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The PID Tuning Procedure for Performance Optimization of the Underdamped Second-Order Processes

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***Abstract*— In this work, it is proposed the procedure for tuning the PID controller to the underdamped second-order systems, that offers the possibility to optimize the performance of the system. The analytical expressions for calculation the tuning parameters of the PID controller were obtained according to the maximum stability degree criterion. These analytical expressions permit to calculate the tuning parameters according to the values of the damping ratio, natural frequency and transfer coefficient, that can be determinate from the step response of the underdamped system. The proposed procedure for performance optimization permits to optimize the value of rise time and percentage of overshoot. To demonstrate the efficiency of proposed method the computer simulation was performed.**

Keywords—PID controller; maximum stability degree criterion; underdamped systems; performance of the control system

I. INTRODUCTION

The practice of the automation of the technological processes demonstrates that most of the industrial processes can be controlled by PID control algorithms and its variation. The are a lot of tuning methods of the PID controller that permits to achieve the imposed performance and good robustness to the automatic control systems. There is a relevant issue, as there are many processes (robot arms, cranes, power system electronics etc.) in automation practice that exhibit oscillatory behaviour. These processes are described by the second order models and can required the transient response of the closed loop system with minimum overshoot and perturbation rejection [1].

There are proposed several tuning methods of the PID controller for underdamped systems as Ziegler-Nichols method, frequency methods, Posicast Input Command Shaping (PICS) method. Some of these methods require to be known the mathematical model that approximates the dynamics of the process [1-3].

In this paper, there are proposed the analytical expressions for calculation the tuning parameters of the PID controller for the processes that are described by the second order model of objects. These analytical expressions were developed based on the maximum stability degree criterion and they depend on the values of the damping ratio, natural frequency and transfer coefficient of the system, which are obtained from the step response of the open-loop system.

II. THE TUNING PROCEDURE OF THE PID CONTROLLER TO THE UNDERDAMPED SECOND-ORDER PROCESSES

It is considered given the conventional structure of the automatic control system (Fig. 1), which includes the control object with inertia second order and the controller that represents the generalized form of the PID controller, which it is described by the following transfer function [1]

$$H_R(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_i + k_p s + k_d s^2}{s}, \qquad (1)$$

where k_p, k_i, k_d are the tuning parameters of the controller.

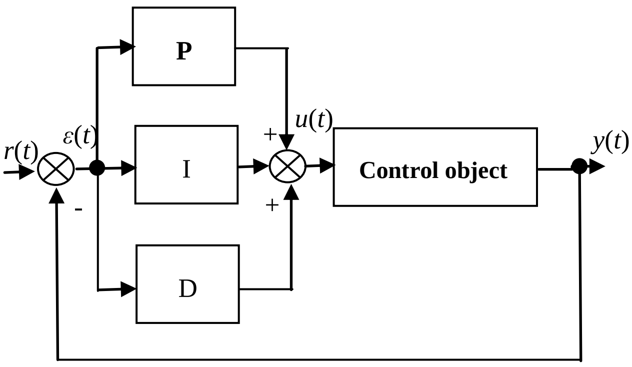

Figure 1. Block scheme of the automatic control system.

The control object is described by the transfer function with inertia second order [1]:

$$H_F(s) = \frac{k\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{k}{a_0 s^2 + a_1 s + a_2}, \quad (2)$$

where k is the transfer coefficient of the control object; ω_n - is the natural frequency, ξ - is the damping ratio, and $a_0 = \frac{1}{\omega_n^2}; a_1 = \frac{2\xi}{\omega_n}; a_2 = 1.$

The value of transfer coefficient, natural frequency and damping ration can be calculated from the step response of the open system:

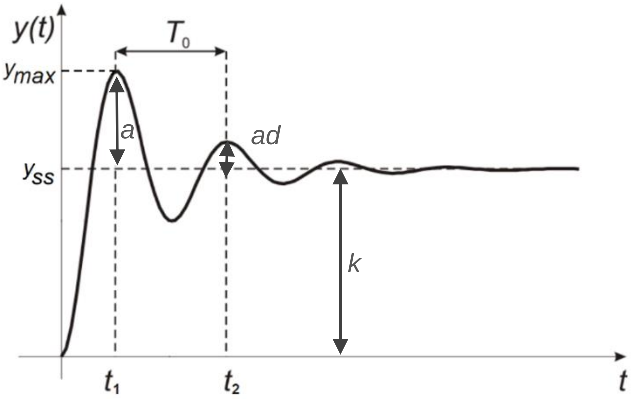

Figure 2. Step response of the underdamped system.

From Figure 2, the damping ratio is calculated according to the [6, 9]:

$$\xi = \frac{1}{\sqrt{1 + (2\pi / \log d)^2}} \quad (3)$$

where d is decay ratio.

The value of the natural frequency is given by:

$$\omega_n = \frac{2\pi}{T_0\sqrt{1-\xi^2}} \quad (4)$$

where $T_0 = t_2 - t_1$ is period of oscillations.

One of the criterion that is used for tuning the PID controller is maximum stability degree (MSD) criterion [4, 8]. This criterion supposes the maximum displacement in the complex half-plane of the nearest characteristic equation's roots of the designed system to the imaginary axe $Re\, p_i \leq 0$.

