

Model of inherent mechanisms of perception

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Abstract

Formalization aspects of decision making procedure under the condition of a priori uncertainty of the analyzed images are considered. The analytical schemes of decision making process are constructed. They have the same organization as pyramidal visual machines.

1 Introduction

One of the most fundamental problems of a science is the problem of recognition. The methods of recognition are formulated because of accumulated knowledges about regularities of the nature and society and form one of the most valuable sections of the uman knowledge. They represent the tool of cognition of properties and regularities of a reality, i.e. all that forms the main objective of a science.

Each scientific direction and sphere of practical activity have the methods of recognition. One of the most interesting directions is the visual pattern recognition. It is explained by that the image is: the natural tool of interaction of the person and the world; the natural tool of dialogue between the person and machine in any system of processing, analysis and control; a natural model of representation of many-dimensional signals (fields) practically in all range of electromagnetic waves.

The image has become a subject of a research of exact sciences only in a middle of the fiftieth years of the present century and reason to that was the rapid and energetic implantation of cybernetical methods in

the tasks of simulation of biosystems. Then the hypothesis was formulated, that the mechanism of perception is the classifying system and the problem to create the machine capable to be trained is formulated. By a result of such general setting of a problem the two interconnected direction of researches were kept till now: **development of mathematical models of visual perception** (D.Marr, F.Rosenblatt etc.); **development of mathematical methods of information conversions of the image as many-dimensional signal** (M.A.Aizerman, E.V.Braverman, N.G.Zagoryiko, etc.). In course of time researches on the second direction retreated from problems of image processing and nowadays represent independent theoretical discipline, subject of which is the construction of **mathematical models of classification of objects** in a mode of learning.

Pattern recognition process includes the three stages: creation of the initial description, finding of the system of features and construction of a deciding rule (decision making). At a stage of decision making the evaluation of a degree of a like try of the representation with the set of the measurement standards on known (partially or completely) sets of input representations and features are carried out. For today only this section of the mentioned theory, which can be called the theory in a narrow sense, has a rather high-power theoretical basis and rich arsenal of methods. First of all here it is necessary to mark the algebraic theory of recognition developed by the academician Yu.I.Zhuravlev.

However at solution of the task of a pattern recognition there is a number of problems. The known specific properties of any image – **ranking and structuredness** – are not taken into account in the common theory of recognition. Besides the image possesses properties of a **diversity of representations and redundancy of a “pixel structure” on each representation**. The diversity of representations, in its turn, generates a diversity of systems of features of the same image (even in absence of noise). All this requires at first the solution of a **problem of a formalization of any image in independence of representation**, i.e. the consideration of the image in conditions of a priori uncertainty, and it is the problem of identification in a broad sense (“black box” problem). Stage of finding of the sys-

tem of features not less is problematic, as the task of selection of any feature is the task of differentiation of the input description, which, as is known, is connected with a problem of a regularization, solved only for the narrow class of the tasks and unsolved for many-dimensional signals in conditions of a priori uncertainty of the latter.

In modern systems of image processing (SPI) there exists a set of methods of representations of the image as object of processing. It is explained by three reasons: a duality of the image from positions of interpretation (redundancy of set of pixel elements of the image and integrity of its perception); a duality of representation of the image and its processing (the a priori choice of representation of the image determines a method of processing and, vice versa, choice of a method of processing defines representation); **absence of mathematical methods of description and image processing which permit to interpret uniquely any image.**

Now it is theoretically justified (from position the modern mathematicians and physics) system of information transformations for any system of perception — visual, acoustical etc. is missing. Especially in conditions of a priori uncertainty of the signal, coming on as an input (image, sound etc.). However a human, as the system of perception, successfully functions in actual time scale namely in such conditions. Otherwise the human as an object, placed in an environment aggressive to him/her in common case, could not survive.

The known models of perception in general [1] and visual perceptions in particular [2] have purely descriptive character at a level of hypotheses and heuristic methods of information processing.

Perception in general and the visual one in particular we shall understand as a process of feeling and research activity, directed an investigation of a subject or the phenomenon, influencing organs of feelings, a final result of which is reflection of resea ched object in some collection of its properties, circumscribing its objective integrity [3]. Any perception is such, if it has the following properties: objectivity, integrity, constancy and categoryness [3]. Therefore, the system is the system of perception if and only if, when it satisfies to selected above properties. Besides for the system of visual perception (at least) additional prop-

erty of instantly action “singlemomentness”) is known, under which we shall understand process of image analysis at once by large blocks and quickly, on small number of the bases (features) [4].

The following results [4–8] are authentic known for the system of visual perception:

1. The process of visual perception is structured (microgenes is of perception) and consists of several phases, the general functional contents of which more or less is investigated:

- a) phase of detection, takes place at the first (10–15) microsecond,
- b) phase of distinguishing (discrimination) — revealing of a collection of features (space position, orientation, sizes), in the period of (50–70) ms;
- c) phase of identification — construction of form description and relative position of objects;
- d) phase of interpretation — revealing of individual features. The duration of the last phases depends on complexity of the image. On the average a bout after 300ms the process is finished. In usual conditions an eye performs three saccades jumps per second, and it is equal to 10observation of the scene or image.

2. The process of perception during one jump contains the following stages: for first 15 ms after the jump the zone of summation of receptive fields is maximal, the object is perceived as a spot, tending to a round form; after (50–70) ms reorganization of summation zones is completely finished; during consequent (30–50) ms to visual cerebral cortex the information including both low and high space frequencies is transmitted. Small-sized details begin to differ with increase of an exposition duration up to (150–200) ms.

3. During visual perception three mechanisms of identification of the image are functioning: identification under the inherent standard; identification within the limits of the available alphabet of visual images; identification under the produced standard.

4. The structural organization of neuron networks of a visual partition, as well as any other partition of the brain is maximal homogeneous.

5. An eye during observation of the image is fixed at elements containing maximal information.

The offered system, simulating the process of visual perception, is represented at fig.1, where two interconnected hierarchies — hierarchy of functions (on the left) and hierarchy of presentations (on the right) are represented. Under representation the result of processing of the corresponding level of functions is understood. The characteristics and description of hierarchies follows.

2 Initial representation

Initial representation i.e. initial description of any image, coming from the video sensor device (for example, eye, receptor field etc.) A priori there is not enough knowledge of the observable image, but it is known, that the image is a reflection of objects of the surrounding world by some observer. Therefore, on the one hand, the image objectively possesses the properties of the surrounding world, and, on the other hand, limitations, intrinsic to the perception system of the observer, as to any data processing system, are objectively imposed upon the image:

- 1) As the image is the function of intensity of light flux in a visible part of a spectrum, any minimal pixel at a level of a resolving power of the sensor device (as the point) belongs to point set of positive real numbers, satisfying thereby to requirement of observability;
- 2) Light flux as the function of effect on the observer at each instant of time should meet the requirements of a measurability, i.e. requirement of summability (integrability in the sense of Lebesgue [9]);
- 3) As the surrounding world belongs to a Euclidean space, so to topological metric space with base in space on an open sphere of a finite radius [10];

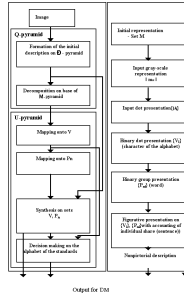


Fig.1. Hierarchy of functions and representations of the system implementing a model of perception, where DM – decision maker

- 4) As the surrounding world a priori is ordered and is structured [11], the image has the same properties. The ordering condition determines any image as partially ordered with the binary relation ρ as \leq , and the condition of structuredness is a condition of availability of upper and lower bounds for any pair of points, belonging to the image [10];
- 5) Information content about the surrounding world is limited by visual space, i.e. visual field of the videosensor of the observer;
- 6) The videosensor of the observer is the system of projection of the 3-dimencional influence function on to a two-dimensional surface (spherical within the limits of clear vision area of the visual analyzer [7]);

- 7) Image, as some function, a priori is discrete in space, since a structure of the image medium is discrete, for example, retina of the visual analyzer;
- 8) The system of perception, as any physical system, has final capacity.

