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## THE USE OF SYNCHRONOUS VECTOR MEASUREMENTS IN THE CALCULATION OF STEADY-STATE MODES OF ELECTRICAL NETWORKS

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**Abstract.** Calculation of steady state mode is the most used tool of modern electrical networks' analysis. The current study is focused on the application of synchronized phasor measurements technology for steady state mode calculation. There is shown that the solely use of synchronized phasor measurements as initial information for calculating the steady state mode or in conjunction with traditional telemetry from Supervisory Control And Data Acquisition system (SCADA) has a number of advantages over classical methods, which is due to the simplification of the computational process, reduced time and required computational memory, and more reliable informational support. This paper presents the novel approach which consists in steady state mode calculations using accelerated and ultra-accelerated algorithms. These algorithms are based on the implementation of the method of determining values and special structure of the matrix of nodal admittances, what allow to reduce the number of equations, that must be solved in order to find the searched nodal voltages. Besides, there is no need to set balancing node, and dependences between searched and known nodal voltages become linear. The calculations performed for test scheme IEEE-14 demonstrate that both algorithms provide high accuracy results, increase the simplicity and speed of calculation by separate solving of linear equations.

**Keywords:** *nodal analysis, matrix of nodal admittances, Phasor Measurement Unit (PMU), SCADA measurements, method of determining values.*

**Abstract.** Calculul regimului permanent de funcționare este cel mai utilizat instrument de analiză a rețelelor electrice moderne. Studiul actual se concentrează pe aplicarea tehnologiei de măsurări fazoriale sincronizate pentru calculul regimului. Se demonstrează, că utilizarea doar a măsurătorilor fazoriale sincronizate ca informație inițială pentru calculul regimului sau simultan cu telemetria tradițională din sistemul de control și achiziții de date (SCADA) are o serie de avantaje față de metodele clasice, ceea ce se reflectă în simplificarea procesului de calcul, reducerea timpului și memoriei de calcul, asigurarea suportului informațional mai fiabil. Această lucrare prezintă o nouă abordare privind calculul regimului permanent de funcționare, care constă în utilizarea algoritmilor accelerați și ultra-accelerați. Acești algoritmi se bazează pe implementarea metodei mărimilor determinate și a structurii speciale a matricei admitanțelor nodale, ceea ce permite reducerea numărului de ecuații, care trebuie

rezolvate pentru a găsi tensiunile nodale căutate. În plus, dispăre necesitatea de a seta nodul de echilibru, iar dependențele dintre tensiunile nodale căutate și cunoscute devin liniare. Calculele efectuate pentru schema test IEEE-14 demonstrează că ambii algoritmi oferă rezultate de înaltă acuratețe, măresc simplitatea și viteza de calcul prin soluționarea separată a ecuațiilor liniare.

**Cuvinte cheie:** *metoda tensiunilor nodali, matricea admitanțelor nodali, unitatea de măsurarea fazorilor (PMU), măsurări SCADA, metoda mărimilor determinate.*

## 1. Introduction

The Kirchhoff's and Maxwell's works gave a tremendous impulse to the development of the analysis of electrical circuits. The development of modern electrical power systems (EPS), which are characterized with a large segment of distributed generation, voltage and power control devices, characterized by multi-mode and complex configuration, is based on the analysis of electrical networks. Calculation of steady state is the most common task in the field of electrical systems analysis. The scope of calculating the steady state mode of an electrical network is to determine the electrical parameters of the state: voltages at the nodes, electric currents and power flows along the branches, power losses in the network, etc. The results of such calculations make it possible to find out:

- the feasibility of this mode of the electrical network (the possibility of transferring the required power from sources of electrical energy to consumers);
- admissible voltage limits in nodes;
- admissible ranges of change of current loadings of elements of electric networks in normal and post-emergency conditions.

In addition, the calculation of the steady state of power networks must be carried out when checking transient, post-emergency modes of network operation, when solving the problem of operational management and control, when calculating urgent and expected modes, when assessing and planning costs of power losses, and any other tasks, that may arise during operation, development and design of electrical systems.

The method of nodal analysis has become widely used in the calculations of steady-state conditions. The advantage of this method is explained by the fact, that this method makes it possible to form relatively simply and definitely a system of nodal equations in which the matrix of nodal admittances is easily corrected when changing the transformation ratios of transformer branches and when switching on / off the power line branches.

At present, a number of mathematical methods are used to calculate the steady state mode using the nodal analysis: simple iteration, Gauss-Seidel, Newton-Raphson, gradient method, modified Newton method (Newton-Kantorovich method), second-order Newton-Raphson methods (using the Hessian matrix), cybernetic modeling, dividing the EPS into subsystems, etc. Iterative methods are most widely used due to the fact that the mathematical description of the electrical network using the nodal analysis is performed using nonlinear dependencies between dependent and independent mode parameters. The essence of various numerical methods for solving nonlinear nodal equations is consistently refining the initial voltages at the nodes, while iterations continue until the specified calculation accuracy is reached. The iterative methods for calculating complex electrical systems require a lot of time and volatile memory even with the current level of development of computer technology.

The requirements set by modern researchers in the field of analysis of electrical modes are [1]:

- reducing the amount of computational work in order to obtain quickly calculation results;
- reducing the amount of volatile memory, which leads to the possibility of increasing the size of the problem being solved;
- solving the problem in any form of informational support for power network parameters.

