DESIGN BASED ON PERFORMANCE. THE SEISMIC RESPONSE OF A STRUCTURE USING A NONLINEAR NUMERICAL ANALYSIS IN DISPLACEMENTS

M. Ieremia
Technical University of Civil Engineering, Bucharest, Romania

ABSTRACT

As the United States, Japan, and Europe move towards the implementation of Performance Based Engineering philosophies in seismic design of civil structures, new seismic design provision will require structural engineers to perform nonlinear analyses of the structures they are designing.

The principal objective of this investigation is to develop a "Push-over" analysis procedure based on structural dynamics theory. The pseudo-static "Push-over" analysis is becoming a popular tool for seismic performance evaluation of existing and new structures.

The purpose of the analysis is to evaluate the expected performance of a structural system by estimating its strength and deformation demands in design earthquakes by means of a static inelastic analysis, and comparing these demands to available capacities at the performance levels of interest. This "Push-over" analysis procedure is obviously based on two major assumptions: – the response of the multi-degree-of-freedom structure can be related to the response of an equivalent single-degree-of-freedom system, implying that the response is controlled by a single mode and this mode shape remains unchanged even after yielding occurs, and – the invariant lateral force distribution can represent and bound the distribution of inertia forces during an earthquake. By including the contributions of a sufficient number of modes of vibration, the height-wise distribution of responses estimated by modal "Push-over" analysis is generally similar to the "exact" results from nonlinear Response History Analysis.

EVALUATING THE DISPLACEMENT OF A STRUCTURE DURING EARTHQUAKE BY USING A "PUSH-OVER" STATIC NONLINEAR ANALYSIS

The main parameter when characterizing the seismic response of a structure, both for meeting the requirements for life-safety, and those of limiting the degradation,-is the **lateral displacement**. Due to this reason, providing a sufficient lateral rigidity during the conception phase becomes a primary factor in the seismic design. Therefore, the seismic response of those buildings which have major torsion vibrations provoked by the coupling of the torsion vibrations modes with those of translation is a disadvantageous one, with significant increases of the lateral displacements.

The way in which structures respond to the seismic excitation is described by the response spectra of structural displacements, S_d, of speed, S_v, of acceleration, S_a, between which the following approximate relationships exist:

$$S_a(\omega,\xi) = \omega^2 S_d(\omega,\xi) = \omega S_v(\omega,\xi) \qquad (1)$$

where, $\omega^2 = k/m = (2\pi f)^2$ – pulsation (circular frequency) of the mass linear oscillator (m) and rigidity (k); $\xi = \frac{c}{2m\omega}$ – damping factor or fraction of the critical damping of the linear oscillator; c – damping coefficient of the internal friction of the viscous material of which the structure is made (for solids – the Voigt model).

The dynamic response (in time) of a nonlinear system is very difficult to model, therefore, in practice; they mainly use "Push-over" static nonlinear calculus methods to determine the structure behaviour. By a conjugated usage of the inelastic response spectra, it is possible to determine the maximal response of the structure, estimated in displacements and then in efforts.

From the practical point of view of dimensioning structures to the seismic action, there are two ways of approaching the problem of their resistance, for which the quantitative factor is the ductility (displacement). On the one hand, we find the constructive provisions of the design codes that are doubled by the appropriate reduction of the conventional calculus seismic forces.

On the other hand, we find the theoretical approach and the modeling of the finite element, where they start from a fine mesh of the structure and, after a nonlinear analysis; they obtain the “real” response in case of a certain type of seismic event. Although it is more difficult to apply it, this last solution is the one which should be chosen for the projects with a special technicality.

The dynamic calculus of structures can not be generally done on a *linear* model. This type of model is appropriate for the study of vibrations of those systems whose amplitudes remain moderate. But this is not the case of structures that are placed in seismic areas. As the amplitudes become significant, the material of which the studied structure is made passes over the elastic field and the system’s behaviour becomes nonlinear.

Although a real structure is generally more complex than an oscillator with one dynamic degree of freedom, we can often reduce its study to such a model, and a simple model allows us to approach the basic concepts of ductility, inelastic spectra and numerical solving of nonlinear systems.

For the linear system, there is a biunique correspondence between force and displacement. The dimensioning of such a structure traditionally needs only an estimation of the forces which act upon it. The present thinking way which is based on the equilibrium of forces is generally preferred by engineers.

For the elastoplastic or *nonlinear* system, there isn’t any biunique relationship between force and displacement. Force is not the significant parameter anymore, because the maximal force which the system can assume is still limited by its resistance characteristics, but this maximal force corresponds to infinity of displacement vales, of which some can go beyond the stability limit of the system. For the nonlinear system, the main parameter which has to be studied during dimensioning is the maximal displacement D_m, or, in an equivalent manner, the ductility μ.

