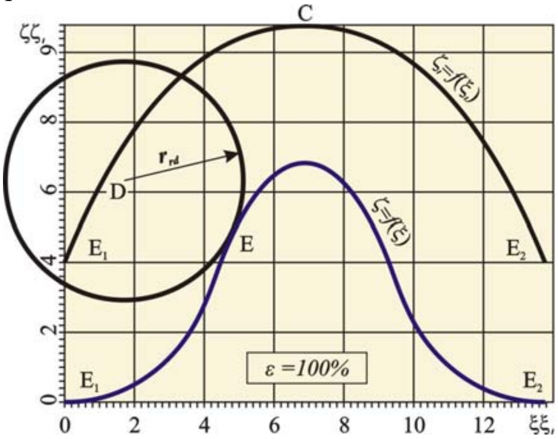


Figure 5. Interaction teeth to a precession cycle the gear multiplicity ε <100%.

Determination of tooth surface area will allow us calculate then the volume of the tooth and the gap between two adjacent teeth in order to determine the size of the semi product to be subjected to plastic deformation. Tooth geometry is more convenient to analyze in normal section, so tooth profile is projected from the sphere on a plane P_1 perpendicular to the tooth [1]. Therefore the tooth surface was divided into simple geometric shapes to obtain the total surface of the tooth fig.6.

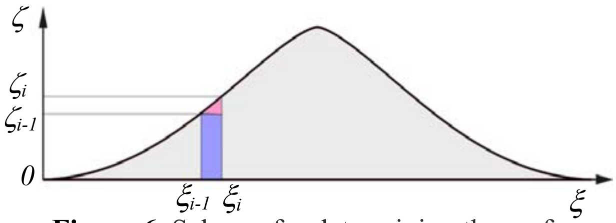


Figure 6. Scheme for determining the surface area in normal tooth section.

Each elementary surface area obtained is expressed:

$$S_0=0;$$
$$S_1=(\xi_1-\xi_0)\zeta_0+1/2(\zeta_1-\zeta_0)(\xi_1-\xi_0);$$
$$S_2=(\xi_2-\xi_1)\zeta_0+1/2(\zeta_2-\zeta_1)(\xi_2-\xi_1)+S_1;$$
$$S_3=(\xi_3-\xi_2)\zeta_0+1/2(\zeta_3-\zeta_2)(\xi_3-\xi_2)+S_2; \qquad (5)$$
$$S_i=(\xi_i-\xi_{i-1})\zeta_{i-1}+1/2(\zeta_i-\zeta_{i-1})(\xi_i-\xi_{i-1})+S_{i-1}.$$

Relation (5) is available only for the case when gearing multiplicity is $\varepsilon=100\%$. Applying the tooth profile description relations presented largely in [1], we define the tooth surface area in normal section depending on the geometric parameters of the precessional gearing:

- gearing multiplicity ε;
- taper angle of roller β;
- angle of nutation θ;
- conical axoid angle δ;
- number of teeth Z.

Using *MatchCAD* software, a number of calculations for various parameters of precessional gear have been done for their research on tooth surface in the normal section.

The following is a sequence of calculating the tooth surface area in normal section, using equation (5). Admit the following values of precessional gear parameters:

- gearing multiplicity $\varepsilon = 100\%$;
- taper angle of roller $\beta=1,1^o$;
- angle of nutation $\theta=3^o$;
- conical axoid angle $\delta=0^o$;
- number of teeth $Z=29$.

Tooth surface in normal section will be:

$S_0=0$

i = 1…360 where i – represents the number of points researched on the tooth profile.