The steady state of the electrical system is determined by the values of the parameters of its nodes. Each node is characterized by four regime parameters: active  $P$  and reactive  $Q$  power, module  $U$  and argument  $\delta$  of the complex voltage, while in practice information is known only about two of the four parameters [1].

Depending on the nature of the assignment of the initial information, the nodes can be classified as follows [1]:

1. A node of type  $U - \delta$ , for which the module and argument of the complex voltage are specified, and it is necessary to determine the active and reactive powers. As such a node, a generation node is selected, which is assigned as the base node.
2. Node of type  $P - U$ , for which the active power and the module of the complex voltage are specified, and it is necessary to determine the reactive power and the argument of the complex voltage. In practice, nodes of this type are usually only generation nodes.
3. Node of  $P - Q$  type, for which active and reactive power are set, and it is necessary to determine the module and argument of the complex voltage. Nodes of this type can be both generation and load nodes.

If it is used the standard approach to information support for calculating the steady state using the nodal analysis, then the parameters of nodes of the type  $P - U$  and  $P - Q$  can be determined from the data of traditional measurements (telemetry of the SCADA system), while parameter identification for nodes of the type  $U - \delta$  is more sophisticated problem. The feature of the nodal analysis is the mandatory determination of reference nodes. When solving problems by iterative methods, a reference node is set, this is a node in which the voltage vector is known (its modulus and phase angle which, as a rule, is equal to zero) [2].

If the voltage vector in the reference node is given, then the number of equations solved by the nodal analysis is reduced by one, and any equation can be excluded. A node whose equation is excluded from the system is called a balancing node [3, 4]. When solving a problem using Newton's method, only one reference node is specified. During iterations, the voltage vectors at the all nodes are oriented relative to the voltage vector at the reference node. Thus, from the choice of a node as a reference, balancing or reference-balancing one, the results of the calculation may change.

Classical methods for state estimation and steady state mode calculation were based on the initial data provided by SCADA [5]. Current researches in the field of solving problems of optimizing methods for calculating the steady state, as well as eliminating the disadvantages of iterative methods of nodal analysis, are focused on the application of synchronized phasor measurements technology. This technology is based on the tracking of voltage phasors in certain system nodes, correlated often with measurements of current phasors in incident branches [4]. As traditional SCADA system is built on Remote Terminal Units (RTU), so and the system of synchronized phasor measurements uses Phasor Measurement Units (PMU) as main device for informational support of the wide number of applications, one of them being state estimation and steady state calculation.

The use of synchronized phasor measurements as initial information for calculating the steady state mode independently or in conjunction with traditional telemetry has a number of advantages over classical methods, due to the simplification of the computational process, reduced time and required volatile memory, and more reliable informational support.

The author of this article showed that the methods for PMU optimal placement based on the method of determined values provide both the minimum number of nodes for installing PMU according to the criteria for complete network observability and contribute to performing steady state calculations by accelerated and super-accelerated methods [6]. Minimization of the time spent for calculating the mode is achieved with the method of determined values and the matrix-block transformation of the system of linear equations.

The informational support of the steady state calculation by the accelerated method is provided by using voltage phasors as initial information at the nodes where the PMUs are installed, as well as the scalar values of the power flows ( $P$  and  $Q$ ) received from the SCADA system.

The steady state calculation by the ultra-accelerated method is achieved by using voltage phasors as initial information at nodes where PMUs are installed, as well as current phasors in all incident branches [7]. When calculating by ultra-accelerated method, scalar values of power flows ( $P$  and  $Q$ ) obtained from the SCADA system can also be used [8].

This paper is dedicated to the novel approach in steady state calculations, based on the application of PMU measures. In this study, the algorithm and technique for calculating steady state by the accelerated method, as well as the ultra-accelerated method, will be described in detail. The main features of the steady state calculation technique based on these algorithms will be shown.

## 2. Materials and Methods

### 2. 1 Algorithm for accelerated steady state mode calculation (A1)

All accelerated methods for calculating the steady state mode of the EPS are based on dividing the column matrix  $[\underline{U}_\Sigma]$  into 2 submatrices  $[\underline{U}_p]$  and  $[\underline{U}_q]$ , respectively [9]. The first submatrix includes the determining nodal voltages, that is, independent values, and the second submatrix includes dependent variables. The submatrix  $[\underline{U}_q]$  is determined depending on  $[\underline{U}_p]$ . After separating the matrix-column, the system of nodal voltage equations that describes the EPS mode can be written as follows [6]:

$$\begin{cases} [\underline{Y}_{pp}] \cdot [\underline{U}_p] + [\underline{Y}_{pq}] \cdot [\underline{U}_q] = [\underline{J}_p]; \\ [\underline{Y}_{qp}] \cdot [\underline{U}_p] + [\underline{Y}_{qq}] \cdot [\underline{U}_q] = [\underline{J}_q]. \end{cases} \quad (1)$$

If the elements of the submatrix  $[\underline{U}_p]$  are known, then they can be considered independent variables (these are the determining nodal voltages), then the dependent variables of the submatrix  $[\underline{U}_q]$  are determined from the second equation of system Eq.(1) using the expression:

$$[\underline{Y}_{qq}] \cdot [\underline{U}_q] = [\underline{J}_q] - [\underline{Y}_{qp}] \cdot [\underline{U}_p]. \quad (2)$$

Reducing the matrix  $[\underline{Y}_{qq}]$  to a diagonal form ( $[\underline{Y}_{qq}] = [\underline{Y}_d]$ ) makes it possible to obtain easily its inverse matrix and to find the required nodal voltages using the formula:

$$[\underline{U}_q] = [\underline{Y}_d]^{-1} \left\| [\underline{J}_q] - [\underline{Y}_{qp}] \cdot [\underline{U}_p] \right\|. \quad (3)$$

It is recommended to reduce the matrix  $[\underline{Y}_{qq}]$  to a lower triangular or band form, since in this case the number of PMU devices will be minimal. Thus, on the basis of the matrix-block transformation, it is possible to solve 2 problems at once: the problem of the optimal placement of the PMU in the EPS, as well as the problem of calculating the steady state by a simplified method.