The simplest method used by seismic engineering for calculating the nonlinear displacement of a structure subject to a earthquake is to use Newmark’s rule (1960) concerning the conservation of displacements: the maximum of the relative displacement of a simple oscillator with a nonlinear behaviour (represented by a “elastic-perfect plastic” model) is equal to that of a simple equivalent elastic linear oscillator with the same proper frequency and damping (but with a rigidity that is lower than the elastic rigidity of the initial oscillator). It’s worth mentioning that the displacements equivalence is justified only for long periods, that is, for oscillators which are flexible enough related to the frequential content of the seismic excitation.

Unfortunately, the analysis of displacements is not very clearly explained in all seismic design codes which try to keep the possibility of using the engineering logic of the force-based dimensioning. The forces are estimated by an elastic calculus and then reduced by a reduction coefficient, R, whose value is given in the regulations. This value depends on the material which is used for the structure (steel, concrete, bricks) and on its structural scheme (frames, shear walls…), in other words, on the maximum admitted value of the displacement that the system can accept or by the maximum accepted ductility.

That is why the name of the “Push-over” nonlinear analysis comes from the substance of the method: setting a unique “effort-displacement” curve, which should characterize the behaviour of the resistance structure which is subject to a “push”-excitation which is monotonously increasing, stronger and stronger. The checking criteria of the “primary” structural elements and “secondary” nonstructural ones are defined by comparing the maximal deformation effectively produced by the earthquake to their maximal deforming capacity.

This deals with a static calculus applied on a nonlinear or linear equivalent model, driven by a series of gravitational loads (deadweight, service loads, and climatic loads – snow) which remain constant throughout the numerical experiment and lateral horizontal loads – earthquake, which increase incrementally. These final loads are multiplied by the increasing factor λ until you obtain the plastic deterioration considered the acceptable limit for the security of that structure. The lateral loads are applied at the level of the structural model’s masses and simulate the significant inertia forces of the seismic action, having a distribution largely similar to that of the displacements which have their origins in fundamental vibration mode I (the proper vector diagram, Φ_i). This distribution is rigorously accurate for a monomodal system, in the elastic area of behaviour of the material.

But the nonlinear static equivalent “Push-over” analysis based on the distribution of the lateral forces correspondent to the fundamental vibration mode is not appropriate for a seismic calculus, unless the fundamental mode becomes predominant, that is, the mass which is driven into vibration represents more than 80% of the total structure mass. This requires a quasi-regular disposition on the plane and on the vertical, both for masses, and for rigidities.

The repartition of the horizontal efforts in the earthquake is also sometimes (arbitrarily) chosen according to the type of the structure, in this way: inverse linear-triangular disposition for regular structures in frames with the height $\boldsymbol{H}$ (of the type $\Phi = \frac{\boldsymbol{x}}{\boldsymbol{H}}$), parabolic disposition for regular structures with carrying shear walls ($\Phi = \left(\frac{\boldsymbol{x}}{\boldsymbol{H}}\right)^{1,5}$), uniform disposition for multiple stage structures made in frames with a "piloti"-type floor (e.g. flexible ground floor).

In the case of a structure which has a certain distribution of masses and rigidities, Chopra (1995) suggested a specific "Modal push-over" analysis, able to also take into account the participation of the superior modes in the structure's vibration. This goes back to elaborating a "Push-over" analysis for each proper vibration mode, the structure being similar to a simple oscillator and using a repartition of lateral efforts similar to the modal deformation characteristic for each of these modes.

Taking into account of each vibration mode, the displacements spectrum directly offers the appropriate modal displacement, obeying the rule of equivalence of the altered displacements. In conclusion, these displacements are combined by considering the modal participation factors, according to the rule of modal composition SRSS-Square Root of Square Sum, or the CQC-Completely Quadratic Combination.

The efforts (normal and shearing forces, bending and twisting moments) are then determined according to the displacements obtained and the nonlinear constitutive behaviour laws of the used elements.

The total maximal structural response (in efforts or displacements) is more accurately estimated by using SRSS or CQC rules when the seismic movement has a spectral composition with a wide frequency band and an effective duration which is longer than the proper fundamental vibration period of the structure. The SRSS rule will be applied when the modal responses with significant contributions can be considered independent, the proper vibration modes being clearly separated. The CQC rule will be applied when the vibration modes correspondent to certain oscillations $\boldsymbol{j}$ and $\boldsymbol{k}$ can not be considered independent; in this case, a modal correlation coefficient will be taken into account.

