NORMALIZED REPRESENTATION OF SPIN VALVE RESISTANCE VALUE BY THE HYPERBOLIC METRIC

Alexandr Penin[1] and Anatolie Sidorenko[1,2]

[1]D. Ghitu Institute of Electronic Engineering and Nanotechnologies, Chisinau, MD-2028 Republic of Moldova

[2]Technical University of Moldova, Chisinau, MD-2004 Republic of Moldova

Emails: aapenin@mail.ru, anatolie.sidorenko@mib.utm.md

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Abstract

Magnetoresistive sensors for measuring the magnetic field strength in a wide range up to saturation are considered. Normalized or relative expressions are introduced to change the resistance of the sensor to evaluate the qualitative characteristics of the mode, increase the informativity of the mode, and provide the possibility of comparing different sensors. Magnetoresistors are characterized by three characteristic values (minimum, nominal, and maximum). Therefore, the proposed relative expressions include these characteristic values. To justify these expressions, a hyperbolic straight line geometry and a cross ratio of four points are used. The results are of interdisciplinary interest in view of similarity of characteristics of sensors of different physical nature.

Keywords: magnetoresistance, spin valve sensor, magnetic field, cross ratio, hyperbolic geometry.

Rezumat

Se analizează utilizarea senzorilor magnetorezistivi pentru măsurarea intensității câmpului magnetic într-un interval larg până la saturație. Pentru a modifica rezistența senzorului, a evalua caracteristicile calitative ale regimului, a spori informativitatea acestuia și a oferi posibilitatea de a compara diferiți senzori sunt introduse expresii normalizate sau relative. Magnetorezistoarele sunt caracterizate de trei valori specifice (minimă, nominală și maximă). De aceea, expresiile relative propuse includ aceste valori caracteristice. Pentru a justifica aceste expresii, se utilizează geometria hiperbolică a liniei drepte și raportul transversal a patru puncte. Rezultatele obținute prezintă un interes interdisciplinar, având în vedere similitudinea caracteristicilor senzorilor de natură fizică diferită.

Cuvinte cheie: magnetorezistență, senzor pe baza valvei de spin, câmp magnetic, raport transversal, geometrie hiperbolică.

1. Introduction

Currently, magnetoresistive or spin valve sensors are commonly used to measure magnetic field strength and electric current value [1–4].

All the listed magnetoresistive sensors are characterized by a typical dependence of the resistance value on the strength of the external magnetic field. The zero value of the strength corresponds to the initial resistance. Increasing the intensity in one or another direction of the magnetic field leads to an approximately linear change in the resistance. At fairly high strength values, the saturation of the characteristic is manifested and the resistance of the sensor takes the minimum and, accordingly, maximum value.

Manufacturers offer a wide range of sensors of different types [5–8]. A commonly used full sensor bridge circuit (four identical sensors) provides the maximum sensitivity and linearity of the transient characteristic [9]. Therefore, the change in the resistance value is set as an increment.

On the other hand, with large changes in this resistance, it is necessary to compare the current value of the resistance with the minimum and maximum value to evaluate the qualitative capabilities of the mode and thus increase the information value. This is important in actual practice, if one system uses, for example, two sensors to measure the load current and voltage [10].

Generally, normalized or relative values are introduced. There is no problem with using one characteristic value. However, in the case of a magnetoresistive sensor, there are three characteristic resistance values, namely, minimum, initial, and maximum. Therefore, the deviation or change in resistance relative to the initial value should be expressed taking into account the minimum and maximum value. It is obvious that the formal increment does not give a qualitative idea of the current mode.

In the geometric sense, a change in resistance relative to the initial value corresponds to the concept of the distance or length of the respectrive segment on a straight line. However, three methods or metrics are known on the straight line [11]. In particular, the value increment will correspond to parabolic, or Euclidean geometry. However, the Lobachevsky or hyperbolic geometry determines the distance taking into account the boundaries of the variable value. To do this, a cross ratio of four straight line points is used. In this case, three points can correspond to three characteristic resistance values, and the fourth point will be the current value.

Further, various interpretations of hyperbolic geometry are known. Thus, in the Beltrami interpretation, the respective bounded coordinate of the point is introduced through the hyperbolic tangent function from the most unlimited distance [12]. This geometric interpretation is of interest in the context of the known approximation of the dependence of resistance on magnetic field strength through this hyperbolic tangent [13].