There are presented below the rules of the algorithm for the formation of the matrix of nodal admittances, in which the submatrix  $[\underline{Y}_{qq}]$  has a band structure.

The illustration of the specified algorithm is based on a dot diagram, which is a conditional form of representation of the matrix of nodal admittances. In it, non-zero elements of the matrix are replaced by dots.

The following steps must be performed to divide the nodes of the EPS into two subsets:

1. The ranks of all nodes of the system are determined, i.e. the number of incident branches for each node.

2. The node of maximum rank is excluded from the scheme, as well as all branches adjacent to it. In the case when there are several nodes with the maximum rank, the first node is selected in the order of numbering of nodes in the scheme. Moving a node of maximum rank  $q$  to a subset  $P$  leads to the maximum decomposition of chains  $g-1$  at this step.

3. After the elimination of any node, the ranks of the nodes remaining in the scheme are recalculated.

4. The presence of looped circuits in the studied scheme is checked. If so, steps 2 and 3 are repeated until the scheme has been converted to a tree or multiple trees.

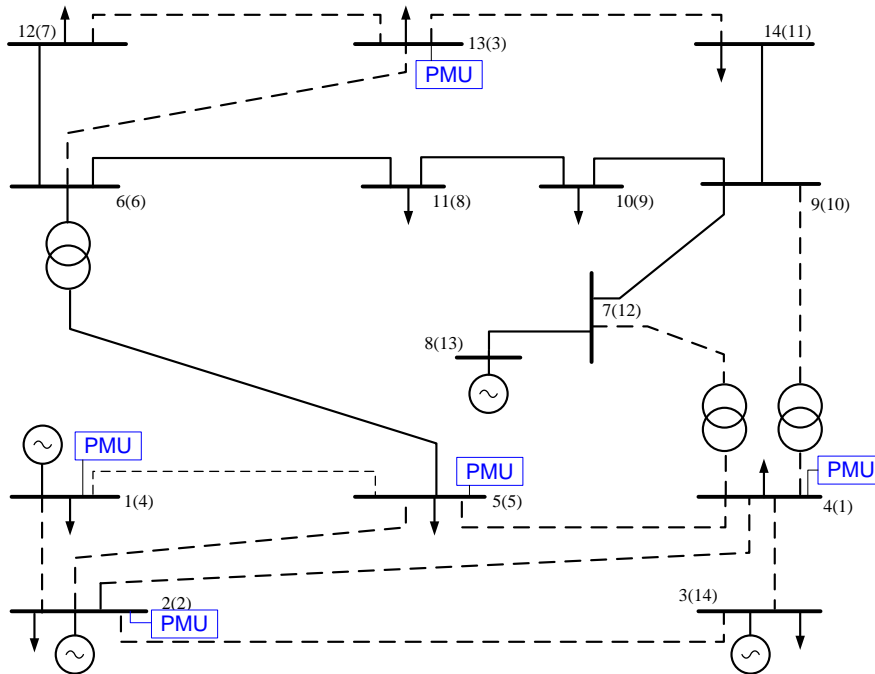
5. First, the nodes of the subset  $P$  are randomly numbered, which, in addition to the nodes selected for the installation of the PMU, will also include one node each from the obtained trees and leaf nodes. (Leaf nodes are nodes associated with a tree with only one branch).

6. Then the nodes of the subset  $Q$  are numbered, which includes the remaining nodes of the tree, as well as isolated nodes. Isolated nodes are numbered last.

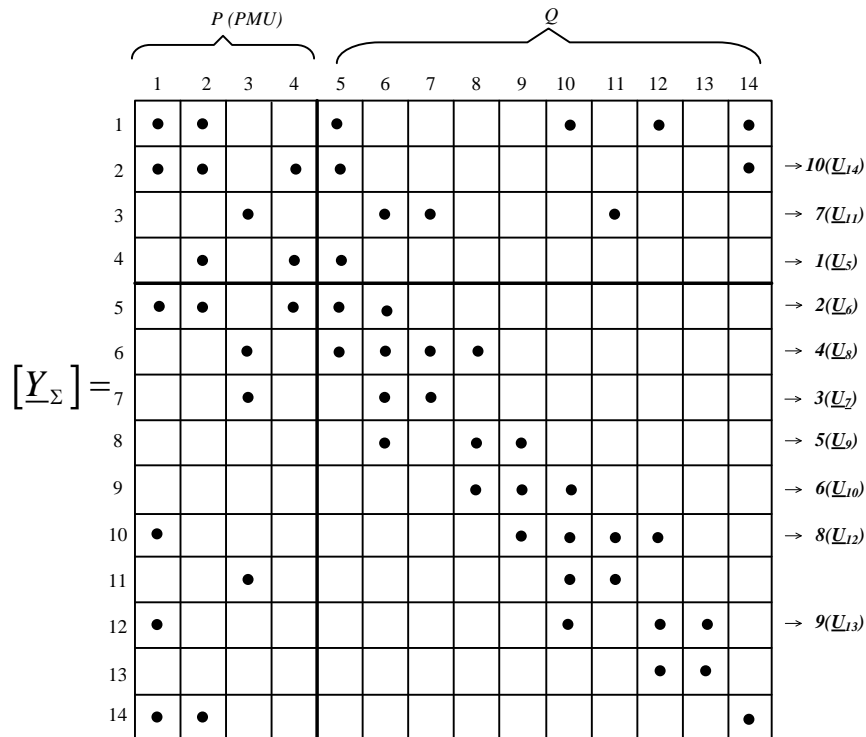
Figure 1 shows the IEEE-14 test network [5], where nodes were renumbered (numbers after transformation are indicated in brackets) according to this algorithm. Figure 2 shows dot diagram of the matrix of nodal admittances obtained by this algorithm.