Therefore the image, as input effect on the system of perception, is necessarily discrete in time.

Therefore, the image M in conditions of a priori uncertainty is a set, each element of which in the fixed instant $t \in T$ is a non-negative real-valued function of the valid arguments.

$$M_t = \begin{cases} \mu(x, y), & \text{if } (x, y) \in \overline{G} \subset R^2 \\ 0, & \text{if } (x, y) \notin \overline{G}, \end{cases} \quad (1)$$

determined on a final point set of closed two-dimensional area of a Euclidean space, summable

$$\int \int_{x, y \in G} \mu(x, y) dx dy < \infty$$

on this set, with the listed above properties.

As the field of sight is closed and possible amplitude of brightness of each point of the image is finite, then set of all possible images M with maximum number of realizations $k_{max} = P^{N \times N}$ is finite too, where $N \times N$ is the size of a visual field, and P is the number of gradation of brightness of the quantified image .

3 Formation of the initial description on P -pyramid

P -pyramid is a finalite set of mappings of set M , represented as $\mu(x, y)$, where $(x, y) \in G$, in an infinitely smooth manifold (C^∞ -manifold)

$$\mathbf{W} : M \rightarrow \mathbf{P} \in C \quad (2)$$

with use of a filter

$$W_i = \begin{cases} 1(x, y), & \text{if } (x, y) \in \overline{G}_i \subset \overline{G} \\ 0, & \text{if } (x, y) \notin \overline{G}_i, \end{cases} \quad (3)$$

In any subregion of a visual field, covering both the set M , and any its subset with a result of covering like

$$m(G_i) = \int \int_{x,y \in G_i} \mu(x, y) dx dy \quad (4)$$

In work of the authors [12] it is shown, that D - pyramid can be constructed just starting from its top, i.e.

$$m(G) = \int \int_{x,y \in G_i} \mu(x, y) dx dy = m_0$$

In the same work it is shown, that in conditions of a priori uncertainty the transformation (4) is optimal under the criterion of maximal produced entropy is authentic, is ϵ -exact (gives minimum of an root-mean-square error of approximation), is unique accurate up to isomorphism, is necessary and sufficiently, since the mapping (2) has the converse mapping $\mathbf{W} = \mathbf{V}^{-1}$, obtained with the help of a filter

$$V_i = \begin{cases} 1(x, y), & \text{if } (x, y) \in \overline{G}_i^1 \\ -1(x, y), & \text{if } (x, y) \in \overline{G}_i^2, \end{cases} \quad (5)$$

where G_i^1, G_i^2 are nonintersected of subregions of area G_i , detecting the binary relation between subregions with the help of unique operation

$$\mathbf{V}(M) = \mathbf{W}(M_2) - \mathbf{W}(M_1), \quad (6)$$

where $\mathbf{W}(M_1), \mathbf{W}(M_2)$ are determined by (2).

We shall name value $m(G_i)$ as a structural element of subregion G_i . Then the operation (6) reveals the relation, that is structural relationship between structural elements. In works [12, 13] it is shown, that each structural element from the position of mathematical physics

as the meaning of masses, which is called by the authors by a visual mass. In works [14, 15] from the position of the group theory and mathematical physics a finiteness of set $\{m(G_i)\}$, hence a finiteness of **P**-pyramid is proved. Thus, due to transformation (2) on an output **P**-pyramid we have final set $\{m(G_i)\}$, each element of which can be ordered on a matrix $\| m_{ij} \|$ of a size 4×4 . As the values m_{ij} can be unrestricted within the limits of real numbers set, a matrix $\| m_{ij} \|$ is a gray-scale representation of the input image M on **P**-pyramid.

4 Decomposition on base of *I*-pyramid

The *I*-pyramid is a finite set of filters (5), implementing operation (6) on the whole visual field, represented after **P**-pyramid as a set of visual masses, ordered on a matrix $\| m_{ij} \|$ of a size 4×4 . In work [13] it is shown, that this set of filters determines the set of linearly independent transformations

$$\nabla_i = \frac{d}{d_i} = \frac{\partial}{\partial x^i} \mathbf{e}^i, \quad (7)$$

revealing the relation between masses $\{m_i^1, m_i^2\}$ authentically and ϵ -exactly, where the set \mathbf{e}^i is basis of 16-dimensionial space (including a zero – direction). There a construction of filters is reduced and it is shown, that the superposition of transformations (2), (6) over a visual field will is a single pyramid – *Q*-pyramid, consisting of finite set of filters $F = \{F_i\}$, implementing integro-differential transformation of a kind

$$q = \mathbf{V} \circ \mathbf{W} \equiv d \circ \int_G \quad (8)$$

The result of operation of a *Q*-pyramid is a finite set $\{\mu_i : i = \overline{0, 15}\}$ of components of a vector $\mathbf{M} = \mu(x, y)$ of the image (1), as points in space with basis $\{\mathbf{e}_i\} : \mu(x, y) = \mu_0 e^0 + \mu_1 e^1 + \dots + \mu_{15} e^{15}$. It is natural, that two images M_1, M_2 are indistinguishable (are identical), if their decompositions in basis $\{\mathbf{e}_i\}$ are equal.

5 Mapping onto V

The set $V = \{V_i\}$ is a set of binary operators, ordered on a lattice $V(x, y)$ (fig.2), structurally determined up accurate to their serial number according to filters of set $F = \{F_i\}$, but distinguished from the latter by weight factors (values of weight factors for $F : (+1, -1)$, for $V : (+1, 0)$).

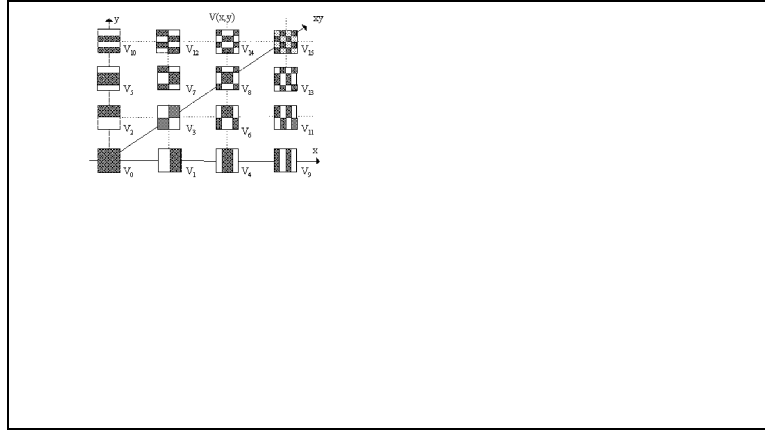


Fig.2. Set of operators $\{V_i\}$ on a lattice $V(x, y)$. The indexing of operators is conventional

The order of imaging of set $\{\mu_i : i = 0, 1, 2, \dots, 15\}$ on set $\{V_i : i = 0, 12, \dots, 15\}$ is the following:

$$V_i = \begin{cases} V_i, & \text{if } \mu_i > 0, \\ \bar{V}_i, & \text{if } \mu_i < 0, \\ 0, & \text{if } \mu_i = 0, \end{cases} \quad (9)$$

Let $\{\mu_i\}$ be the result of decomposition of any image. Each value μ_i carries the information on a mass of subregion of the image, covered with a filter. Then, the more is $|\mu_i|$ on set $\{|\mu_i|\}$, the higher is the weight of an appropriate filter, so of an operator V_i in general decomposition $\mu(x, y)$. Therefore individual share of each V_i , belonging

to set $\{V_i : i = \overline{1, 15}\}$ is equivalent to a mass distribution $|\mu_i| \in \{|\mu_i|\}$. Then a condition

$$M \cong V_i \Leftrightarrow |\mu_i| = \mu_0, \quad \mu_j = 0, \quad \forall j \neq i, \forall i \neq 0, \quad (10)$$

where \cong is the ϵ -exact identity, is a condition of detection of a structure (mass distribution) of a corresponding organization of an operator V_i on the image M (kind of an operator – direct or inverse, is determined according to (9)). In particular the fulfillment of a condition (10) for an operator V_0 , is a condition of availability of the image in the visual field. From neurophysiological positions of a neuron model it corresponds to summation on all receptor field of a neuron, and since removal of uncertainty begins at once in the whole field, we have direct analogy to initial perception of the image, as a “spot”. Fulfillment of (10) for any other operator is the regularity condition (antisymmetry) of image at the given level of decomposition and of the image description (with neurophysiological position of a neuron model for each filter, implementing (6), a set of neurons with own **on** and **off** receptor fields, correcting a structure of the image — “spot”, is analogous).