This study develops a geometric interpretation of the dependence of resistance on magnetic field strength based on hyperbolic geometry. A well-founded normalized expression for changing resistance leads to convenient expressions for calculating the field strength.

2. Typical Characteristic of Magnetoresistance

A typical asymmetrical $R(H)$ dependence of the spin valve resistance with giant magnetoresistance (GMR) on magnetic field strength [8] is shown in Fig. 1.

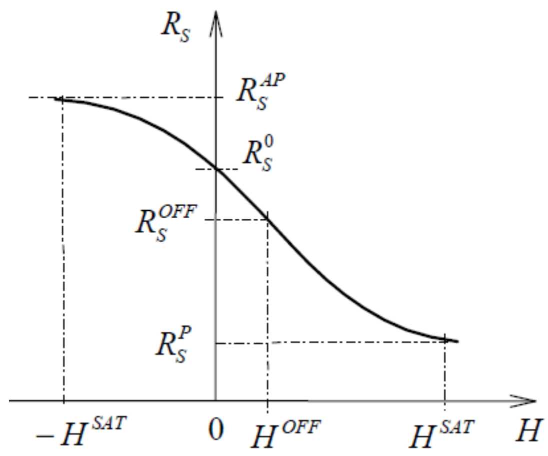

Fig. 1. Typical *R*–*H* curve of a GMR spin valve.

The zero value of strength $H = 0$ determines the nominal R_S^0 sensor resistance. As the strength increases, the resistance decreases up to saturation. In this case, the saturation resistance R_S^P corresponds to the parallel case of magnetizing the sensor layers. At negative strength values, resistance increases up to saturation. In this case, the saturation resistance R_S^{AP} corresponds to the antiparallel case of magnetizing the sensor layers.

The magnetoresistive ratio is defined by the relative expression

$$r = \frac{R_S^{AP} - R_S^P}{R_S^P}.$$

Most frequently, the inflection point of the characteristic

$$R_S^{OFF} = \frac{R_S^{AP} + R_S^P}{2} \qquad (1)$$

does not correspond to the strength $H = 0$.

For characteristic values of the sensor resistance, such as R_S^P, R_S^0, R_S^{AP}, the question arises of a reasonable normalized expression for both the value of the resistance and the value of the change in this resistance. Therefore, we consider a possible normalized representation of the characteristic in Fig. 2.

For the current resistance value, we introduce a deviation from the inflection point in the usual form as follows:

$$R_S - R_S^{0FF} = R_S - \frac{R_S^{AP} + R_S^P}{2}.$$

Then, we normalize by the value of full resistance change $R_S^{AP} - R_S^P$. In this case,

$$\frac{2R_S - (R_S^{AP} + R_S^P)}{2(R_S^{AP} - R_S^P)}.$$

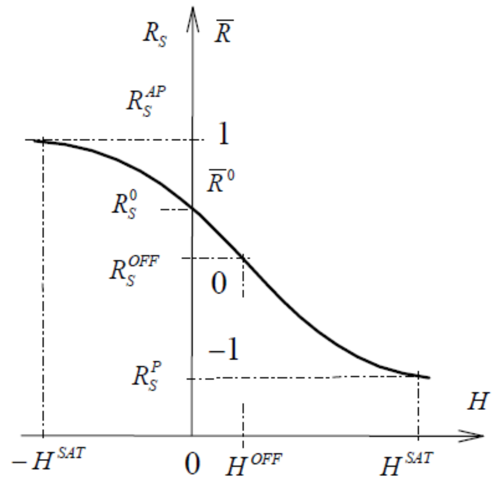

Fig. 2. Correspondence of the actual and normalized magnetoresistance values.

The substitution of variables or the transform $R_S \to \overline{R}$ is followed from here according to the expression

$$\overline{R} = \frac{2R_S - (R_S^{AP} + R_S^P)}{R_S^{AP} - R_S^P}. \tag{2}$$

In particular, the R_S^0 characteristic point gets the value

$$\overline{R}^0 = \frac{2R_S^0 - (R_S^{AP} + R_S^P)}{R_S^{AP} - R_S^P}. \tag{3}$$

In this case, for sensors with different values R_S^0 of characteristic points, the normalized values $\overline{R}^0$ will also differ. Therefore, the above mentioned problem of comparison of the modes of different sensors arises.

3. Geometry of a Straight Line

Further, we consider some provisions about different geometries of a straight line or determining the segment length [11] according to Fig. 3.