The "Push-over" curve, representing the resistance capacity of a structure subject to earthquake action, can be obtained starting from a static nonlinear calculus applied on a F.E.- finite element based on a specific constitutive law. Therefore, the reinforced concrete requires a nonlinear behaviour of the material due to the stretching cracking and compression crashing of the concrete and the plastic flow of soft steel used in building. These constitutive nonlinearities can be taken into account by laws of monotonous or cyclic structural behaviour which can be classified in three major families:

– *Global models*, based on bilinear elasto-plastic laws, with/without hysteretic behaviour (e.g. for steel) or trilinear laws – Takeda (e.g. for reinforced concrete) which link the bending moment to the curvature or the shear force to the specific angular strain. These models require a small number of parameters to define the curve of the first load (the bending rigidity $\boldsymbol{K = EI}$ or the shear rigidity $\boldsymbol{K = GA}$) or the cyclic behaviour (useful for the dynamic temporal calculus);

– *Local models* by means of which one can describe the behaviour of each constitutive material of which the structure elements are made: steel, concrete, steel-concrete cooperate in shearing strain, bricks etc;

– *Semi-global models*, multiple-layer models or fiber models. Therefore, in the bended beam theory, they consider the bi-dimensional geometric description of a section to behave on the cinematic plane according to J. Bernoulli's hypothesis about plane and normal sections, without shearing deformations or according to S. Timoshenko's hypothesis concerning the deplaned (contorted) sections due to the effect of the tangential shear stresses.

Concerning this issue, CASTEM 2000, the numerical calculus program, improved as CAST3M, created by Pegon/1993, Guedes/1997 and Combescure/2001, is mainly used by the structuralist engineers in the European Community to model the response of the buildings which are subject to earthquake action.

According to the presented considerations, one can plot the „Push-over" capacity curve which translates the behaviour (response) of the structure under the excitatory load (mainly the earthquake). Therefore, the displacement $\boldsymbol{D}$ determined on top of the structure is set in the abscissa and the shear force $\boldsymbol{V}$ (reaction) calculated at the base of the structure is set in the ordinate. This shear effort represents the sum of the exterior lateral forces multiplied by an increasing parameter, $\boldsymbol{\lambda}$ (fig.1). The total lateral force shall be distributed to the various vertical elements of the lateral-force-resisting system in proportion to their rigidities considering the rigidity of the horizontal bracing system or diaphragm. Rigid elements that are assumed not to be part of the lateral-force-resisting system may be incorporated into buildings, provided that their effect on the action of the system is considered and provided for in the design. The rigidity and dynamic characteristics of the structure should be determined:

– The initial rigidity, K_i, corresponds to the slope of the capacity curve in its elastic area; $tg\alpha = K_i \cong const$.

– The effective rigidity, K_e, corresponds to 60% of the value of the plastic flow shear force; $\boldsymbol{K_e = 0,6\frac{V_y}{D_1}}$ **[kN/m]**

One obtains the initial vibration periods, $\boldsymbol{T_i}$, and effective vibration period, $\boldsymbol{T_e}$:

$$\boldsymbol{T_i = T_1 = \frac{1}{f_1}} \quad \textbf{[s]}$$

$$\boldsymbol{T_e = T_i\sqrt{\frac{K_i}{K_e}} = 2\pi\sqrt{\frac{M}{K_e}}} \quad \textbf{[s]}$$

So, the “effort $\boldsymbol{V}$ – displacement $\boldsymbol{D}$” curve represents an intrinsic characteristic of the structure regarding the effect of the lateral horizontal actions (wind, seism, braking on the bridge crane etc.) of a static or dynamic type. The “$\boldsymbol{V-D}$” curve emphasizes the structure’s ability to dissipate the energy and, consequently, it provides an estimation of the expected plastification mechanism, as well as the distribution of the progressive structural deterioration, depending on the increasing intensity of the excitatory seismic forces and the size of the corresponding horizontal displacements.

To concern: story drift-the lateral displacement of one level relative to the level above or below and story drift ratio-the story drift divided by the story height.

As presented so far, the “Push-over” capacity curve can be obtained by starting from a nonlinear static calculus which is sometimes difficult to make due to the problems regarding the convergence of solving differential equations which characterize the physical phenomenon and the obstacles met in the F.E. modeling.
Sometimes it is easier and less expensive to use simplified methods which are based on a series of linear calculations and constitutive laws of “elastic-perfect plastic” behaviour.