To the right of the dot diagram (figure 2), the sequence for determining the voltages of dependent nodes is indicated. If the voltage vectors of these nodes are known from the PMU measurements installed at nodes 1, 2, 3 and 4, then, first of all, the voltage of node 5 can be determined by the expression:

$$\underline{U}_{5calc} = \frac{\underline{Y}_{44} \underline{U}_4 - \underline{Y}_{42} \underline{U}_2 + \frac{\underline{S}_{4,G}^*}{*}}{\underline{Y}_{45}} \cdot \underline{U}_4. \quad (4)$$



**Figure 1.** Single-line diagram of 14 nodes network, IEEE-14 using the A1 algorithm for arranging PMU in four nodes.



**Figure 2.** Matrix of nodal admittances corresponding to the IEEE-14 scheme according to Algorithm A1 with 4 PMU, in which the submatrix  $[Y_{qq}]$  is reduced to a band form.

Similarly, the voltage of node 6 is determined:

$$U_{6calc} = \frac{Y_{55}U_{5calc} - Y_{54}U_4 - Y_{52}U_2 + \frac{S_5^*}{U_{5calc}}}{Y_{56}} \tag{5}$$

The voltage of node 7 is determined from the equation for node 7 by the expression:

$$-\underline{Y}_{73}\underline{U}_3 - \underline{Y}_{76}\underline{U}_{6calc} + \underline{Y}_{77}\underline{U}_{7calc} + \frac{\underline{S}_7^*}{\underline{U}_{7calc}} = 0. \quad (6)$$

As it can be seen from Eq. (6), the voltage of node 7 is in a non-linear dependence on the voltage values of nodes 3 and 6. The voltage value of node 7 is determined iteratively using the Newton-Raphson method. In this case, it took 4 iterations to numerically determine the desired value.

Node 8 voltage is determined from the expression:

$$\underline{U}_{8calc} = \frac{\underline{Y}_{66}\underline{U}_{6calc} - \underline{Y}_{63}\underline{U}_3 - \frac{\underline{Y}_{65}}{k_{56}}\underline{U}_{5calc} + \frac{\underline{S}_6^*}{\underline{U}_{6calc}}}{\underline{Y}_{68}}. \quad (7)$$

Further, according to Eq. (8) - (13), the voltages at nodes 9, 10, 11, 12, 13 and 14 are determined.

$$\underline{U}_{9calc} = \frac{\underline{Y}_{88}\underline{U}_{8calc} - \underline{Y}_{86}\underline{U}_{6calc} + \frac{\underline{S}_8^*}{\underline{U}_{8calc}}}{\underline{Y}_{89}}. \quad (8)$$

$$\underline{U}_{10calc} = \frac{\underline{Y}_{99}\underline{U}_{9calc} - \underline{Y}_{98}\underline{U}_8 + \frac{\underline{S}_9^*}{\underline{U}_{9calc}}}{\underline{Y}_{9,10}}. \quad (9)$$

$$\underline{U}_{11calc} = \frac{\underline{Y}_{33}\underline{U}_3 - \underline{Y}_{37}\underline{U}_{7calc} - \underline{Y}_{36}\underline{U}_{6calc} + \frac{\underline{S}_3^*}{\underline{U}_3}}{\underline{Y}_{311}}. \quad (10)$$