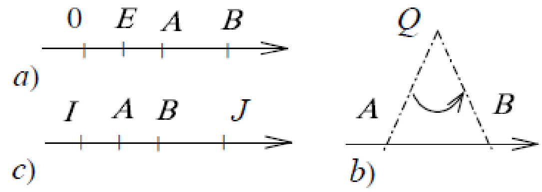

Fig. 3. Three different geometries of a straight line: (a) parabolic Euclidean (ordinary), (b) elliptical Riemann, and (c) hyperbolic Lobachevsky.

For parabolic geometry, the unit of length or scale $0E$ is fixed on a straight line, and the distance d^E_{AB} between points A, B is determined by the formula

$$d^P_{AB} = \frac{AB}{0E}.$$

In the case of elliptical geometry, some point Q is fixed outside the straight line, and the distance is just the "normal" angle $\angle AQB$; that is,

$$d^E_{AB} = \angle AQB.$$

Hyperbolic geometry is defined in a more complex way. In addition to points A, B, two points I, J are fixed. These extreme or base points define an infinitely remote boundary or absolute as the points A, B move. Next, the cross ratio (double proportion) of four points is composed:

$$m = (I\ A\ B\ J) = \frac{AI}{AJ} \div \frac{BI}{BJ}. \tag{4}$$

Then, the distance is as follows:

$$d^H_{AB} = \ln\left(\frac{AI}{AJ} \div \frac{BI}{BJ}\right). \tag{5}$$

If one of the points A, B tends to the base point, then the cross ratio is 0 or ∞. Then, the corresponding distance $d^H_{AB} \to \mu\infty$.

4. Normalized Magnetoresistance Value as a Cross Ratio

The saturation resistances R^P_S, R^{AP}_S correspond to the above base points of hyperbolic geometry. In this case, the distance will correspond to the field strength. In view of the above, we will compose a cross ratio for the resistance existing or current value and a change in it. The R^0_S point is accepted as a scale or unit point.

Current value of resistance

The cross ratio of four points (4) for three characteristic values R^P_S, R^0_S, R^{AP}_S of the sensor resistance and the current value R^1_S leads to a normalized expression of the resistance value [14]. The cross ratio values for the characteristic points are shown in Fig. 4.

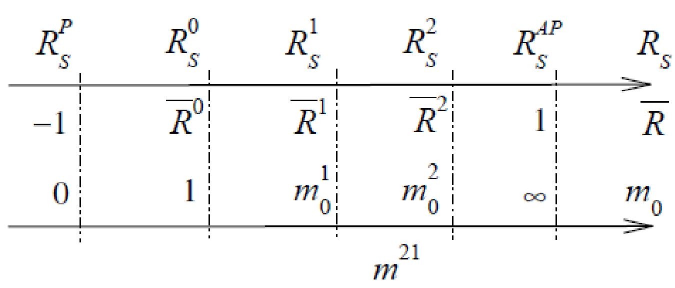

Fig. 4. Correspondence of the resistance and cross ratio values.

Cross ratio for the resistance values or samples is as follows:

$$m_0^1 = \left(R_S^P\ R_S^0\ R_S^1\ \ R_S^{AP}\right) = \frac{R_S^0 - R_S^P}{R_S^0 - R_S^{AP}} \div \frac{R_S^1 - R_S^P}{R_S^1 - R_S^{AP}}. \tag{6}$$

Let us express this cross ratio (6) in terms of a variable $\overline{R}$. In this case,

$$m_0^1 = \left(-1\,\overline{R}^0\ \overline{R}^1\,1\right) = \frac{\overline{R}^0 + 1}{\overline{R}^0 - 1} \div \frac{\overline{R}^1 + 1}{\overline{R}^1 - 1}. \tag{7}$$

Thus, when the sensor resistance varies from the minimum to the maximum value (base values), the relative value of it is quantified by cross ratio m_0. Thus, we get additional information about the capabilities and qualitative characteristics of the mode. A cross ratio is a dimensionless value; therefoe, it can be assumed that the m_0 value is the normalized expression for the sensor resistance, where all characteristic values are used. In other words, the dimensionless coordinate of a resistance value R_S^1 on a straight line relative to the initial R_S^0 value is determined by the cross ratio (6) and (7) or respective distance (5).