The result of mapping $\{\mu_i\} \leftrightarrow \{V_i\}$ is a finite subset of direct and inverse binary operators representing the image M on a lattice $V(x, y)$ or in basis $\{V_i\}$, each element of which is allowed to be considered as the character of the alphabet.

6 Mapping onto P_n

Set $P_n = \{P_{ni}\}$ is a set of full groups on the set $\{V_i\}$, ordered on a lattice $V(x, y)$. The set V accepts two binary operations for elements of matrixes $V_i, V_j \in V$ — addition and multiplying

$$\begin{aligned} [a_{nm}^i] + [a_{nm}^j] &= [a_{nm}^i + a_{nm}^j] \quad \forall n, m; \\ [a_{nm}^i] \cdot [a_{nm}^j] &= [a_{nm}^i \cdot a_{nm}^j] \quad \forall n, m, \end{aligned}$$

such, that for them operations of the Boolean algebra are correct. Then the action of operators V_i, V_j is a covering, i.e. direct sum $V_{ij} = V_i \oplus V_j$

with addition and multiplying, determined above (fig.3). As for V it is correct: $e_1 = V_0 = 1, e_0 = \overline{V_0} = 0; V_i + \overline{V_i} = e_1, V_i \overline{V_i} = e_0; V_{ij} \in V, A(V) = \langle V; +; \cdot \rangle$ is the Boolean algebra. The choice of operation $(+; \cdot)$ is determined by the following rule: condition of individual share

$$\sum_i \frac{|\mu_i|}{\mu_0} = 1, \quad \frac{|\mu_i|}{\mu_0} = 1 \quad \forall i$$

are necessary and sufficient for the description of the image with the help of operations

$$M \cong \sum_i \{V_i\}, \quad M \cong \prod_i \{V_i\},$$

where $\{V_i\}$ is the set of operators, for which $\mu_i \neq 0$.

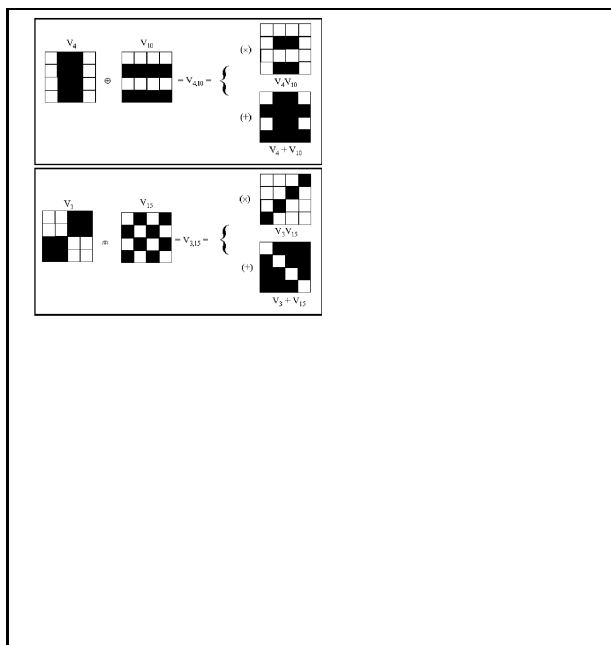


Fig.3. Operation of operators V_i, V_j on operation \oplus on localization of common areas of a homogeneity on e_0 for $(+)$ and e_1 for (\times)

On set V it is possible to select a triple, satisfying to conditions of its definition as group (from the position of the theory of groups), i.e.

$$\sum_i \{V_i\} = e_1, \quad \prod_i \{V_i\} = P = P_n. \quad (11)$$

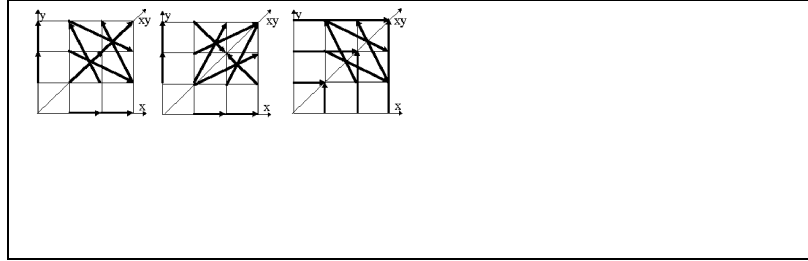


Fig.4. Examples of subset of full groups, which digraphs are not intersected in vertices (operators) of a lattice $V(x, y)$

We shall name P_n the full group. Because set of operators is finite, the number of full groups on this set is finite also. Let $\{V_i\}$ be a set of direct operators. Then: the number of nonintersecting full groups is equal 5 and they form a subset, which is complete on the set of lattice vertices $V(x, y)$ (fig.4); through any lattice passes seven full groups, intersected at this vertex (fig.5); the cardinal number of set of full groups on $\{V_i\}$ is equal 35 (fig.6). It is natural, that the cardinal number set of full groups on the set of direct and inverse operators is equal to 280.

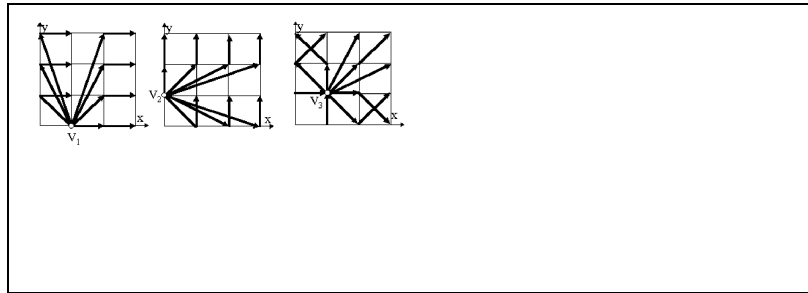


Fig.5. Examples of full groups (represented by the graphs), intersected in the vertex V_i

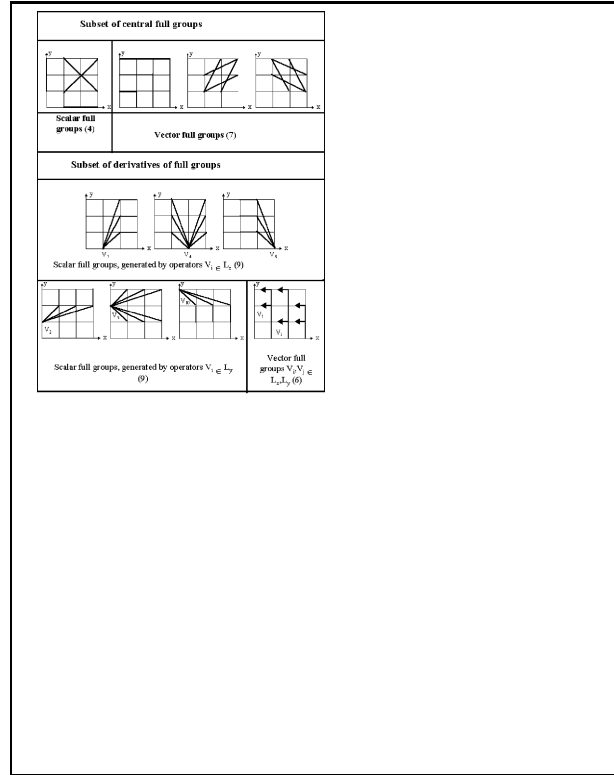


Fig.6. Examples of the images, belonging to the dot class of the image I_1 . Here mass of each point (pixel) of the image is equal to 1