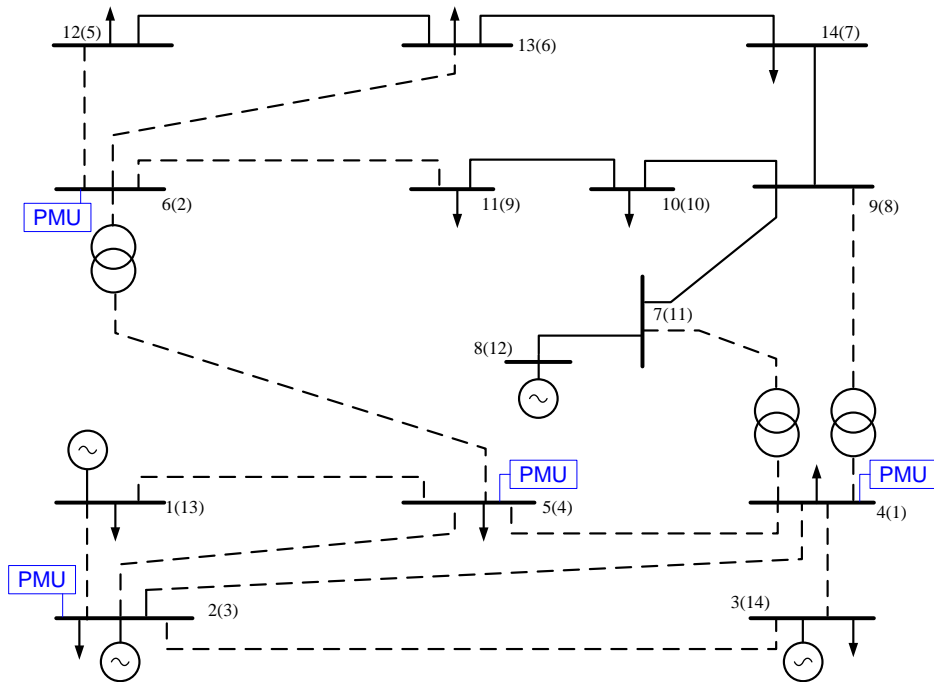
$$\underline{U}_{12calc} = \frac{\underline{Y}_{10,10}\underline{U}_{10calc} - \underline{Y}_{10,9}\underline{U}_{9calc} - \frac{\underline{Y}_{10,1}}{k_{10,1}}\underline{U}_1 - \underline{Y}_{10,11}\underline{U}_{11calc} + \frac{\underline{S}_{10}^*}{\underline{U}_{10calc}}}{\underline{Y}_{10,12}}. \quad (11)$$

$$\underline{U}_{13calc} = \frac{\underline{Y}_{12,12}\underline{U}_{12calc} - \underline{Y}_{12,10}\underline{U}_{10calc} - \frac{\underline{Y}_{12,1}}{k_{12,1}}\underline{U}_1 + \frac{\underline{S}_{12}^*}{\underline{U}_{12calc}}}{\underline{Y}_{10,12}}. \quad (12)$$

$$\underline{U}_{14calc} = \frac{\underline{Y}_{22}\underline{U}_2 - \underline{Y}_{21}\underline{U}_1 - \underline{Y}_{24}\underline{U}_4 - \underline{Y}_{25}\underline{U}_{5calc} + \frac{\underline{S}_2^*}{\underline{U}_2}}{\underline{Y}_{2,14}}. \quad (13)$$

It should be noted that if the voltage of node  $i$  is determined from the expression for the current balance of node  $j$ , then it is necessary to solve a linear equation in function of  $\underline{U}_i$ . If the voltage of node  $i$  is determined from the expression for the current balance of the same node  $i$ , then  $\underline{U}_i$  is determined by solving a non-linear equation by the iterative method, as was done for node 7.

If the number of nodes in which PMUs must be installed is increased, the values of nodal voltages can be determined from linear dependencies. So, if for the IEEE-14 test circuit, the PMU devices are located in 5 nodes, as shown in Figure 3, a matrix of nodal admittances of the band form shown in Figure 4 will be obtained.



**Figure 3.** Single-line diagram of 14 nodes network, IEEE-14 using the A1 algorithm for arranging PMU in five nodes.

		P (PMU)					Q								
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
$[Y_{\Sigma}] =$	1	•	•			•					•		•		•
	2	•	•		•	•									•
	3			•			•	•				•			
	4		•		•	•									
	5	•	•		•	•	•								
	6			•		•	•	•	•						
	7			•			•	•							
	8						•		•	•					
	9								•	•	•				
	10	•								•	•	•	•		
	11			•							•	•			
	12	•									•		•	•	
	13												•	•	
	14	•	•												•

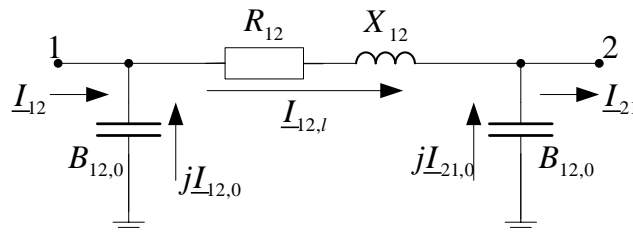
**Figure 4.** Matrix of nodal admittances corresponding to the IEEE-14 scheme according to Algorithm A1 with 5 PMU, in which the submatrix  $[Y_{-qq}]$  is reduced to a band form.



To the right of the dot diagram (Figure 4), the sequence for determining the voltages of dependent nodes is indicated. In this case, the voltages of all nodes are determined by solving linear equations.

## 2.2 Algorithm for ultra-accelerated steady state mode calculation (A2)

This algorithm is based on the joint use of both the voltage vector in the node and the current vectors of all branches connected to the node. This amount of data can be obtained from the PMU system installed in the one node. At the same time, using Ohm's law, it is possible to determine the voltages of adjacent nodes as pseudo-measurements [10]. Pseudo-measurements are determined with the help of current phasors and known branch parameters [11]. Figure 5 and Eq. (14)-(17) illustrate how the voltage vector of node 2 can be calculated with the help of given voltage phasor of node 1 and the current phasor in line 1-2.