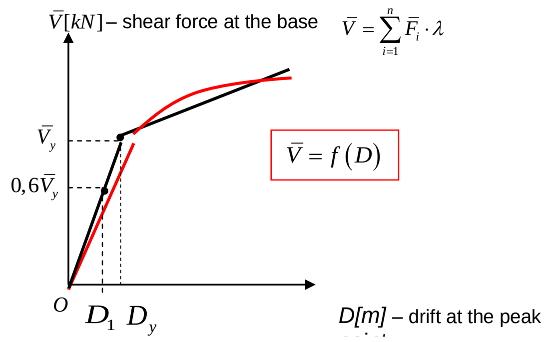

Figure 1. Capacity curve.

In any of the situations, the relevance of the result which is obtained through iterative procedures depends on the capacity of the used algorithm to consider and to adequately model the progressive alteration of the structure behaviour with regard to the rigidity degradation and the increase of the structural elements’ flexibility; this process becomes obvious when the plastic strains occur or, for the structures in frames, when the plastic articulations are formed.

When designing, the successive forming of plastic articulations should be controlled within the phenomenon of structure adaptation. It is recommended that the directed lines for creating plastic articulations be successively formed at the ends of the rows of beams, from bottom to top. The columns will represent the final elastic lines. The directing of the plastification lines along the ends of the girders seems to be the best solution, if one takes into account the fact that the ductility of beams is much higher than that of the columns.

As beams are elements which are subject mainly to bending moments and shear forces, they can be ductilized relatively easily. The increase of the plastification capacity of the beam section is obtained by consolidation of the compressed area, setting reduced reinforcing percents so that the longitudinal reinforcing can arrive at the plastic flow before the crushing of the concrete and using steels with flowing bearing. Beams with a 10÷20 ductility can be obtained under these circumstances.

As columns are elements which are subject to high compression forces, with bending moments and shear forces, they are very difficult to ductilize. The axial compression effort is the main cause of the element’s weakening. The transversal deformation is blocked through an adequate transversal binding, and the axial compression stress is modified into triaxial compression, transforming the brittle and breakable material into a resistant and ductile one. As ductility is reverse proportional to the compression stress, it is recommended that the value of the average compression stress not excel (0,25÷0,35) R_b. With these methods, one can obtain ductility values of 2÷5 for columns.

In order to reduce the distance from the behaviour of the numerical calculus model to the behaviour of the real structure in frames, they promote a simplifying method based on the successive formation of plastic articulations as a consequence of applying the static theorem of the maximum of the yielding load. It is necessary to go through all the phases covered by the structure driven by a load system which increases proportionally from the zero value to that which is appropriate for subsidence, within a biographic calculus-capacity design method.

– Determining the bending moments which have the capacity of plastification in all the critical sections of the resistance elements (columns, beams) of the structure. This will be achieved with the help of the “moment-curvature” constitutive law, resulted from the analyses of the sectional responses. The bending moment diagram has to simultaneously meet the equilibrium and plastification requirements.

– Determining the effective efforts in the structure due to the vertical gravitational loads and to the incremental requirements produced by the applying of a system of unitary horizontal lateral forces representing the seismic action.

– Determining the location of the next plastic articulation and the correspondent lateral load. Modeling the structure with the new articulation and iteration of the bending moments.

– The increment of the lateral load until the next plastic articulation is obtained in the bending moment diagram and, at the end, until they obtain the “failure” limit state – plastic subsidence mechanism (through displacements, shear force etc.).

Remark: following the method application, the repartition of the initial horizontal load can be maintained or modified in order to take into account the mechanism deformation.

Knowing the plastification bending moment, $\boldsymbol{M_{pl}}$, for two values of the normal effort, $\boldsymbol{N}$, one can determine, by linear interpolation, the plastification moment of an element, $\boldsymbol{M_{pl}}(\boldsymbol{N})$, for a normal effort, $\boldsymbol{N}$– fig.2.

$$M_{pl}(N) = M_{pl}(N_1) + \alpha$$

$$\frac{\alpha}{M_{pl}(N_2) - M_{pl}(N_1)} = \frac{N - N_1}{N_2 - N_1}$$

$$M_{pl}(N) = M_{pl}(N_1) + \frac{N - N_1}{N_2 - N_1} \cdot \left(M_{pl}(N_2) - M_{pl}(N_1)\right)$$

The force increment ΔF necessary for the bending moment $\boldsymbol{M}(\boldsymbol{N})$ to reach the plastification moment value.

$$M(N) = M_i + \frac{dM}{dF} \cdot \Delta F \; ; \; N = N_i + \frac{dN}{dF} \cdot \Delta F \quad (2)$$

$$M_i + \frac{dM}{dF} \cdot \Delta F = M_{pl}(N_1) + \frac{N_i + \frac{dN}{dF} \cdot \Delta F - N_1}{N_2 - N_1} \cdot \left(M_{pl}(N_2) - M_{pl}(N_1)\right) \quad (2')$$