The characteristic equation of the closed-loop control system is:

$$A(s) = \frac{1}{k}(a_0 s^3 + a_1 s^2 + a_2 s) + k_d s^2 + k_p s + k_i. \quad (5)$$

In the work [9], was proposed for the case when number of the tuning parameters is equal or less then the characteristic equation order, the maximum stability degree value is calculated by the following expression:

$$J = \frac{a_1}{2a_0}. \quad (6)$$

In the paper [9], has been defined the expression for calculation the value of maximum stability degree of the system in dependency of the tuning parameters:

$$J = \frac{k_p}{2k_d}. \quad (7)$$

In this case, by doing the equalling of the expressions (6) and (7), it is obtained

$$\frac{k_p}{2k_d} = \frac{a_1}{2a_0}. \quad (8)$$

From expression (8), it can be presented the analytical expression for calculation the derivative tuning parameter:

$$k_d = \frac{a_1}{a_0} k_p. \quad (9)$$

Next, according to the maximum stability degree method with iterations [7], there are obtained the analytical expressions for calculation the tuning parameters of the PID controller:

$$k_p = \frac{1}{k}(-3a_0 J^2 + 2a_1 J - a_2) + 2k_d J, \quad (10)$$

$$k_i = \frac{1}{k}(a_0 J^3 - a_1 J^2 + a_2 J) - k_d J^2 + k_p J, \quad (11)$$

$$k_d = \frac{1}{2k}(6a_0 J - 2a_1). \quad (12)$$

In this way, using the (7)-(9) expressions, the dependencies (10)-(12) can be rewritten as:

$$k_p = 2J \cdot k_d, \quad (13)$$

$$k_i = \frac{a_2}{k} J, \quad (14)$$

$$k_d = \frac{a_1}{2k}. \quad (15)$$

According to the expressions (3)-(4) the analytical expressions (13)-(15) can be rewritten in the following form:

$$k_p = 2J \cdot k_d, \quad (16)$$

$$k_i = \frac{1}{k} J, \quad (17)$$

$$k_d = \frac{\xi}{k\omega_n}. \quad (18)$$

The tuning parameters depend on the object parameters that are known and can be determinate from the experimental curve of the open loop system and depend on the value of the maximum stability degree J, that can be calculated from the (6) expression.

The procedure for optimization the performance of the automatic control system supposes the variation of the stability degree value $J>0$. According to this variation it is possible to obtain the transient responses with different system performances as overshoot and settling time.

III. Study Case and Computer Simulation

It is considered that the underdamped second-order process has the step response presented in the Figure 3.

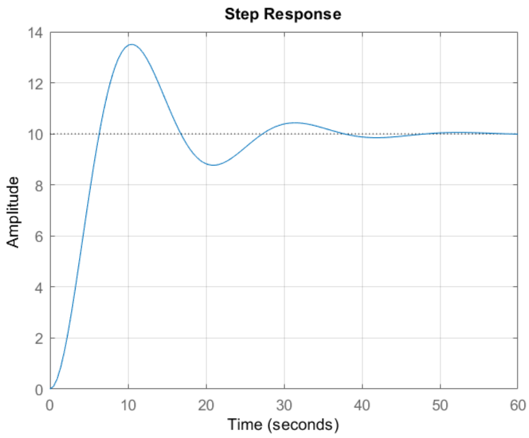

Figure 3. The step responses of the open-loop system.

Based on the expressions (3)-(4) according to the step response, there are calculated parameters of the control object:

$$\omega_n = 0.315,\ \xi = 0.317,\ k = 10.$$

The obtained transfer function, that describes the process is the following:

$$H_F(s) = \frac{k\,\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{0.99225}{s^2 + 0.1997\,s + 0.09922}. \quad (19)$$

Based on the analytical expressions for calculation the tuning parameters of the PID controller (16)-(18), there are obtained the different sets of the tuning parameters presented in the Table I, for the different values of the stability degree.

TABLE I. Tuning Parameters of the PID Controller and Performance of the Control System

No	J	k_p	k_i	k_d	t_s	t_r	σ
1	0.099	0.0199	0.009	0.1006	39.50	39.50	0
2	0.13	0.0262	0.013	0.1006	29.39	29.39	0
3	0.16	0.0322	0.016	0.1006	26.26	9.734	0.81
4	0.2	0.0403	0.02	0.1006	39.93	7.09	3.52
5	0.25	0.0503	0.025	0.1006	47.56	5.66	10.89

The simulation results of the control system with PID controller tuned by the proposed method is presented in the Figure 4, for differents values of the stability degree: curve 1 – transient response for J = 0.099, curve 2 – transient response for J = 0.13, curve 3 – transient response for J = 0.16, curve 4 – transient response for J = 0.2, curve 5 – transient response for J = 0.25.

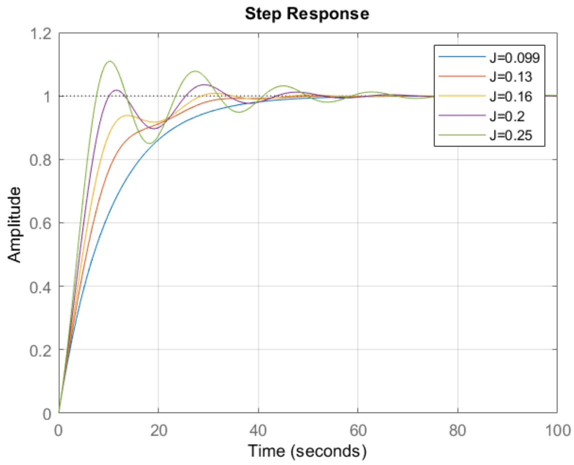

Figure 4. The step responses of the control system.

IV. Conclusions

In this paper, it is proposed the method for tuning the PID controller to the second order underdamped systems. These method can be easily applied for the case then it is known just the step response of the open loop underdamped system and according to the proposed analytical expressions can be calculated the tuning parameters of the PID controller based on the values of the: damping ratio, transfer coefficient and natural frequency.

These expressions were development based on the maximum stability degree criterion, which ensures the good robustness to the control system. The optimization procedure permits to obtain the different performance of the automatic control system by varying the value of the maximum stability degree.

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