**Figure 5.** Equivalent circuit of power lines connecting nodes 1 and 2.

The voltage drop in branch 1-2 is calculated by the expression:

$$\Delta \underline{U}_{12} = \sqrt{3} \underline{I}_{12,l} \underline{Z}_{12}. \quad (14)$$

The current phasor  $\underline{I}_{12,l}$  at the beginning of the line is determined by the expression:

$$\underline{I}_{12,l} = \underline{I}_{12} + \underline{I}_{12,0}, \quad (15)$$

where value of current vector  $\underline{I}_{12,0}$  is calculated as follows:

$$\underline{I}_{12,0} = j \frac{B_{12,0} \cdot U_1}{\sqrt{3}}. \quad (16)$$

If the voltage drop in section 1-2 is known, the voltage of node 2 can be calculated (pseudo measurement):

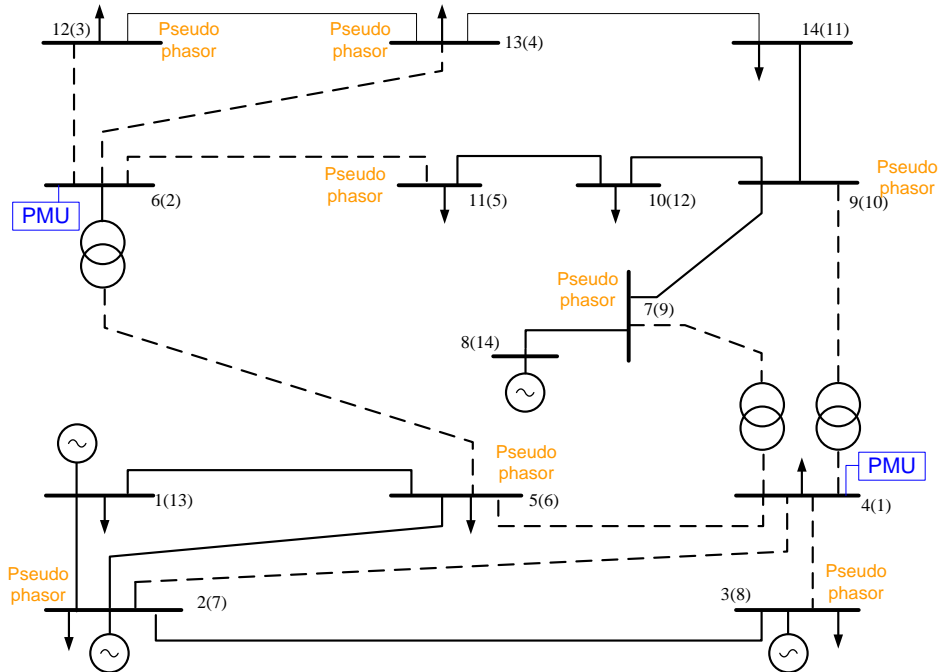
$$\underline{U}_2 = \underline{U}_1 - \Delta \underline{U}_{12}. \quad (17)$$

The essence of the algorithm for ultra-accelerated calculation of the steady state is to form a matrix of nodal admittances, in which nodes will be divided into three subsets: a subset of  $P$  nodes in which PMUs are installed, a subset of  $PP$  nodes in which pseudo-measurements are calculated, and also a subset of  $Q$ , containing all other nodes of the circuit. The following steps must be performed for dividing the nodes of the power system into three subsets:

1. The ranks ( $R$ ) of all nodes of the system are determined, i.e. the number of incident branches for each node.
2. The node of maximum rank, as well as all branches adjacent to it, are excluded from the scheme. In the case when there are several nodes with the maximum rank, the first node is selected in the order of numbering of nodes in the scheme.
3. After eliminating any node, the ranks of the remaining nodes in the scheme are recalculated. Nodes whose ranks have decreased are listed as pseudo-PMUs (a subset of  $PP$ ).
4. Steps 2 and 3 are repeated until all nodes with rank  $R \geq 3$  transform into nodes, that provide PMUs or pseudo-PMUs measurements.

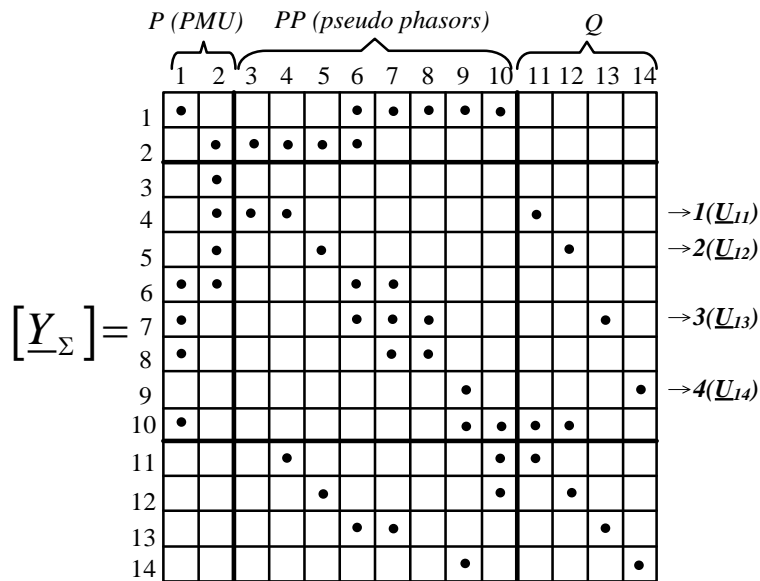
5. If the rank of a node is equal to 1 and 2, then the node is included in the subset  $Q$ , and the complex value of the voltage in this node, as well as the complex values of the currents of the incident branches, will be determined linearly through the voltage drops in the branches and the current balance equations.