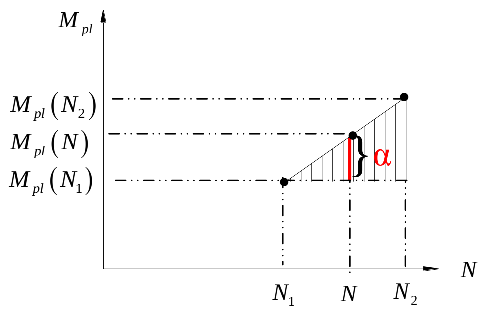

Figure 2. Bending Moment – Axial Force Diagram

In conclusion,

$$\Delta F = \frac{M_{pl}(N_1) - M_i + \frac{N_i - N_1}{N_2 - N_1}\left[M_{pl}(N_2) - M_{pl}(N_1)\right]}{\frac{dM}{dF} - \frac{dN}{dF} \cdot \frac{M_{pl}(N_2) - M_{pl}(N_1)}{N_2 - N_1}} \quad (3)$$

For the multi-floor buildings with a frame structure and brickwork filling that are subject to the seismic action, one should take into account the diaphragm effect, until the filling brickwork gets out of work. In a simplified manner, we can admit a calculus scheme by comparing the brickwork filling frame to a truss beam, the columns taking over the role of soles, the beams taking over the role of the vertical frames, and the filling brickwork the one of the compressed diagonals.

The inherent over strength structure in the total plastic failure form will be estimated by mean of the base shear force correspondent to yield mechanism, related to the force that causes the first yielding.

The need for drawing the "Push-over" curve within the application of nonlinear analysis methods in displacements occurs immediately in the process of determining the *performance point* (functioning point) of a structure by using the quantitative factor in damping, ξ, and elastic response spectra (with increased damping values), or, respectively, a ductility approximation, μ, and inelastic calculus spectra.

For this it is necessary to superpose a curve representing the resistance ability of a structure resulted from a "Push-over" nonlinear static analysis with a curve representing the solicitation imposed by the seism. This excitation is directly emphasized through an ADRS format curve - Acceleration Displacement Response Spectrum. The ADRS curve is obtained by reporting the spectral displacement (S_d [cm]) correspondent to a seism - on the abscissa and the response spectrum in pseudo-accelerations (S_a [g]) – on the ordinate, starting from a 5% damping.

The radial secant straight lines which start from the origin represent isofrequential curves (f = =constant) or isoperiodical ($T = \frac{1}{f}$) and they should be cautiously interpreted when using the inelastic spectra in ADRS format. This happens because the maximal displacement of the oscillator, D_m, and the acceleration, A_y, which produces the effort at the elastic limit (plastic flow), are linked through a relationship that directly depends on the ductility, μ. As a consequence, the secant slope depends on the period through a relationship which changes according to ductility.

$$D_m = \mu A_y \left(\frac{T}{2\pi}\right)^2 \qquad (4)$$

It is worth mentioning that this "Push-over" capacity curve which characterizes the structure behaviour can not be directly superposed with the ADRS spectrum and that is why it is necessary for the curve to be conversed in the capacity spectrum in order to homogenize its parameters into spectral accelerations, S_a, and spectral displacements, S_d.

Therefore,

$$S_a = \frac{V}{\alpha_1 G} \qquad (5)$$

$$S_d = \frac{D}{(PF_1)\Phi_1^V} \qquad (5')$$

where, α_1 – the modal mass coefficient of the first vibration mode; PF_1 – the modal participation factor of the first vibration mode; Φ_1^i – the amplitude of the first vibration mode (the proper vector) at level i; Φ_1^V – the amplitude of the first vibration mode on top of the structure.

In a modern way of thinking, the seismic action can be perceived as an energy supply process of a structure which, if sensibly designed according to the inelastic ductility conception, absorbs, dissipates and gives back the induced energy (back to the foundation ground). The ability of a structure to absorb the mechanical energy through plastic deformations in both directions is characterized by the ductility concept. The attenuation of the structure response to the seismic excitation through nonelastic deformations represents the damping by ductilization.