The rules of detection and description of the image on the set $\{V_i, V_j, V_k\}$

$$M \cong \begin{cases} P_n \Leftrightarrow |\mu_i| = \mu_0 & \forall V_i \in P_n & \mu_j = 0 & \forall j \neq i \\ P_n \Leftrightarrow |\mu_i| = m = \text{const} & \forall V_i \in P_n & \mu_j = 0 & \forall j \neq i \end{cases}$$

$$(V_i, V_j, V_k) \in \begin{cases} P_n, & \text{if } m = \mu_0 \\ \overline{P_n}, & \text{if } m < \mu_0 \end{cases} \quad (12)$$

where P_n is determined by (11), and for obtaining the description $\overline{P_n}$ it is necessary to apply the rule of De Morgan to the description (11) (fig.7)

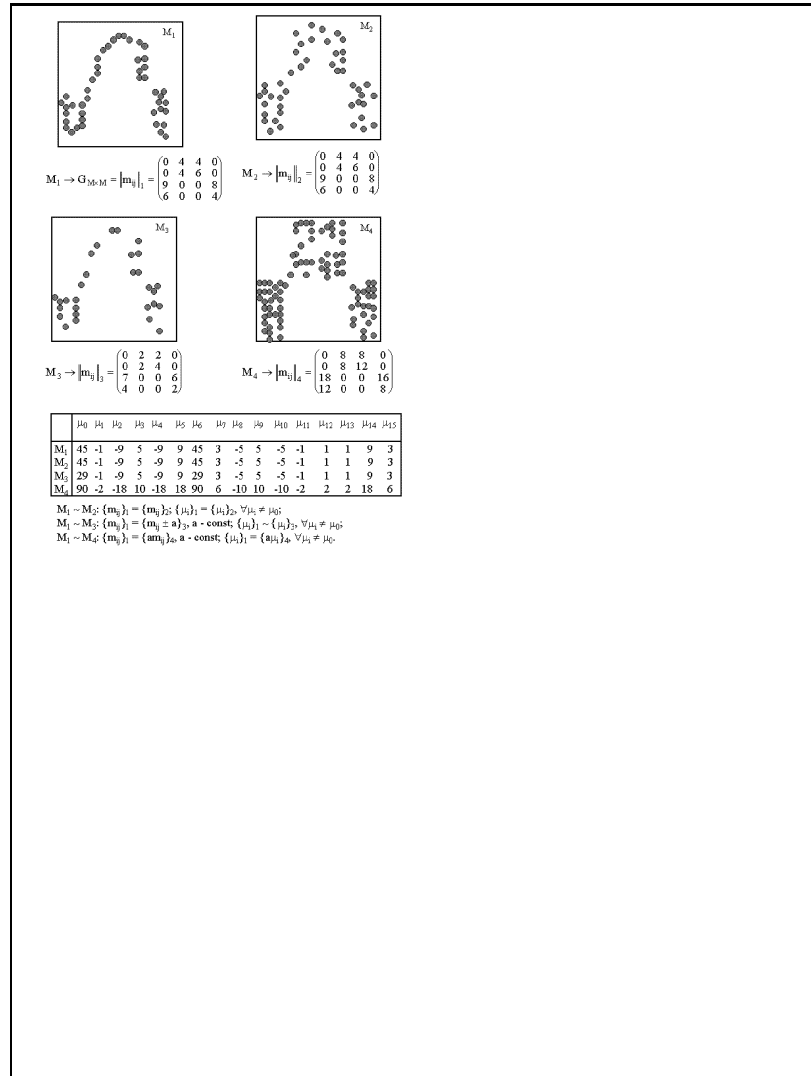


Fig.7. Examples of images, belonging to the dot class of the image M_1 . Here mass of each point (pixel) of the image is equal to 1

$$\overline{P_n} = \overline{V_i} + \overline{V_j} \equiv \overline{V_i} + \overline{V_k} \equiv \overline{V_j} + \overline{V_k} \equiv \overline{V_i} + \overline{V_j} + \overline{V_k}$$

The result of representation of the image on set $\{P_{ni}\}$ is the more compact description of the image as a word in the alphabet $\{V_i\}$.

7 Synthesis on sets $P_n = \{P_{ni}\}$, $V = \{V_i\}$

Let on sets V, P_n the following operations are defined

$$\bigoplus_i V_i = \left\{ \sum_i V_i, \prod_i V_i \right\} \quad \bigoplus_i P_{ni} = \left\{ \sum_i P_{ni}, \prod_i P_{ni} \right\} \quad (13)$$

authentically locating areas of a homogeneity in a variables V_i, P_{ni} by zeros and ones accordingly, where the choice of elements and operations depends on individual share of elements in creation of the description of the image M on a matrix 4×4 . Then the sentence (the predicate) $P = P(\{V_i\}, \{P_{ni}\})$ on operations (13) is named as **nonpictorial** description of binary (black-and-white) pattern $O(P)$, represented on a matrix 4×4 . The predicate is an result of synthesis of the pattern on sets V, P_n . And the first description (left-side in (13)) is the description of the pattern in base $\{V_i\}$, second, correspondingly in base $\{P_{ni}\}$.

Set of patterns on a matrix 4×4 is finite and its cardinality is equal to $2^{4 \times 4}$. Any image M can be mapped on this matrix with use of transformation (2). Then any M from set of cardinality $P^{N \times N}$ (or $P^{N \times M}$ in case of a rectangular visual field) it is possible to put in correspondence an pattern from the set of cardinality $2^{4 \times 4}$. Just on this set will be realized both problem of synthesis of an pattern, and problem of creation of the measurement standard for the image from set $P^{N \times N}$. For example, if the number of gradation of brightness (P) equals 256, and size of the image ($N \times N$) is 1024×1024 , then to the set of the images $256^{1024 \times 1024}$ (and it is indeed the "cosmic" number) is put in correspondence total 2^{16} patterns. The obtained result is one of explanations of superhigh (relative to the low speed of distribution of physiological processes) speed of recognition in the systems of visual perception.

8 Decision making on the alphabet of standards

By definition the distribution of visual masses $\{m_{ij}\}$ on a matrix 4×4 is representation of an image $O(M)$, obtained as a result of action of **P**-pyramid. Let a set of filters $\{w_i\}$, belonging to a **P**-pyramid, is applied to the whole area G of existence of the image V (where G is the visual field). Let each mass distribution $\{m_{ij}\}$ on a matrix $M_{4 \times 4}$ is the standard for a subset of the images, represented by the value m_{ij} in the subregion $G_{ij} \subset G$. Let set $A = \{A_i : A_i = \{m_{ij}\}, A_i \leftrightarrow W_i\}$ is the set of representation of the image M on set $\{W_i\} \in \mathbf{P}$. Then any two images from M are equivalent, if the following holds:

$$M_i \sim M_j \Leftrightarrow \{A_i\}_i = \{A_i\}_j. \quad (14)$$

We shall name a subset of the images $\{M_k\} \subset M$, satisfying (14), dot class $Y_k(O_T(M))$ of images with the standard $O_T(M_k) = \{m_{ij}\}$ on a matrix 4×4 .