4. DETERMINATION OF TOOTH SURFACE IN NORMAL SECTION FOR GEARING MULTIPLICITY ε < 100%

As mentioned above, equation (5) is valid only if the gearing multiplicity is ε = 100%. If the multiplicity *ε <100%* it is necessary to remove points that describe the figure formed by points *EFB* (fig7). Because the interaction period determines teeth gearing multiplicity tooth [7], it follows that the gearing multiplicity is determined by *EFB* figure size, which perimeter length characterizes the lack of contact between the roller - tooth. The bigger size of this figure, the lower multiplicity is, and vice versa. Without this figure the gearing multiplicity is ε = 100%, because teeth constantly interact with each other (fig. 5). In the presence of this figure (fig. 7) gearing multiplicity will be *ε <100%.* Tooth profile with geometric parameters is presented in fig. 8, a:

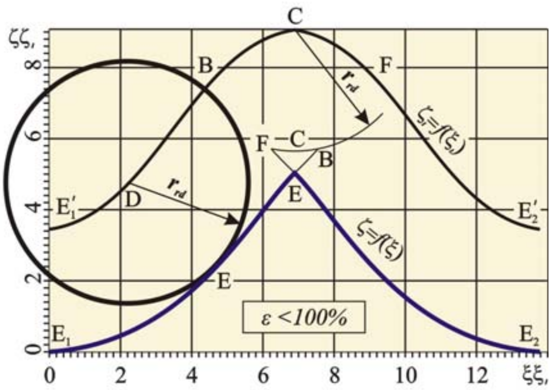
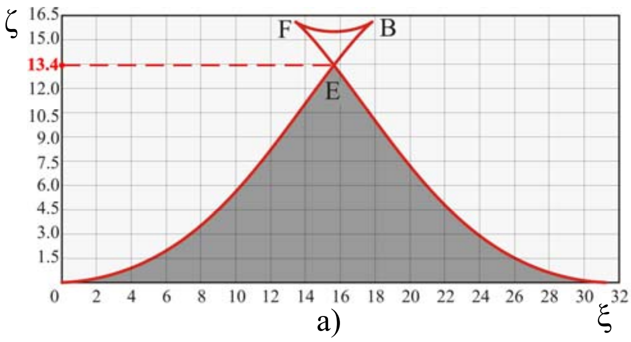
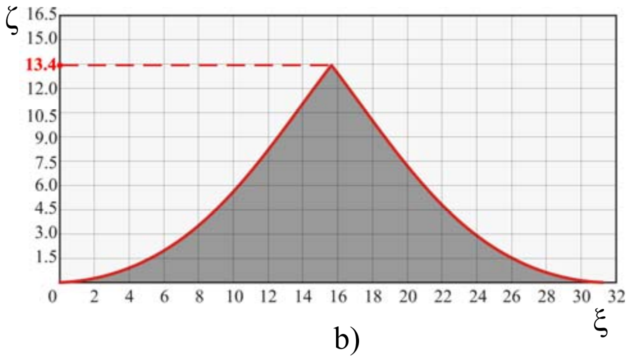


Figure 7. Interaction teeth to a precession cycle the gear multiplicity *ε <100%.*

- gearing multiplicity *ε= 60%*;
- taper angle of roller *β=3,024°*;
- angle of nutation *θ=3°*;
- conical axoid angle *δ=0°*;
- number of teeth *Z=29*.

For the equation 5 to "work" in the MathCAD software, points forming *EFB* figure are extracted, with subsequent obtaining of tooth profile (fig.8. a).





b)

Figure 8. Tooth profile shape parameters:
R_{out}=147,5 mm; h=13,4 mm; δ=0°; β=3,024°; θ=3,0°; ε=60%.

It is observed that figure *EFB* is missing in fig. 8. b, which will allow the use of the equation 5 to obtain the teeth surface area five teeth in the normal section. Further the profile surface area is calculated similarly to equation 5 as in the previous case.

CONCLUSIONS

As basis for the manufacturing technology of gears with convex-concave profile in the precessional gear by plastic deformation by rolling motion, the similarity of the tool (roller deformation) with roller motion in precessional gear was used.

The constructive cinematic features of precessional gear were considered at the development of manufacturing technology of the central wheel teeth for precessional transmissions via knurling.

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