The actual ductility, μ, is the ratio between the maximal elastoplastic strain, D_m, and the strain on the elastic threshold (at the yield point), D_y. The response attenuation through plastic strains is emphasized by the subunitary ratio ψ_D between the yield force, F_y and the force which provides a high elastic resistance to the structure, F_e.

$$\mu = \frac{D_m}{D_y} > 1; \qquad \psi_D = \frac{F_y}{F_e} < 1 \qquad (6)$$

To keep the maximum displacement, if we equalize the induced energies (elastic and elasto-plastic) in the dynamic "Force-Displacement" model (fig.3) according to the two design conceptions: elastic and inelastic (without hysteretic behaviour), we obtain the ψ_D coefficient which characterizes the response attenuation (damping) by

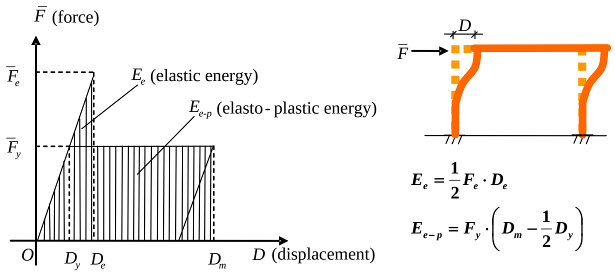

Figure 3. Force-displacement diagram. Absorbtion of energy.

inelastic deformations, due to ductility.

$$\psi_D = \frac{1}{\sqrt{2\mu - 1}} \tag{6'}$$

The proper seismic forces of the inelastic behaviour of the structure will be obtained if we reduce the elastic seismic forces by multiplying them by factor ψ_D. The higher plastic displacement capacity of the structure, the lower the seismic calculus forces will be. Thus, for a ductile reinforced concrete structure ($\mu = 5$), the calculus forces will be reduced to a third (ψ_D =**1/3**), compared to the elastic forces.

So, the damping which evaluates the energy dissipated by the structure in the post-elastic field depends on the displacements felt by the structure during a seism.

To conclude, the intersection between the “Push-over” curve and the ADRS spectrum provides a *performance point* PP which designates the nonlinearities that affect the structure; the result will be an damping which, in most practical cases, will not be the same with the initial one. It is therefore necessary to update this damping in order to determine a new ADRS spectrum. This spectrum will be used in an iterative calculus to determine another performance point and, consequently, a new displacement. The equivalent damping of this displacement has to be compatible with the representation of the seismic solicitation. This method of determining the performance point of the structure is called the damping method, because the quantitative factor is the damping. Then, one has to evaluate the ***M***, ***N*** and ***V*** efforts, the linear and angular displacements, and check the resistance and rigidity of the structure.

The calculus of the equivalent damping, ξ_{eq}, is based on representing the dynamic behaviour of the idealized structure, that is, the energy dissipated by an elastoplastic oscillator with hysteretic behaviour (fig.4).

$$\xi_{eq} = \xi_0 + 0{,}05$$

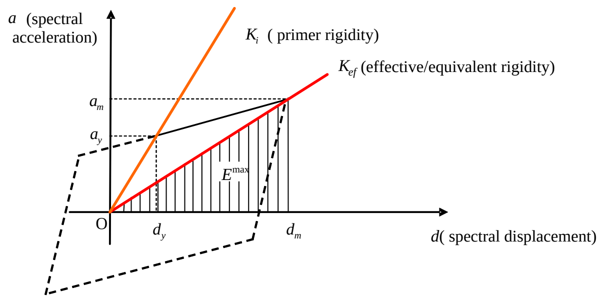

Figure 4. Idealized structure behaviour image

The 0,05 coefficient designates the viscous damping through internal frictions of the material the structure is made of (for example, reinforced concrete). The critical damping fraction which globally characterizes the damping of a certain material varies from 2% for steel to 18% for brickwork or prefabricated materials.

In order to consider the dynamic behaviour specific to a certain type of structure made of a certain material, an effective damping, ξ_{ef}, is used in the practical calculus, $\xi_{ef} = k\xi_0 + 0{,}05$, where, ***k***– empirical coefficient which depends on the energy dissipation capacity, that is on the dynamic behaviour (with hysteretic damping) of a certain type of structure (with a ductile-breakable behaviour); the coefficient depends on the typology and age of structure, as well as on the earthquake period; ξ_0 – the equivalent viscous damping which is correspondent to the hysteretic damping, β_0;

$$\xi_0 = \frac{1}{4\pi} \cdot \frac{E_D}{E^{max}} = \frac{2\left(a_y d_m - d_y a_m\right)}{\pi a_m d_m} \tag{7}$$

E_D – the energy dissipated through damping;

$$E_D = 4\left(a_y d_m - d_y a_m\right) \tag{8}$$

E^{max} – the maximal deformation energy.

$$E^{max} = \frac{1}{2a_m d_m} \tag{8'}$$

The calculus of the performance point can also be done by comparing the curve concerning the energy dissipation ability of the structure with the curve having the form of a spectrum which concerns the energy needed to be dissipated. This is why the method is also called the ductility method, as ductility is the quantitative factor. The energy request is characterized by the PGA point – Peak Ground Acceleration on the curve and the corner period, T_c, adequate to the end of the spectral plateau.

