THE DISCREET REPRESENTATION OF REAL MATERIAL IN STRUCTURAL MODEL

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INTRODUCTION

The governing equations at macroscopic level have more modifications. As a result, the analysis of behavior of structural model with infinite number of subelements is complicated. The analysis of behavior of material in function of exterior history is simplified in the one discreet model.

1. CALCULUS OF ELASTICITY LIMITS OF SUBELEMENTS

Let us examen the calculation scheme of subelements characteristics. The deviator modules of stress and strain tensor in *k* number subelement we note through $\bar{\sigma}_k, \bar{\varepsilon}_k$. The interaction among subelements is defines through relationship

$$\bar{\sigma}_i - \sigma = A(\varepsilon - \bar{\varepsilon}_i), \quad i = 1,2,3,...,N \tag{1}$$

During analysis it is easy to replace stress through elastic strain relative to the elastic limit of the first subelement. These elastic strains will note through $\bar{e}_i$. The report among total strain at elasticity limiter of this subelement will note though $\bar{\varepsilon}_i$.

In this case relationship (1) can be written under shape:

$$\bar{e}_i - e = b(\varepsilon - \bar{\varepsilon}_i), \quad i = 1,2,3,...,N \tag{2}$$

Strain diagram for this subelement is presented in figure 1.

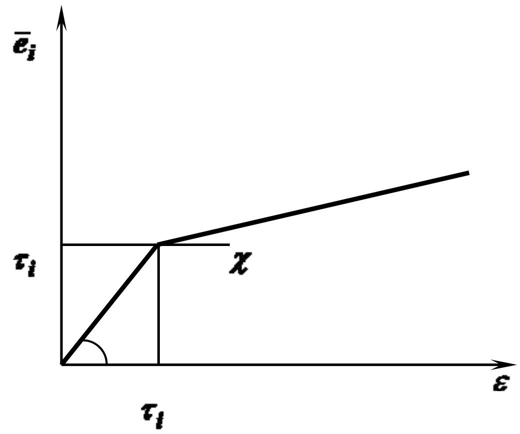

Figure 1. Strain diagram for subelement.

In scleronome processes each subelement will be characterized though flow limit τ_i and its share in system - ψ_i. The relationships for respective characteristics at macroscopic level are presented as:

$$e = \sum_{i=1}^{N} \bar{e}_i \psi_i, \quad \varepsilon = \sum_{i=1}^{N} \bar{\varepsilon}_i \psi_i, \tag{3}$$

where

$$\sum_{i=1}^{N} \psi_i = 1. \tag{4}$$

The characteristic diagrams for different subelements are presented in figure 2.

Further we establish the calculus relationships for elasticity limits and shares of subelements. In this purpose relation (1) will be written as:

$$\bar{e}_i + b\bar{\varepsilon}_i = e + b\varepsilon . \qquad (5)$$

The relationships among $\bar{e}_i$ and $\bar{\varepsilon}_i$ we establish in base of diagram which is presented in figure 3.

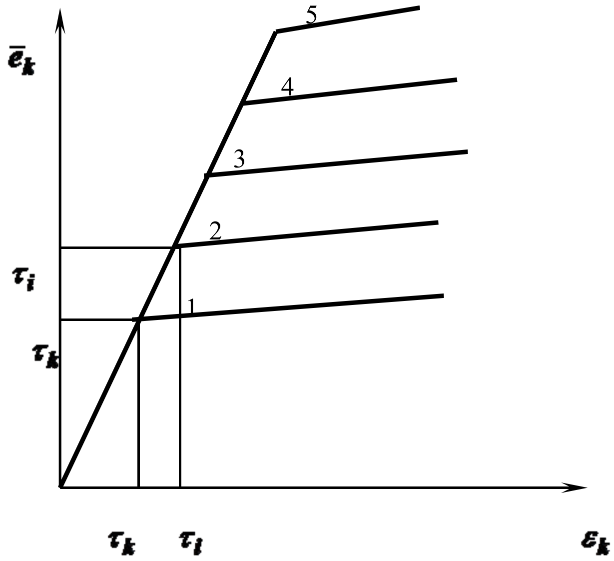

Figure 2. The characteristic diagrams for different subelements.

