Abel quadratic differential systems of second kind

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The Abel differential equations of second kind, named after Niels Henrik Abel, are a class of ordinary differential equations studied by many authors (see, for instance, [1-3]). Here we consider the Abel quadratic polynomial differential equations of second kind denoting this class by $\boldsymbol{QS_{Ab}}$. Firstly we split the whole family of non-degenerate quadratic systems in four subfamilies according to the number of infinite singularities. Secondly for each one of these four subfamilies we determine necessary and sufficient affine invariant conditions for a quadratic system in this subfamily to belong to the class $\boldsymbol{QS_{Ab}}$. Thirdly we classify all the phase portraits in the Poincaré disc of the systems in $\boldsymbol{QS_{Ab}}$ in the case when they have at infinity either one triple singularity (21 phase portraits) or an infinite number of singularities (9 phase portraits). Moreover we determine the affine invariant criteria for the realization of each one of the 30 topologically distinct phase portraits. To obtain these criteria we apply the theory of algebraic invariants of polynomial differential systems, developed by Sibirsky and his disciples (see for instance [4-8]).

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