To each distribution $\{m_{ij}\}$ there corresponds a vector \mathbf{M} , as a result of operation of a **M**-pyramid to the initial description. As two images M_i, M_j , satisfying (14), are ϵ -close on the standard (are indistinguishable) by use of a **P**-pyramid to the whole visual field, then, at first, for them it is correct that

$$M_i \sim M_j \Leftrightarrow O(M_i) = O(M_j) \quad (15)$$

where $O(M_i), O(M_j)$ is the description of patterns in base V ; secondly, it is natural to refine the closeness of operators $O(M_i), O(M_j)$ with each other. Let G_i be a subregion of a visual field G . Then the application of a **Q**-pyramid to the image, determined in G_i , with obtaining of a vector $M(G_i)$, is named ϵ -exact decomposition of the image $M(G_i) \subset M(G)$ for level i . Therefore the application of a **Q**-pyramid to the whole field G is decomposition of level 0 (or 1). Because the refinement of the image M can be (in general case) continued up to a limit of the resolution (to a pixel of the initial image) then:

- 1) we obtain a multilevel hierarchy of pyramidal decompositions, where a universal element is a **Q**-pyramid

$$U(Q) = U_0(U_1(U_2(\dots U_k(Q)) \dots)); \quad (16)$$

- 2) the decomposition of the image M will be fulfilled as a whole (at first a **Q**-pyramid applies to the whole field G , then to its subregions);
- 3) the decision making according to (14), (15) begins at once.

In expression (16) $U_k(Q) = \{M(G_k)\}$ is the set of decompositions of the image M , determined in area $G, G_k \subseteq G; M(G_k) = \{\mu_i\}_k$ is the decomposition of the image M , determined in subregion G_k , on set F of a **Q**-pyramid; $G_{k=0} \equiv G$. The hierarchy (16) is named a **U**-pyramid.

Let μ_{i_1}, μ_{i_2} are representations of the images M_1, M_2 , for which $\mu_0(M_1) = a\mu_0(M_2)$ holds, where a is constant, $a > 0$. Then

$$M_1 \sim M_2 \Leftrightarrow \{\mu_i : i \neq 0\}_1 \sim \{\mu_i : i \neq 0\}_2 \Leftrightarrow O(M_1) \equiv O(M_2) \quad (17)$$

where sign \Leftrightarrow denotes one-to-one correspondence. Thus, if conditions (14), (15) (first for representation $\{m_{ij}\}$, second – $\{\mu_i\}$) are necessary and sufficient to refer the image to the class, then (17)) is necessary and sufficient condition of equivalence of the images, which is correct not only for a pair of the images, presented by patterns of one level of decomposition, but also for different levels of hierarchy (16)).

Let M_i, M_j be images of levels i, j of a **U**-pyramid. Let $Y_i(O_T(M)) \equiv Y_i(M) \in Y(M)$ is the class of the image M_i on set of classes $Y(M)$ and for M_i, M_j (17) it is correct. Then

$$M_i \sim M_j \Leftrightarrow (M_i, M_j) \in Y_k(M). \quad (18)$$

The conditions (17), (18) are necessary and sufficient conditions of invariance of a pair of the images both concerning a mass (invariance to transformation of a homothety), and a area of definition (invariance to affine mappings as homogeneous deformations – expansion and compression). Besides all obtained conditions are conditions of

decision making, when one of the images is the standard (standard image). Because each pattern can be described on a matrix in bases V, P_n with the help of operations (13), is allowable to define set of images $\{O^2(S_i)\} = (\{O_i^2(V)\}, \{O_i^2(P_n)\})$, as set of the standards of a set S . Then, if the patterns $O_i(M), O_j(M)$ of images M_i, M_j do not satisfy to a condition (17), but satisfy to the condition

$$O_i(M) \sim O_j(M) \Leftrightarrow (O_i(M), O_j(M)) \in O_T(S_i). \quad (19)$$

they belong to the class $Y_i(O_T(S_i))$ of standard $O_T(S_i)$ from a set S . Each class of the measurement standard (the fuzzy class) integrates be it self some subset of images (dot classes (18)). A general property for selected classes is their belong ng to a space with base V , which we shall name space of classes. A measure in this space is a mass (visual) image and its elements, displayed on a matrix 4×4 . Let $\{O_T(S_i)\} = \bar{M}$ is the set of the standards. In this case the standard $O_T \in \bar{M}$ and pattern $O \cong$ are compact if and only if, the condition is fulfilled

$$\min_{O_T \in \bar{M}} |\mu_0 - \mu_T| \Rightarrow \{ \min_{O_T \in \bar{M}} \mathbf{P}_p, \max_{O_T \in \bar{M}} \mathbf{P}_c \}, \quad (20)$$

where the measure of a deviation of an pattern O of the image M from the standard O_T is a predicate of distinction $= O\bar{O}_T + \bar{O}_T O_T$, and the measure of concurrence of an pattern O with O_T is a predicate of concurrence $P = O O_T \equiv O_T$; the sign \Rightarrow means “follows”; μ_0, μ_T is mass of the pattern and the standard accordingly. The condition (19) is a condition of decision making against a standard (standards) from a set S on sets of patterns V, P_n according to the criterion (20).

The set V sets a hierarchy of levels of the description of the image M and the corresponding hierarchy of decision making within the limits of one i -th level of a \mathbf{U} -pyramid on a criterion (20) on set of the standards $\{O_T(S_i)\}$. We shall name a hierarchy of levels of the description a \mathbf{U} -tree. Then $U_i(M)$ is the tree of the description of the image M on i -th level of a \mathbf{U} -tree. It is natural, that two images M_i, M_j , represented by set of patterns on set of levels of a \mathbf{U} -pyramid are isomorph c (are indistinguishable), if they are identical under the description on $\{U_i(M)\}$. It corresponds to their belonging to one dot

class of an pattern on a condition (17). Similarly, two images M_i, M_j , having decompositions $\{U_i(M_i)\}, \{U_i(M_j)\}$ on a \mathbf{U} -pyramid (16) and sets $\{V_i\}, \{P_{ni}\}$, belong to one class of the standard, if (19) is fulfilled. As groups from $\{P_{ni}\}$ are functions of operators from V , we have direct analogy of process of creation of the description on (13) and process of creation of the description of visual patterns on pyramidal cells [5, 7].

As any pattern on a matrix can be presented on sets $\{V_i\}, \{P_{ni}\}$ with operation (13), it is allowable to consider sets $\{V_i\}, \{P_{ni}\}$ as the alphabets of the description of the image M . The first is the alphabet of letters (symbols), the second is the alphabet of words. Then any image M is the sentence on sets $\{V_i\}, \{P_{ni}\}$, integrated operations (13), and also logical operations “and”, “or”. As a result we obtain the system of transformations of the image M , represented by set $\{\mu_i\}$ with the enumerated below properties.

P1. Finiteness . Because the description of the image M_j is a set of U_i -trees on a \mathbf{U} -pyramid, determination of the accurate (accurate to a pixel of the initial image) correspondence of the image M_j to the class of an image (18) or class of the standard (19) is their correspondence at all levels of a \mathbf{U} -pyramid on all subsets of set of U -trees. Thus two variants are natural - availability or absence of set of the standards $\{O_T(Si)\}$ for the analyzed image. In the latter case the image M_j on set of decompositions (16) is the dot class (new dot class) in the space of classes. Because for sets M, F, V conditions of entirety, defining a finiteness \mathbf{U} -pyramids (16), are correct, the processes of decomposition and decision making are finite.

P2. Homogeneity. It is ensured by the same type of procedures of creation of a \mathbf{U} -pyramid.

P3. Integrity. Because process of decomposition of the image M in conditions of its a priori uncertainty spreads above - downwards (from top to the basis) of pyramids, by virtue of (16) the process of its description will be fulfilled from general (whole) to partial(individual).