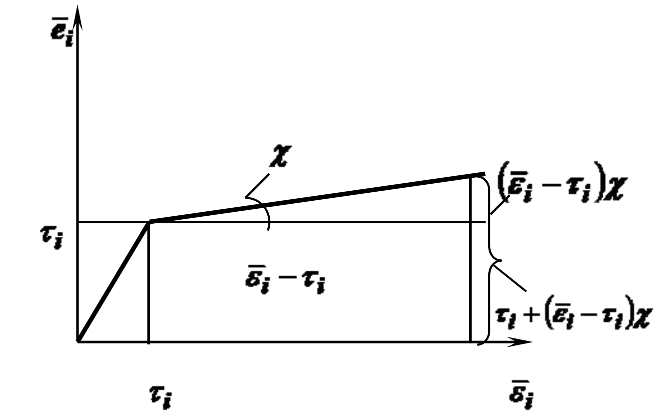

Figure 3. The dependence among stress and strain tensor deviator.

For $\bar{\varepsilon}_i$ and $\bar{e}_i$ the following relationships are obtained

$$\bar{\varepsilon}_i = \frac{e + b\varepsilon - (1-\chi)\tau_i}{\chi + b}, \qquad (6)$$

$$\bar{e}_i = (1-\chi)\tau_i + \frac{\chi}{\chi + b}\left(e + b\varepsilon - (1-\chi)\tau_i\right),$$

$$\bar{e}_i = (1-\chi)\tau_i - \frac{\chi(1-\chi)}{\chi + b}\tau_i + \frac{\chi}{\chi + b}e + \frac{\chi}{\chi + b}b\varepsilon$$

$$\bar{e}_i = \left(\frac{\tau_i b(1-\chi)}{(\chi + b)} + \frac{\chi}{\chi + b}(e + b\varepsilon)\right),$$

$$\bar{e}_i = \left(\frac{\tau_i b(1-\chi) + \chi(e + b\varepsilon)}{(\chi + b)}\right). \qquad (7)$$

In figure 4 the characteristic diagram at conglomerate level is presented (at macroscopically level). In „*i*" point the elasticity limite is reachen in subelement with number *i*.

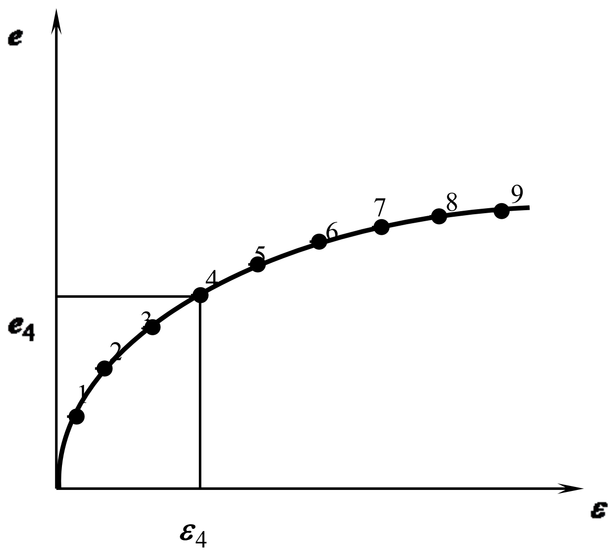

Figure 4.

The subelements with $i < k$ number are required in irreversible domain, but $i \geq k$ number in reversible domain. From (5) relationship results that elasticity limit in *k* subelement obtained values

$$\tau_i = \frac{e_k + b\varepsilon_k}{1+b}. \quad (8)$$

2. DETERMINATION OF ELEMNTS SHERES IN BASE OF CHARACTERISTIC DIAGRAM OF MATERIAL

The elastic macroscopic strain will be equal with sum of elastic strains of all subelements

$$e_k = \sum_{i=1}^{n} \bar{e}_i \psi_i = \sum_{i=1}^{k-1} \bar{e}_i \psi_i + \sum_{i=k}^{n} \bar{e}_i \psi_i . \quad (9)$$

In the subelements which continuing to work in elastic domain strains are equal. The (7) relationship can be written as

$$\bar{e}_k = \frac{b\varepsilon_k + e_k}{1+b}, \Rightarrow b\varepsilon_k + e_k = \tau_k(1+b). \quad (10)$$

Substituting (10), (7) in (9), we obtain

$$e_k = \sum_{i=1}^{k-1} \frac{\tau_i b(1-\chi)}{(\chi+b)} \psi_i + \frac{1+b}{\chi+b} \chi \tau_k \sum_{i=1}^{k-1} \psi_i + \left(1 - \sum_{i=1}^{k-1} \psi_i\right) \tau_k \quad (11)$$

or

$$e_k - \tau_k = \sum_{i=1}^{k-1} \frac{\tau_i b(1-\chi)}{(\chi+b)} \psi_i - \tau_k \sum_{i=1}^{k-1} \psi_i \left(1 - \frac{\chi + b\chi}{\chi+b}\right) \quad (12)$$

