The comitants of Lyapunov system with respect to the rotation group and applications

Pricop Victor

"Ion Creangă" State Pedagogical University,
Institute of Mathematics and Computer Science,
Chişinău, Republic of Moldova
e-mail: pricopv@mail.ru

Let us consider the Lyapunov system $s\mathcal{L}(1, m_1, ..., m_\ell)$

$$\dot{x} = y + \sum_{i=1}^{\ell} P_{m_i}(x, y),\ \dot{y} = -x + \sum_{i=1}^{\ell} Q_{m_i}(x, y), \tag{1}$$

where P_{m_i} and Q_{m_i} are homogeneous polynomials of degree m_i with respect to phase variables x and y. The set $\{1, m_1, ..., m_\ell\}$ consists of a finite number of distinct natural numbers. With A is denoted the set of coefficients of P_{m_i} and Q_{m_i}.
We investigate the action of the rotation group $SO(2, \mathbb{R})$ on the system (1).
Following [1] analogically were defined the comitants of differential systems with respect to the rotation group.
The Lie operator of the representation of the group $SO(2, \mathbb{R})$ in the space $E^N(x, y, A)$ of the system (1) was defined [2].
Using this Lie operator was determined the criterion when a polynomial is a comitant of Lyapunov system with respect to the rotation group.

Theorem 1. *The number of functionally independent focus quantities θ in the center and focus problem for the Lyapunov system $s\mathcal{L}(1, m_1, ..., m_\ell)$ does not exceed the number*

$$2\left(\sum_{i=1}^{\ell} m_i + \ell\right) + 1.$$

Bibliography

[1] Sibirsky K. S., *Algebraic invariants of differential equations and matrices*, Shtiintsa, Kishinev, 1976 (in Russian).

[2] Popa M. N., Pricop V. V., *The center-focus problem: algebraic solutions and hypotheses*, Kishinev, ASM, Institute of Mathematics and Computer Science, 2018 (in Russian).