Using elastic spectra (the damping method) is a method which is highly differentiated from the method based on using inelastic spectra (the ductility method) by the calculus of the R factor, the one of reducing the elastic spectrum; the factor allows the evaluation of adequate reduced spectra (super damping spectrum).

The effort reduction factor can be expressed as:

a) $R_\xi = \dfrac{S_{ae}}{S_a}$ – for the quantification in damping method. R_ξ can be expressed in accordance with the elastic damping, ξ_e and the equivalent damping, ξ_0.

$$R_\xi = \sqrt{\frac{\xi_{el}}{\xi_{el} + kk'\xi_0}} \tag{9}$$

The k' coefficient depends on the parameters of the capacity curve - A_y, T_e, and on those of the energy request curve - PGA, T_c.

b) $R_\mu = \dfrac{S_{ae}}{A_y}$ – for the quantification in ductility method.

To be noted: S_{ae} – acceleration of the initial elastic spectrum (ξ=5%); S_a – acceleration of the reduced spectrum; $A_y = \omega^2 D_y$ – acceleration at yielding of the inelastic oscillator; T_0 - elastic period.

In this case, the reduction factor depends on the value of the vibration period in comparison with the corner period value. Thus,

– for $T < T_c$, $$R_\mu = \frac{T(\mu - 1)}{T_c + 1}; \tag{10}$$

– for $T \geq T_c$, $$R_\mu = \mu. \tag{10'}$$

The reduction factor is graphically described depending on the period for various ductility values.

The main difference between the two methods is noticed by drawing the reduced spectra.

– In the quantification in damping method, these spectra are determined with the constant period – fig.5.

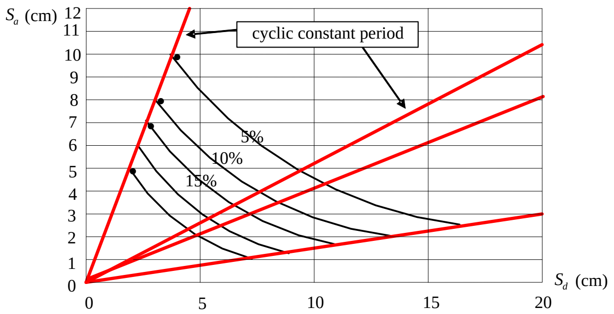

Figure 5. The effect of increase in damping upon T_c–transition period for damping method.

– In the ductility method, these spectra are determined by considering a constant displacement (or constant ductility) – fig. 6.

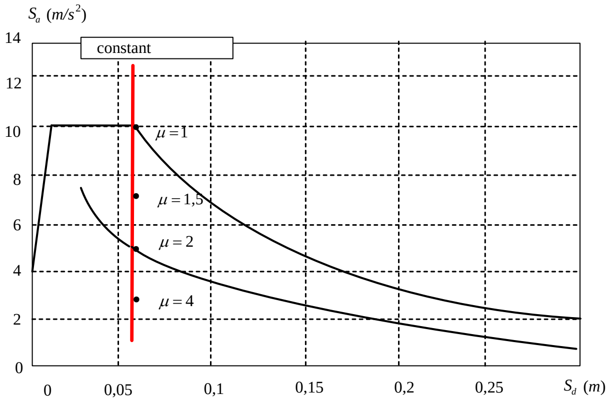

Figure 6. The effect of increase in ductility upon T_c–transition period for ductility method.

For the same structure and elastic acceleration spectrum, there is the possibility of obtaining different performance points in comparison with the reduced spectra – fig.7. In case of a unitary energy dissipation coefficient ($k = 1$; $\mu = 2,7$), the approximation in ductility method (inelastic spectra) provides the same result with the damping method. But if $k \neq 1$, the damping method offers a superior displacement. At the end, another comparison between the two calculus methods, another way of determining the *performance point* of the structure, is to represent the damping variation curves $\beta = \xi_{eq}$ (the factor of hysteretic damping is equivalent to viscous damping), according to the ductility μ.