The (12) relationship can be presented as

$$\frac{(\chi+b)(e_k - \tau_k)}{(1-\chi)b} = \sum_{i=1}^{k-1} \psi_i \tau_i - \tau_k \sum_{i=1}^{k-1} \psi_i =$$

$$= \sum_{i=1}^{k-1} (\tau_i - \tau_k) \psi_i . \quad (13)$$

We will replace in (13) relationship *k* though *k+1,* as a result we will obtain

$$\frac{(\chi+b)(e_{k+1} - \tau_{k+1})}{(1-\chi)b} = \sum_{i=1}^{k} (\tau_i - \tau_{k+1}) \psi_i =$$

$$= \sum_{i=1}^{k-1} (\tau_i - \tau_{k+1}) \psi_i + (\tau_k - \tau_{k+1}) \psi_k , \quad (14)$$

Subtract form (14) (13)

$$\frac{(\chi+b)}{(1-\chi)b} (e_{k+1} - e_k - \tau_{k+1} - \tau_k) =$$

$$= \sum_{i=1}^{k-1} (\tau_i - \tau_{i+1}) \psi_i + (\tau_k - \tau_{k+1}) \psi_k - \sum_{i=1}^{k-1} (\tau_i - \tau_k) \psi_i$$

$$\frac{(\chi+b)}{(1-\chi)b} (e_{k+1} - e_k - \tau_{k+1} - \tau_k) = (\tau_k - \tau_{k+1}) \sum_{i=1}^{k} \psi_i$$

$$\frac{(\chi+b)}{(1-\chi)b} \frac{(e_{k+1} - e_k - \tau_{k+1} - \tau_k)}{(\tau_k - \tau_{k+1})} = \sum_{i=1}^{k} \psi_i \quad (15)$$

From (8) we obtain

$$\tau_{k+1} - \tau_k = \frac{e_{k+1} + b\varepsilon_{k+1}}{1+b} - \frac{e_k + b\varepsilon_k}{1+b}$$

$$\tau_{k+1} - \tau_k = \frac{e_{k+1} - e_k + b(\varepsilon_{k+1} - \varepsilon_k)}{1+b} \quad (16)$$

Replace (16) in (15), we obtain

$$\frac{(\chi+b)}{(1-\chi)b} \left(1 - \frac{(e_{k+1} - e_k)(1+b)}{(e_{k+1} - e_k + b(\varepsilon_{k+1} - \varepsilon_k))}\right) = \sum_{i=1}^{k} \psi_i \quad (17)$$

After one number of transformations we find

$$\frac{(\chi+b)}{(1-\chi)} \left(\frac{(\varepsilon_{k+1} - \varepsilon_k - e_{k+1} + e_k)}{(e_{k+1} - e_k + b(\varepsilon_{k+1} - \varepsilon_k))}\right) = \sum_{i=1}^{k} \psi_i \quad (18)$$

Replace in (18) *k* through *k-1*, we obtain

$$\frac{(\chi+b)}{(1-\chi)}\left(\frac{(\varepsilon_k-\varepsilon_{k-1}-e_k+e_{k-1})}{(e_k-e_{k-1}+b(\varepsilon_k-\varepsilon_{k-1}))}\right)=\sum_{i=1}^{k-1}\psi_i \quad (19)$$

Subtract from (18) (19), we obtain

$$\psi_k=\frac{(\chi+b)}{(1-\chi)}\left(\frac{(\varepsilon_{k+1}-\varepsilon_k-e_{k+1}+e_k)}{(e_{k+1}-e_k+b(\varepsilon_{k+1}-\varepsilon_k))}-\right.$$
$$\left.-\frac{(\varepsilon_k-\varepsilon_{k-1}-e_k+e_{k-1})}{(e_k-e_{k-1}+b(\varepsilon_k-\varepsilon_{k-1}))}\right) \quad (20)$$

$$\tau_k=\frac{b\varepsilon_k+e_k}{1+b}. \quad (21)$$

Using the relationships (20) şi (21) we can calculate the subelements characteristics in base of characteristic diagram of material at macroscopic level.

3. CONCLUSIONS

In base of (20), (21) relations the limits of elasticity and shares of subelements are determined in base of characteristic diagram of material. In established relationships for elements characteristics one single intern parameter appearing which characterizes the cinematic interactions among subelements. The method of calculation of this parameter will be specified in base of discordance principle.

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