Change in resistance

Let the subsequent value of the sensor resistance correspond to R_S^2. In this case, the subsequent value of the cross ratio is as follows:

$$m_0^2 = \left(R_S^P\ R_S^0\ R_S^2\ \ R_S^{AP}\right) = \frac{R_S^0 - R_S^P}{R_S^0 - R_S^{AP}} \div \frac{R_S^2 - R_S^P}{R_S^2 - R_S^{AP}}. \tag{8}$$

The cross ratio has a group property if a subsequent m_0^2 value is expressed relative to an initial m_0^1 value through change m^{21} by a group operation (addition or multiplication). The expression structure (6) or (7) shows the execution of the group multiplication operation

$$m_0^2 = m^{21} \cdot m_0^1.$$

Then, we introduce the change in resistance m^{21} by the cross ratio

$$m^{21} = m_0^2 \div m_0^1 = \frac{R_S^1 - R_S^P}{R_S^1 - R_S^{AP}} \div \frac{R_S^2 - R_S^P}{R_S^2 - R_S^{AP}} = \left(R_S^P\ R_S^1\ R_S^2\ R_S^{AP}\right),$$

$$m^{21} = \frac{\overline{R}^1 + 1}{\overline{R}^1 - 1} \div \frac{\overline{R}^2 + 1}{\overline{R}^2 - 1} = \left(-1\ \overline{R}^1\ \overline{R}^2\,1\right). \tag{9}$$

It is evident that the change in resistance in the resulting expressions does not depend on unit points.

Parameter unsymmetry of a magnetoresistance characteristic

Let us consider the meaning of the cross relation for inflection point R_S^{0FF}. Using (1), we obtain

$$m_0^{0FF} = \left(R_S^P\ R_S^0\ R_S^{0FF}\ \ R_S^{AP}\right) = \frac{R_S^0 - R_S^P}{R_S^{AP} - R_S^0}.$$

This expression can serve as a reasonable parameter, for example, the value of usymmetry of the characteristic.

Hyperbolic model of the change in resistance

The proposed expression (9) for slight changes in the resistance value should result in a commonly used increment in parabolic geometry. To do this, we convert (9) to the following form:

$$m^{21} = \frac{\overline{R}^1 + 1}{\overline{R}^1 - 1} \div \frac{\overline{R}^2 + 1}{\overline{R}^2 - 1} = \frac{1 + \dfrac{\overline{R}^2 - \overline{R}^1}{1 - \overline{R}^2\overline{R}^1}}{1 - \dfrac{\overline{R}^2 - \overline{R}^1}{1 - \overline{R}^2\overline{R}^1}}.$$

Therefore, the expression

$$\frac{\overline{R}^2 - \overline{R}^1}{1 - \overline{R}^2\overline{R}^1} = R^{21}$$

can be considered as another definition of the change in resistance. The R^{21} value of this change corresponds to the hyperbolic tangent of the difference between two angles. Therefore, at low values $\overline{R}^2 << 1$, $\overline{R}^1 << 1$, it is found that $R^{21} = \overline{R}^2 - \overline{R}^1$.

5. Geomertic Interpretation of the *R–H* Curve of a Spin Valve

Hyperbolic tangent aproximation of the R–H curve

The hyperbolic tangent model [13] is known

$$R_S(H) = R_S^{OFF}\left(1 + \frac{r}{r+2}\,th\frac{H^{OFF} - H}{H^{SAT}}\right) = R_S^{OFF} + \frac{R_S^{AP} - R_S^P}{2}\,th\frac{H^{OFF} - H}{H^{SAT}}. \qquad (10)$$

This model fits closely the *R–H* data of a typical GMR spin valve [5].

If $H = 0$, then

$$R_S(0) = R_S^0 = R_S^{OFF} + \frac{R_S^{AP} - R_S^P}{2}\,th\frac{H^{OFF}}{H^{SAT}}.$$

In this case, the R_S^{OFF} inflection point corresponds to the symmetry point of the hyperbolic tangent and is defined by expression (1).

Then, expression (10) takes the form

$$R_S(H) = \frac{R_S^{AP} + R_S^P}{2} + \frac{R_S^{AP} - R_S^P}{2}\,th\frac{H^{OFF} - H}{H^{SAT}}. \qquad (11)$$

Beltrami coordinate of a point on a straight line

The Beltrami coordinate is determined by the known expression [12]

$$x = th\frac{\xi}{C}, \qquad (12)$$

The $\xi \le \infty$ value is the hyperbolic distance of a point from the origin, and the coordinate of this point is a bounded value, $-1 \le x \le 1$. Constant C corresponds to a scale.

