On boundedness of the operator with Cauchy kernel on the real axis

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Studying boundary value problems and singular integral equations the class of functions $E(\Gamma)$ defined on some curve Γ is considered. The main questions that arise in relation to the classes $E(\Gamma)$ are the following: 1) find the class of functions to which the singular integral $(S_\Gamma\varphi)(t)$ belongs when $\varphi \in E(\Gamma)$; 2) find the classes of functions that are invariant with respect to the singular integral; 3) find the Banach spaces $E(\Gamma)$ in which the operator S_Γ is bounded, that is

$$||S_\Gamma\varphi||_{E(\Gamma)} \leq const \cdot ||\varphi||_{E(\Gamma)}, \ \forall\varphi \in E(\Gamma). \tag{1}$$

It should be noted that if the functional class E is fixed, then different problems depending on the class of curves to which belongs Γ arises.

For example, it is known that a singular integral $(S_\Gamma\varphi)(t)$ exists if Γ is a circle and $\varphi \in L_p(\Gamma),\ 1 < p < \infty$. But if Γ is an arbitrary smooth curve, then up to now it is not possible to find out whether the noted result remains valid or not.

In this paper we study the case when the curve Γ is unbounded, "admissible" curve, in particular, Γ is the real axis. As $E(\Gamma)$ we take the space of all measurable functions φ on Γ with the norm

$$||\varphi|| = \left(\int_\Gamma |\varphi(t)|^p \rho(t)|dt|\right)^{1/p}. \tag{2}$$

It is proved that under certain restrictions on the weight $\rho(t)$ the operator S_Γ is bounded in the space $E(\Gamma)$, that is the inequality (1) holds. The proof of this proposition is based on the results from [1].

References

[1] Khvedelidze B.V. The method of Cauchy type integrals in discontinuous boundary value problems of the theory of holomorphic functions of a complex variable. Itogi Nauki i Tekhniki. Ser. Sovrem. Probl. Mat., vol. 7, 1975, p. 5–162.

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