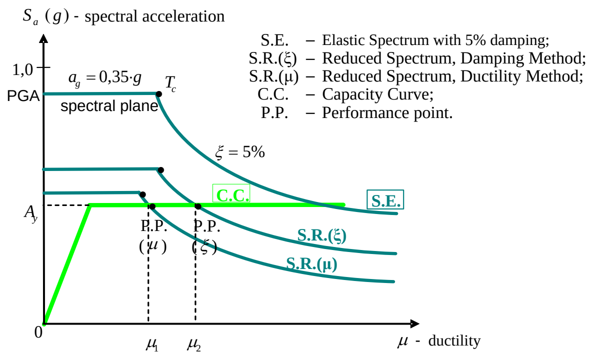

Figure 7. Comparison between performance points indicated by various methods, when T_0<T_c.

– Thus, the damping of the capacity curve which depends on the energy dissipated by the inelastic oscillator for $k = 1$, is:

$$\beta = \frac{2(\mu - 1)}{\pi\mu}; \qquad (11)$$

– The damping of the curve describing the necessary energy is determined as:

$$\beta = \frac{2}{\pi \frac{1-\mu}{\mu_p^2}}, \qquad (11')$$

where,

$$\mu_p = \frac{a_s R_p T_c T}{4\pi^2 D_y} \qquad (12)$$

a_s – nominal ground acceleration; R_p – amplification coefficient ($R_p \approx 2{,}5$).

The intersection of the two damping curves is represented in the *performance point* of the structure.

References

1. Aldea, A., Arion, C., Vacareanu, R., Lungu, D. *Evaluarea vulnerabilitatii la cutremur a cladirilor.Metode de calcul si prioritati de consolidare/, Buletin AICPS nr. 1/2003, p. 14-25.*

2. Chen, A.C., Chen, F.W. *Constitutive Relationship for Concrete. Journal of the Engineering Mechanics Division, vol. 101, nr. EM 4, 1975 , p. 465-481.*

3. Chopra, A.K., Goel Rakesh, K. *Capacity-Demand-Diagram Methods for Estimating Seismic Deformation of Inelastic Structures: SDF System // PEER, pacific Earthquake Engineering Center, University of California, Berkeley PEER 1999/02, April 1999.*

4. Chopra, A.K. *Dynamics of Structures Theory and Applications to Earthquake Engineering. New Jersey, Prentice Hall, Second Edition, 2000.*

5. Cismigiu, Al. *Dupa cutremurul moldavic din 4 Martie 1877. Spre noi tipuri de structuri antiseismice. „Arhitectura", nr.4, 1977, p. 1-23.*

6. Combescure, D. *Modélisation des structures de génie civil sous chargement sismique à l'aide de CASTEM 2000 // Rapport CEA-DEMT/EMSI/RT/01-008, 2001.*

7. Gulkan, P., Sozen, M.A. *Inelastic Responses of Reinforced Concrete Structures to Earthquake Motions. Journal of the American Concrete Inst., 71(12), 604-10, 1974.*

8. Ieremia, M., Sidorenco, E., Gînju, S. *Analiza numerica neliniara a structurilor, vol II - Modelarea raspunsului structural, Ed. Conspress, Bucuresti, 2005.*

9. Newmark, N.M., Veletsos, A.S. *Effects on inelastic behavior on the response of simple system to earthquake motions // Proceedings of the 2nd World Conference on Earthquake Engineering, Japan 2:895-912.*

10. Papanikolaou Vassilis, K., Elnashai Amr, S., Pareja Juan, F. *Limits of Applicability of Conventional and Adaptive Pushover Analysis for Seismic Response Assessment // Mid-America Earthquake Center and Environmental Engineering Department University of Illinois at Urbana-Champaign 2005.*

11. Toba, S. *Consideratii asupra ductilitatii cadrelor din beton armat // Referat doctorat: „Concepte moderne de elasticitate, plasticitate si element finit", Universitatea Tehnica de Constructii Bucuresti, 14.02.2008.*

12. Tulei, E., Cretu, D., Topa, N. *Controlul performantelor unei structuri metalice multietajate supuse la actiuni seismice // A 3-a Conferinta Nationala de Inginerie Seismica, Universitatea Tehnica de Constructii Bucuresti, 09.12.2005.*

13. ** * * Cahier technique AFPS, Nr. 26, 2006: „Méthodes en déplacement: Principe- Codification- Application".*

14. ** * * Eurocode8, pr.EN1998: „Calcul des structures pour leur résistance aux séismes" Règles générales, actions séismiques et règles pour les bâtiments", 2003.*

15. ** * *Applied Technology Council: "Seismic Evaluation Retrofit of Concrete Buildings", ATC-40, Vol. 1 and 2, Report No. SSC 96-01, Seismic Safety Commission, Redwood City, CA, 1996.*

16. ** * *"Computers and Structures", Inc.: ETABS Computer Program v 8.5.4. Berkeley, California, USA.*

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