The inverse expression is as follows:

$$\xi = \frac{C}{2} Ln\frac{1+x}{1-x}. \qquad (13)$$

According to (5), the hyperbolic distance is determined by the cross ratio. Therefore, expression (13) is expressed in terms of the following cross ratio:

$$\xi = \frac{C}{2} Ln\frac{x+1}{x-1} \div \frac{0+1}{0-1} = \frac{C}{2} Ln(-1\; x\; 0\; 1) = \frac{C}{2} Ln\, m_x^0. \qquad (14)$$

This relationship gives rise to Beltrami coordinate (12) for expression (11). Using substitution of variables (2) and (3), we obtain the Beltrami coordinate

$$\overline{R}(H) = th\frac{H^{OFF} - H}{H^{SAT}}. \qquad (15)$$

Thus, the $(H^{OFF} - H)$ value corresponds to the hyperbolic distance, and the H^{SAT} value is the scale.

Field strength calculation

Similarly to (13) and (14), we represent the inverse relation to (15) by the cross ratio using Fig. 2. In this case, we obtain

$$H^{OFF} - H = \frac{H^{SAT}}{2} Ln\frac{\overline{R}+1}{\overline{R}-1} \div \frac{0+1}{0-1} = \frac{H^{SAT}}{2} Ln(-1\;\; \overline{R}\;\; 0\;\; 1) = \frac{H^{SAT}}{2} Ln\, m_R^0. \qquad (16)$$

If $H = 0$,

$$H^{OFF} = \frac{H^{SAT}}{2} Ln(-1\;\; \overline{R}^0\;\; 0\;\; 1) = \frac{H^{SAT}}{2} Ln\, m_{R0}^0. \qquad (17)$$

According to expressions (16) and (17), the field strength itself takes the form

$$H = \frac{H^{SAT}}{2}\left[Ln\, m_{R0}^0 - Ln\, m_R^0\right] = \frac{H^{SAT}}{2} Ln\frac{m_{R0}^0}{m_R^0}.$$

In turn,

$$\frac{m_{R0}^0}{m_R^0} = \left(\frac{\overline{R}^0+1}{\overline{R}^0-1} \div \frac{0+1}{0-1}\right) \div \left(\frac{\overline{R}+1}{\overline{R}-1} \div \frac{0+1}{0-1}\right) = \frac{\overline{R}^0+1}{\overline{R}^0-1} \div \frac{\overline{R}+1}{\overline{R}-1} = (-1\;\; \overline{R}^0\;\; \overline{R}\;\; 1).$$

This resulting cross ratio corresponds to (7). Then, according to (6),

$$(-1\ \ \overline{R}^0\ \ \overline{R}\ \ 1) = \left(R_S^P\ R_S^0\ R_S\ R_S^{AP}\right) = m_0\ .$$

Finally,

$$H = \frac{H^{SAT}}{2} Ln\, m_0 = Ln\left[\frac{R_S^0 - R_S^P}{R_S^0 - R_S^{AP}} \div \frac{R_S - R_S^P}{R_S - R_S^{AP}}\right].$$

This expression fundamentally differs from the inverse expression directly to formula (11), in which the cross ratio is first calculated by the measured resistance values.

In turn, the structure of cross ratio (6) and (8) shows the mutual reduction of possible additive and multiplicative errors of measurement of resistance samples.

6. Discussion

In fact, the study addressed two issues. The first concerns the expression to represent the existing or current value of resistance and a change in it relative to three characteristic values. Hyperbolic metric makes it possible to justify the use of a cross ratio of four points. Thus, any formal expressions are excluded.

The second issue concerns the use of this cross ratio to calculate the magnetic field strength by the measured resistance using the known *R–H* characteristic approximation. The described approach provides a basis for consideration of symmetric characteristics [6], in particular, for superconducting spin valves [15].

7. Conclusions

(i) The use of a cross ratio of four resistance samples for normalized or relative deviation and a change in resistance taking into account minimum and maximum values has been justified.

(ii) The proposed geometric interpretation of the resistance characteristic has made it possible to obtain a formula for calculating the field strength, in which the measurement errors of four resistance samples are mutually reduced.

(iii) The results are of interdisciplinary interest in view of similarity of sensor characteristics of different physical nature.

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