- P4. Singlemomentness.** The **U**-pyramid, obtained as a result of construction process of the image description assumes decision making on any i -th level of a **U**-pyramid and on any j -th level of the description of a U_i - tree, including level U_0 , implementing thus the single-moment mechanism of perception.
- P5. Universality.** The property is fulfilled by virtue of fundamentality of transformation (8) and optimality of a pyramidal model of removal of a priori uncertainty.
- P6. Inherency.** The property is correct, since all set of elements (filters, operators), the set of their communications on set of pyramids of a model of removal of a priori uncertainty are strictly ordered on a lattice $V(x, y)$, so a priori are given.

Therefore sets $\{V_i\}, \{P_{ni}\}$ are the set of inherent standards, and model (system) the removal a priori uncertainty of the observable image is an information model of inherent mechanisms of visual perception, where rules (15), (17) will implement identification (recognizing) within the limits of available alphabet of visual patterns; rules (10), (12) will implement identification within the limits of the inherent standards; rule (19) on a criterion (20) will implement identification under the produced standard according to (13).

We shall consider correctness of definition of an model of removal a priori uncertainty as an information model of process of visual perception from the position of generally accepted concept of perception. The property of objectness is ensured by objectivity of properties of the image. The property of integrity is fulfilled by virtue of a property **P3**. Constantness is ensured by the nature of set of filters F , as of elements of algebra of invariancy (7) [13], and also condition of invariancy of the images on a **U**-pyramid (17). Categoryness assumes fulfillment of properties of partitionedness and generalization [3]. The first is a property of selection and fixing of rather stable properties of objects and their relations and is fulfilled by virtue of a nature of obtained

system of filters. Second is a property of ordering of a diversity of objects in their classification and assumes both inherent mechanisms of generalization, and is obtained by experience, due to learning process. According to **P6** the fulfillment of a considered property is ensured at a level of inherent mechanisms of decision making. So, the model (and the system implementing it) of a priori uncertainty removal on the basis of the system $(Q - U)$ – of pyramids is **an information model of inherent mechanisms of visual perception.**

As the **Q**-pyramid on set of filters $\{F_i\}$ is universal and it is fundamental, the following is correct: let $\{B_i\}$ be a set of systems of perception. Then $B_i \cong B_j$ on sets of inherent operators $\{V_i\}$ and full groups $\{P_{ni}\}$, where \cong is the of isomorphism.

9 Examples of solution of the application problems

1. Identification within the limits of the alphabet of visual patterns. By definition the image is digital and can be represented on a square (or rectangular) matrix, $\|\mu(i, j)\|_{N \times N}$, where each element is a pixel of the original image. After transformation on finite set of filters P – pyramids we obtain a matrix, where each element is a pixel of the transformed image. After transformation on finite set of filters of a **M**-pyramid we obtain the set $\{\mu_i : i = \overline{0, 15}\}$. Because the fulfilled transformations are linear concerning transformations, $a\|a\mu(i, j)\|, \|a\mu(i, j) \pm a\|$ where a is constant, and transformation, fulfilled on a **M**-pyramid is invariant to transformations, $a\|a\mu(i, j)\|, \|a\mu(i, j) \pm a\|$ two images M_1, M_2 on selected transformations of pixels are not distinguishable, i.e. according to (15), (17) belong to one class of the images (fig.7), one of which can belong to the alphabet of standard patterns. The exception makes a filter F_0 , following changes of the image in a area of definition.

The invariance to changes on M is kept and in the case of any (not necessarily linear) changes on M within the limits of preservation of a visual mass of subregions of a filter F_i . The given case is shown on

the image M_2 fig.7 by change ordering of pixels in subregions (case of “fuzzy” M_1). In remaining cases images M_i, M_j according to (18) belong to single, or different classes of the standards (fig.8, 9). Just so a problem of recognition of continuous patterns in two-dimensional space on diverse points can be solved.

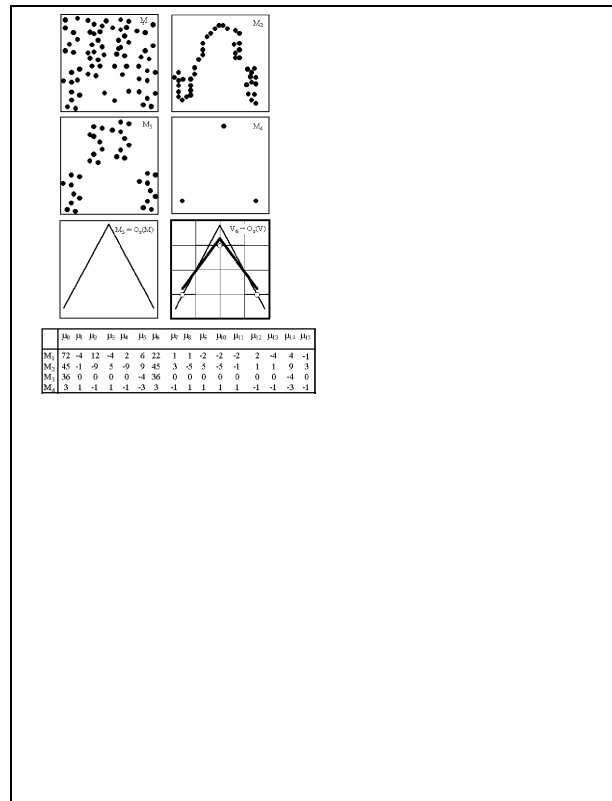


Fig.8. Examples of the belonging to the class of the standart $O_T(V_6) \cong O_T(M_5)$, M_5 is the standart image, having ϵ -close representational description be the operator V_6 .

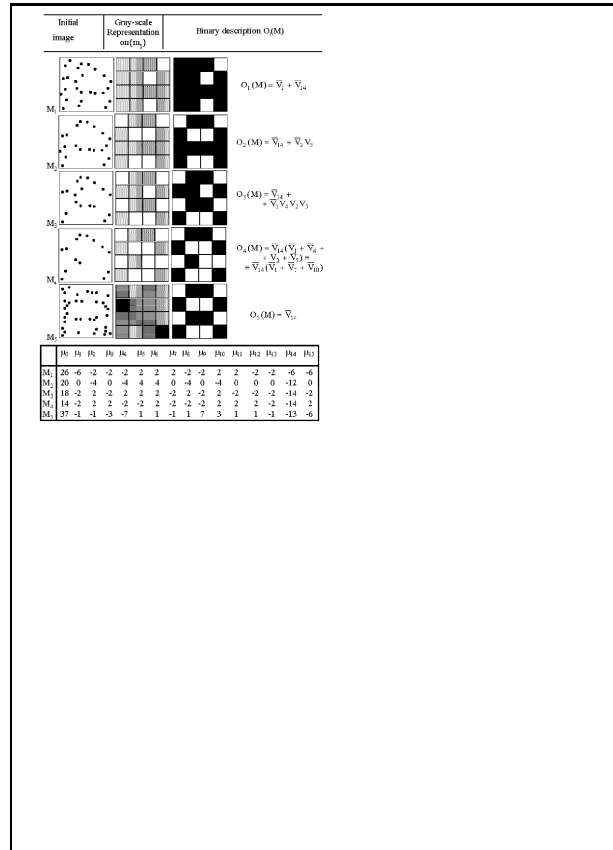


Fig.9. Examples of the images, belonging to the class of the standart $O_T(V_14)$

2. Detection of movement. The predicate of a kind

$$P = P\left(\sum_i V_i \vee \sum_i P_{ni}\right)$$

is the predicate of detection of object availability in a visual field. Because for its implementation (as well as any filter) it is necessary and sufficient only summation operation on the area of definition, the procedure of detection is minimal in the sense of computing complexity.

