Family of subspaces with bases of countable order

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The families of subspaces of spaces are considered, more precisely families that have bases of countable order. The results obtained for a spaces with such families are applied in selection theorems.

Namely the following notions are considered:

A base $\mathcal{B}$ of a space X is called a base of a countable order if for any infinite perfectly decreasing sequence $\{U_n \in \mathcal{B} : n \in \mathbb{N}\}$ and a point $x \in \cap\{U_n : n \in \mathbb{N}\}$ the sequence $\{U_n : n \in \mathbb{N}\}$ is a base for X at the point x.

A sieve-base on a space X is an A-sieve $(\gamma, \pi) = (\{\gamma_n = \{U_\alpha : \alpha \in A_n\} : n \in \mathbb{N}\}, \{\pi_n : A_{n+1} \to A_n : n \in \mathbb{N}\})$ of the space X with the following property:

(BCO) For any c-sequence $\alpha = \{\alpha_n \in A_n : n \in \mathbb{N}\}$ and any point $x \in \cap\{U_{\alpha_n} : n \in \mathbb{N}\}$, the sequence $\{U_{\alpha_n}; n \in \mathbb{N}\}$ is a base for X at the point x.

A space X has a sieve-base if and only if the space X has a base of countable order (see [1]).

A family $\mathcal{A}$ of subsets of the space X is called a family of subspaces with (complete) base of countable order if $\mathcal{A}$ is a family of (complete) $A(\Omega)$-subspaces of the space X.

Theorem 1. *Let $\mathcal{A}$ be a family of $A(\P)$-subspaces of a regular space X. The following assertions are equivalent:*

(1) $\mathcal{A}$ is a family of subspaces with a complete base of countable order.

(2) There exist a regular space Z, a continuous pseudometric d on Z and an open continuous mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that the family $\mathcal{A}_f$ is complete metrizable by the pseudometric d.

(3) There exist a regular space Z and an open continuous mapping $f : Z \longrightarrow X$ of the space Z onto the space X such that $\mathcal{A}_f$ is a family of subspaces with a complete base of countable order.

Theorem 2. *Let $\mathcal{A}$ be a family of subspaces with a complete base of countable order of the space Y, $\theta : X \longrightarrow Y$ be a lower semicontinuous mapping of a paracompact space X into a space Y and $\theta(x) \in \mathcal{A}$ for each $x \in X$. Then:*

(1) There exists an upper semicontinuous mapping $\psi : X \longrightarrow Y$ *such that* $\psi(x) \subseteq \theta(x)$ *and* $\psi(x)$ *is a subspace with property ¶ for each* $x \in X$.

(2) If $dim X = 0$, *then there exists a continuous single-valued mapping* $g : X \longrightarrow Y$ *such that* $g(x) \in \theta(x)$ *for each* $x \in X$.

References

[1] A. V. Arhangel'skii and M.M. Choban, *Spaces with sharp bases and with other special bases of countable order*, Topology and its Applications. 159 (2012) 1578-1590.

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