6. The nodes of each subset are numbered in direct but random order.



**Figure 6.** Single-line diagram of 14 nodes network, IEEE-14 using the A2 algorithm for arranging PMU in two nodes and having pseudo-measurements in 8 nodes.

Figure 6 shows the IEEE-14 test circuit, for which nodes were renumbered (numbers after transformation are indicated in brackets) according to the algorithm A2. Figure 7 shows a dot diagram of the matrix of nodal admittances obtained by this algorithm.



**Figure 7.** Matrix of nodal admittances corresponding to the IEEE-14 scheme according to Algorithm A2.

The calculation of the steady state according to the A2 algorithm contains two stages. At the first stage, using the above formulas, the nodal voltages of the nodes adjacent to the PMU installation site are calculated (pseudo-measurements), and the second stage, the voltage phasors in the dependent nodes are determined from the current balance equations with the help of the PMU data and pseudo-measurements. At the same time, it is important to satisfy condition described for the Algorithm A1, the voltage phasor for node  $i$  is determined from the expression for the current balance of node  $j$ . Then it will be necessary to solve linear equation in function of searched value  $\underline{U}_j$ .

The progress of the calculation according to the A2 algorithm can be described as follows:

1. The PMU installed in node 1 measures and provide the following information:  $\underline{U}_1^{PMU}$ ,  $\underline{I}_{1-6}^{PMU}$ ,  $\underline{I}_{1-7}^{PMU}$ ,  $\underline{I}_{1-8}^{PMU}$ ,  $\underline{I}_{1-9}^{PMU}$  и  $\underline{I}_{1-10}^{PMU}$ . Then the voltage drops in branches adjacent to node 1 are determined:  $\Delta\underline{U}_{1-6}$ ,  $\Delta\underline{U}_{1-7}$ ,  $\Delta\underline{U}_{1-8}$ ,  $\Delta\underline{U}_{1-9}$  and  $\Delta\underline{U}_{1-10}$ .
2. The PMU installed in node 2 measures and provide the following information:  $\underline{U}_2^{PMU}$ ,  $\underline{I}_{2-3}^{PMU}$ ,  $\underline{I}_{2-4}^{PMU}$ ,  $\underline{I}_{2-5}^{PMU}$ ,  $\underline{I}_{2-6}^{PMU}$ . Then the voltage drops in branches adjacent to node 2 are determined:  $\Delta\underline{U}_{2-3}$ ,  $\Delta\underline{U}_{2-4}$ ,  $\Delta\underline{U}_{2-5}$ ,  $\Delta\underline{U}_{2-6}$ .
3. There are calculated the pseudo-measurements of voltage phasors in nodes, 4, 5, 6, 7, 8, 9 and 10:  $\underline{U}_3^{pP}$ ,  $\underline{U}_4^{pP}$ ,  $\underline{U}_5^{pP}$ ,  $\underline{U}_6^{pP}$ ,  $\underline{U}_7^{pP}$ ,  $\underline{U}_8^{pP}$ ,  $\underline{U}_9^{pP}$ ,  $\underline{U}_{10}^{pP}$ .
4. After calculation of the phasors at nodes 1-10, knowing traditional SCADA power flow measurements at nodes 4, 5, 7 and 9, the voltage vectors for dependent nodes 11, 12, 13, 14 are determined.

Thus, there is no need to solve a system of equations with size 14x14 within the steady state mode calculation. Within joint application of PMU data, pseudo-measurements and traditional SCADA measurements, it is necessary to solve only four linear equations similar to Eq. (7)-(13).

### 3. Results and discussion

The placement of PMU in nodes of EPS makes the system model observable and as a result ensures the linear dependence between unknown and measured node voltages. Although, in [12] there are indicated sites, where PMU must be installed obligatory (intersystem lines, generation nodes, plants of frequency control, etc.), it is unfeasible to install PMU in each node of the network. Thus, many studies deal with the searching of the minimal number of PMU that should be placed in EPS' nodes to ensure the complete observability. As it was noted in [13] and developed in [8] and [14], the optimal placement of PMU implies the existence of at least one spanning tree in each network, which connects all the nodes trough branches with measured or calculated phasors. This principle underlies the rules of the accelerated algorithm for the formation the matrix of nodal admittances, which are specified in this paper.

In [15] there is clearly described main data necessary for steady state mode calculation: topology data, data about parameters of the branches and nodes. All these data are used within the process of accelerated calculation of steady state mode.

There is noted in [16] the feasibility of partition of nodal admittances matrix in some regions (submatrices). According to described here algorithms this partition and matrix transformation bring to the possibility to reduce the calculation time. Both proposed algorithms A1 and A2 are based on transformation of nodal admittance matrix. But essential difference in comparison with [13] is that there are developed specific rules for implementation and the nodes in the graph of EPS are renumbered. These algorithms can be applied without considering the presence of reference or reference-balancing nodes.

For the IEEE-14 test circuit, the steady state calculation was performed using the A1 and A2 algorithms proposed in this paper. The similar calculation of the mode was performed for the IEEE-14 model in the specialized program RastrWIN in order to demonstrate the correctness of the calculations and compare the results. Table 1 presents the summary results of steady state calculations for the IEEE-14 scheme.