Besides by virtue of linearity of summation operation, use of integers (equivalents of pixels brightnesses of the image) is allowable.

Let S be the system of detection with a task of detection of any object in a field of sight on a priori known scene M_1 . Let $\{\mu_i\}_1$ is the result of decomposition M_1 on a \mathbf{Q} -pyramid. Then any changes on $\{\mu_i\}_1$ are found out at once (instantly) at a level U_0 of a decision pyramid. It explains a high resolution power of the visual system on detection of dot objects on a homogeneous background. Really, let M_1 is a homogeneous background to which there corresponds a mass μ_0 . Then any changes in one pixel of the image M_1 result in changes of μ_0 .

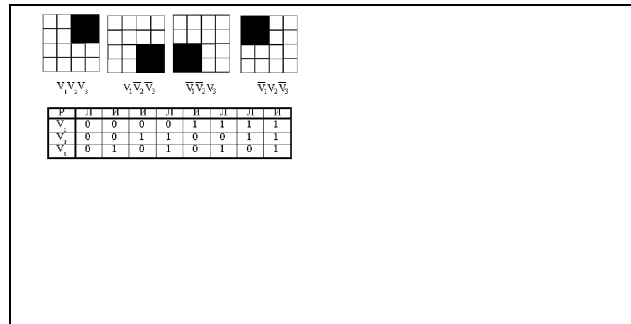


Fig.10. Equation $P_1 = \bigvee_{p=1}^4 P_{np}$ describes the set of position of an pattern group in a visual field G represented by the matrix 4×4 , and equation of king $P_2 = \bigwedge_{p=1}^4 P_{np}$ describes motion of this image in G .

Let a task of the system S is detection of movement in a visual field on a priori known image M_1 , which corresponds to $\{\mu_i\}_1$. Then any movements of dot (accurate up to a pixel of the original image) object, intersecting boundary of section of subregion of the filter F_i , realizing operation (6), are find out at once at a level U_0 of a decisions pyramid through change of the sign of the corresponding value μ_i . It explains high sensitivity of the visual system (in particular, peripheral sight) to movements in the visual field. Thus for detection of movement there is enough set $\{\mu_i\}_1$, and for its description — set $\{P_{ni}\}$ (fig.10). It is natural, that any movement in subregion of a homogeneity of a

filter cannot be detected by it and requires the analysis at the following levels of a **U**-pyramid. This property of a filter F_i , on the one hand, ensures the filtering function, and on the other hand, justifies a known principle of life-support, realised by the visual system, – the large mass is gone, the more it is dangerous.

Set $\{P_{np}\} = \mathbf{P}$ for $P_n = (V_1, V_3, V_3)$, where 0 is the condition of inversion of an operator; T, F – Truth, False

3. “Vision” of a shielded object. In practice each system S has some alphabet of standard images (visual images) and will fulfil the identification within the limits of this alphabet. Therefore each standard image, represented by set $\{\mu_i\}$, is the dot class, and the belonging to it can be defined on a condition (17) (fig.7). However, the belonging of the observable image to standard is kept and within the limits of the class of the standard. Really, let $O_T(M_i) = O_T(M)$ be the alphabets of the standard images and $O_T(M_i) = V_i$. Each operator belongs to a subset P_{ni} with cardinal number 7 (fig.5). We shall consider full group $V_i V_j V_k$, for which the following condition (fig.11) is executed

$$\{(V_i V_j V_k), (V_i \bar{V}_j \bar{V}_k), (V_i \bar{V}_j V_k), (V_i V_j \bar{V}_k)\} \in O_T(M_i) \equiv V_i$$

Therefore, all four descriptions of images of full group belong to the standard image V_i : first two are variant of belonging to the dot class; last two belong to the class of the standard. Obtained variants are possible to consider as conditions of shielding of an image of the standard V_i by images of operators from structure of full group. From the point of view of interaction of sets we have a condition

$$O_T \subset O(M_i) \forall V_i \in \{V_i\} \in P_{ni}, \quad (21)$$

where $O(M_j)$ is the image of shielding object. Thus, the condition (21) does not exclude the fact of availability of the standard $O_T(M_i)$ in the observable image $O(M_j)$. Therefore the rule is correct: let $O_T(M_i), O_T(M_j)$ are images of the standard an observable image in a field of sight. Then

$$O(M_j) \in O_T(M_i) \Leftrightarrow \{V_i\} \sim \{V_i\}_i \quad (22)$$

where the condition of equivalence means concurrence on set (black-and-white) operators provided that for some their subset (21) is correct.

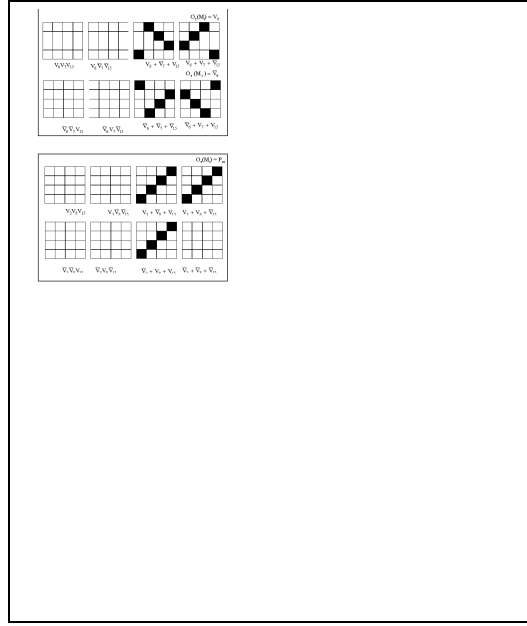


Fig.11. Examples of equivalent patterns of the standart images from sets V and P_n

As any full group is triple (11), each full group contains three shielding (masking) one another of an image of the operators.

4. Detection of object on the image. Each filter $F_i \in \{F_i\}$ on operation (6) reveals concentration of a mass in subregions of a field of sight, appropriate to the design construction F_i . Each V_i , as the binary operator, uniquely saves individual share in creation of an image (black-and-white) image. Therefore a condition (22) is the solution of a problem of the analysis of any gray scale image through synthesis on sets V, P_n , generating binary images (and standards). The example of implementation of algorithm (22) is represented on fig.12, where the standard and deformed images are taken from [16]).

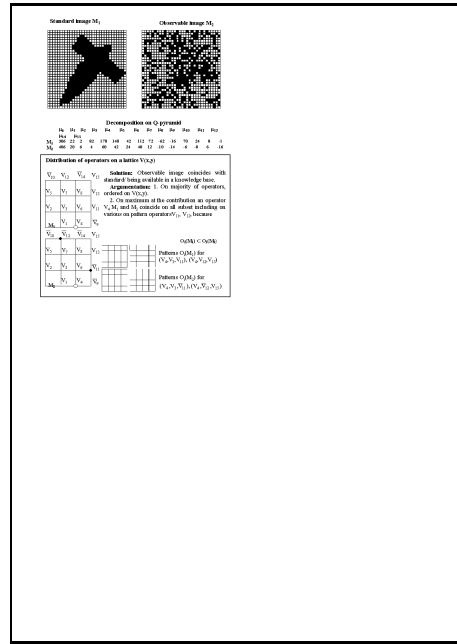


Fig.12. An example of formation of solution in the object detection system

5. Predicting properties of set of the inherent standards. Let $O_T(M_i) = Pni = V_i V_j V_k$. On the one hand, as the set of full groups is created on set of operators, the condition (22) is correct for any full group of set P_n . On the other hand, each full group on set of direct and inverse operators has 8 patterns (fig.11), three of which satisfy to a condition (22) (on fig.11 are highlighted, and one is the variant of an inverse background, requires additional information (for example, knowledge of conditions of observation – day, night)). Four patterns on operation (11) belong to two classes of the images of various orientation on a plane, but using one and the same subset of operators. Therefore, if $V_i V_j V_k$ is a standard, then $\bar{V}_i \bar{V}_j \bar{V}_k$ is the standard for that the images too, but in another way oriented on a plane. Therefore each full group is full on the set of variants of orientations of the standard,

belonging to the class of the standard. The given property ensures invariance of the system of detection to possible positions of object of detection in a visual field, i.e. solves a problem of the prognosis. The similar property is correct and for the standard (for example, produced standard), described by several full groups on operation (13) – fig.13.