Table 1

**Results of the calculation of the steady state by the accelerated and ultra-accelerated methods and performed in RastrWIN software**

Node	RastrWIN	Algorithm A1		Algorithm A2
		4 PMU	5 PMU	
1	$243.8 \cdot e^{j0^\circ}$	$243.8 \cdot e^{j0^\circ}$	$243.8 \cdot e^{j0^\circ}$	$243.262 \cdot e^{j0.6^\circ}$
2	$230 \cdot e^{-j4.433^\circ}$	$230 \cdot e^{-j4.433^\circ}$	$230 \cdot e^{-j4.433^\circ}$	$230.075 \cdot e^{-j4.43^\circ}$
3	$224.521 \cdot e^{-j13.095^\circ}$	$224.523 \cdot e^{-j13.098^\circ}$	$224.523 \cdot e^{-j13.098^\circ}$	$224.937 \cdot e^{-j12.99^\circ}$
4	$223.202 \cdot e^{-j10.234^\circ}$	$223.202 \cdot e^{-j10.234^\circ}$	$223.202 \cdot e^{-j10.234^\circ}$	$223.202 \cdot e^{-j10.234^\circ}$
5	$224.409 \cdot e^{-j8.609^\circ}$	$224.409 \cdot e^{-j8.608^\circ}$	$224.409 \cdot e^{-j8.609^\circ}$	$224.409 \cdot e^{-j8.61^\circ}$
6	$115 \cdot e^{-j14.681^\circ}$	$114.998 \cdot e^{-j14.681^\circ}$	$115 \cdot e^{-j14.681^\circ}$	$115 \cdot e^{-j14.681^\circ}$
7	$113.94 \cdot e^{-j13.666^\circ}$	$113.928 \cdot e^{-j13.681^\circ}$	$113.942 \cdot e^{-j13.669^\circ}$	$113.929 \cdot e^{-j13.66^\circ}$
8	$115 \cdot e^{-j13.666^\circ}$	$114.975 \cdot e^{-j13.706^\circ}$	$115.006 \cdot e^{-j13.675^\circ}$	$115.03 \cdot e^{-j11.55^\circ}$
9	$113.353 \cdot e^{-j15.483^\circ}$	$113.344 \cdot e^{-j15.49^\circ}$	$113.353 \cdot e^{-j15.483^\circ}$	$113.353 \cdot e^{-j15.48^\circ}$
10	$112.728 \cdot e^{-j15.669^\circ}$	$112.72 \cdot e^{-j15.675^\circ}$	$112.728 \cdot e^{-j15.669^\circ}$	$112.746 \cdot e^{-j15.65^\circ}$
11	$113.423 \cdot e^{-j15.326^\circ}$	$113.418 \cdot e^{-j15.329^\circ}$	$113.423 \cdot e^{-j15.326^\circ}$	$113.423 \cdot e^{-j15.32^\circ}$
12	$113.176 \cdot e^{-j15.659^\circ}$	$113.174 \cdot e^{-j15.659^\circ}$	$113.176 \cdot e^{-j15.659^\circ}$	$113.176 \cdot e^{-j15.66^\circ}$
13	$112.588 \cdot e^{-j15.751^\circ}$	$112.588 \cdot e^{-j15.751^\circ}$	$112.588 \cdot e^{-j15.751^\circ}$	$112.588 \cdot e^{-j15.75^\circ}$
14	$110.794 \cdot e^{-j16.752^\circ}$	$110.798 \cdot e^{-j16.749^\circ}$	$110.794 \cdot e^{-j16.752^\circ}$	$110.794 \cdot e^{-j16.75^\circ}$

Taking into account the obtained results, it can be concluded that the proposed algorithms don't concede the results obtained by the iterative Newton method in the RastrWIN software package in terms of the accuracy of calculating the steady state mode. It is worth mentioning that the results in the RastrWIN were performed after 4 iterations.

**4. Conclusions**

In the context of using the technology of synchronized vector measurements in conjunction with traditional SCADA measurements, new algorithms for calculating the steady state mode were proposed. These algorithms have a number of advantages over the widely used iterative calculation methods:

- there is no need to select a balancing node (although it must be performed in iterative methods),

- minimizes or completely eliminates the necessity of solving nonlinear equations,
- in the process of selecting nodes for PMU placement, a matrix of nodal admittances is formed, which is easy to correct in case of a change in the transformation ratio of transformer branches or switching lines,
- minimizing the amount of time and the amount of volatile memory required to perform calculations using these specified algorithms.

The acceleration of the calculation process is achieved by reducing the number of calculations, since the values of the voltage phasors are known in advance for the nodes in which PMUs are installed. For other nodes, for which only information about traditional measurements is available, voltages are calculated using the current balance equations (algorithm A1), or partially according to Ohm's law and current balance equations (algorithm A2). Calculations are mainly performed by solving linear equations. In order to obtain a linear relationship between the known and searched voltages, it is necessary to calculate the unknown voltage according to the balance equation, compiled not for one's own, but for an adjacent node.

A significant reduction in the number of calculations is achieved by using the method of determining values and matrix-block transformation of the system of linear equations, according to which the matrix of nodal admittances includes a submatrix of a band or lower triangular structure. This in turn ensures that there is a minimum set of nodes where a PMU needs to be installed, and also provides accelerated or ultra-accelerated way to calculate steady state mode by reducing the number of required variables, whose dependence on known parameters is predominantly linear.

The proposed algorithms for accelerated and ultra-accelerated steady state mode calculation based on the joint use of PMU and SCADA measurements might become a convenient tool for analyzing the operating modes of modern EPS.

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