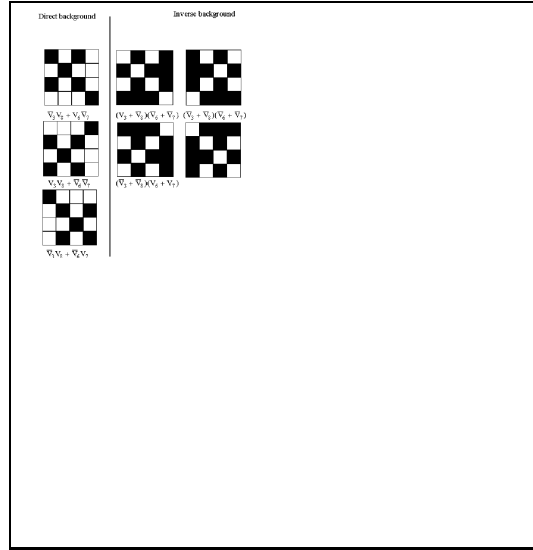


Fig.13. An example of implementation predicting properties of description on a subset of full groups

6. Creation of the representational description. In work of the authors [15] is shown, that each filter, implementing transformation (7), reveals on C^∞ manifold an integrated curve in a visual field concerning center of this field A_0 , and the distribution of elements in structure of each filter corresponds to an integrated curve of i -th of solution of system of equations

$$\frac{dx}{dt} = f_x(x, y) \equiv f_x, \quad \frac{dy}{dt} = f_y(x, y) \equiv f_y$$

and this correspondence is ϵ -exact (fig.14). Besides the tangents to an integrated curve form a skeleton of an appropriate operator from set V

on a lattice $V(x, y)$ (fig.15). The obtained result can be distributed to patterns of full groups and their a sociations (fig.16).

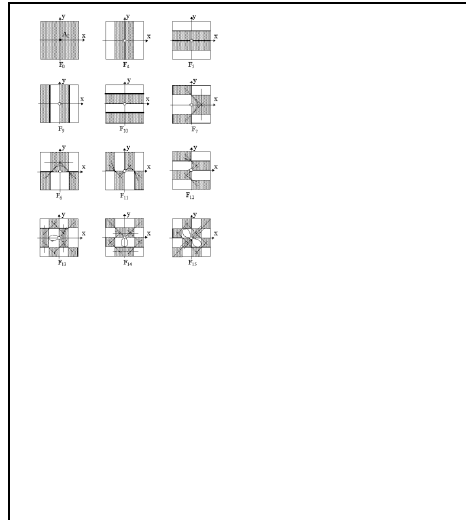


Fig.14. Envelopes (integral curves) for filters of a Q-pyramid

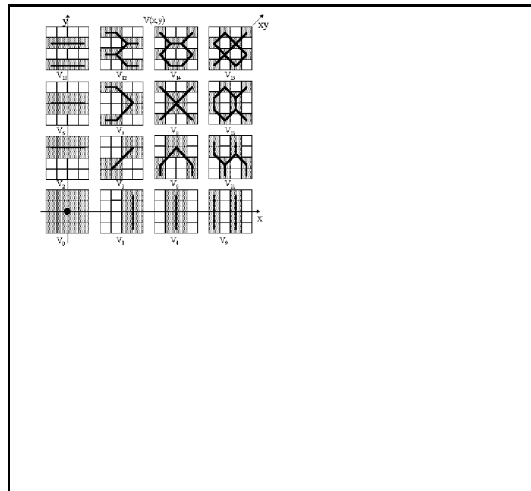


Fig.15. A diversity of sceletons for set $\{V_i\}$ on a latice $V(x, y)$

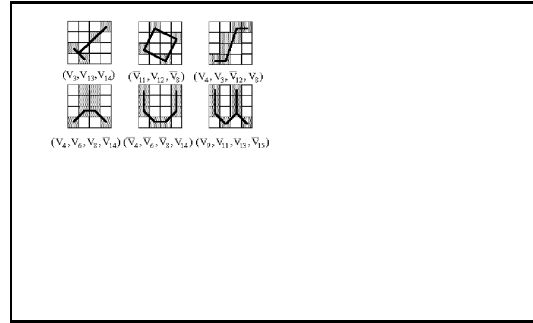


Fig.16. Examples of skeletons for patterns of full groups and their association

7. Restoring of 3-dimensional object using its single two-dimensional image. According to design definition each filter F_i , implementing the differential operator (7), locates i -th differential structure. Let $\phi(x, y) \in C^\infty$ manifold. As the function $\phi(x, y)$ is two-dimensional, and each filter is an operator of a kind [15]

$$\frac{\partial}{\partial x} \left(\frac{\partial^{j-1}}{\partial x^{k-1} \partial y^r} \right) \quad \frac{\partial}{\partial y} \left(\frac{\partial^{j-1}}{\partial x^k \partial y^{r-1}} \right) \quad \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial^{j-r}}{\partial x^{k-1} \partial y^{r-1}} \right),$$

where $k, r = \overline{1, 3}, k + r = 6, j = \overline{1, 6}$, each filter F_i solves a differential partial equation

$$\frac{\partial^j \phi(x, y)}{\partial x^k \partial y^m} = \begin{cases} c - const \\ 0 \end{cases} \quad (23)$$

Solution for (23) is a three-dimensional surface $z = \phi(x, y)$. Therefore each two-dimensional filter F_i is possible to consider as a result of orthogonal projection of a 3-dimensional surface on a plane xOy , dissecting this surface, with allowance for la outs of this surface above a plane xOy (fig.17) – if the part of a surface is located above a plane, to it the sign is assigned, for example a plus (assigning of that or other sign is arbitrary – ambiguity of solution). For a uniqueness of solution additional information is necessary (for example, result of analysis of shadows on object).

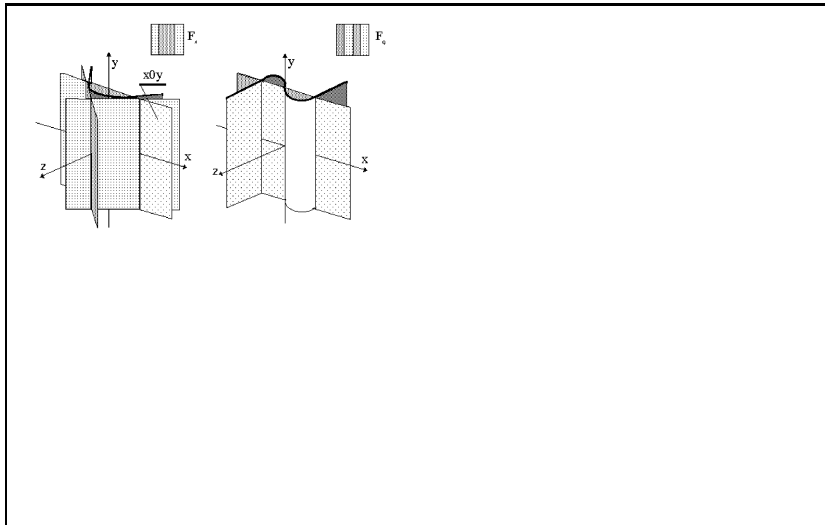


Fig.17. A three-dimensional model of surfaces $z = \varphi(x, y)$ for filters F_4, F_9

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Key words: **recognition, image, algebra, description, standard, class**

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