Synthesis of the Control Algorithm for Third-Order Object Models with Additional Zero and Time Delay

Sinteza algoritmului de reglare la modele de obiecte de ordinul trei cu zerou suplimentar și timp mort

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Drd. Dumitru MORARU,
Școala doctorală a Universității Tehnice a Moldovei,
Specialitatea: 122.03 – Modelare, metode matematice, produse program
https://orcid.org/0000-0001-8957-6046

Conducător științific:
Conf. univ. dr. Bartolomeu IZVOREANU,
Universitatea Tehnică a Moldovei
https://orcid.org/0000-0001-9886-5878

ABSTRACT

In this paper, the tuning procedure for the standardized PID controller for third-order object models with additional zero and time delay using the maximum stability degree method with iteration is analyzed. Analytical calculation expressions for the tuning parameters of the controller were obtained as nonlinear functions of the argument - the degree of system stability, and linear functions of the known parameters of the object model. By varying the degree of stability, these functions are graphically constructed, after which, for the same argument value, sets of controller tuning parameter values are chosen, and the performances of the automatic control system for the control object are verified. The proposed method was compared with other known methods and tested for tuning the PID controller for the temperature control in the extruder of a 3D printer. The advantages of the elaborated procedure were highlighted.

Keywords: *automatic control system; PID controller; mathematical model; control algorithm; tuning method; maximal stability degree method with iterations.*

REZUMAT

În lucrare se analizează procedura de acordare a regulatorului tipizat PID la modele de obiecte de ordinul trei cu zerou suplimentar și timp mort după metoda gradului maximal de stabilitate cu iterații. S-au obținut expresiile analitice ale parametrilor de acord ai regulatorului ca funcții neliniare de argumentul − gradul de stabilitate și liniare de parametrii cunoscuți ai modelului obiectului. Variind gradul

de stabilitate, aceste funcții se construiesc în formă grafică după care, pentru aceeași valoare a argumentului, se aleg seturi de valori ale parametrilor regulatorului și se verifică performanțele sistemului automat pentru obiectul real. Metoda propusă a fost comparată cu alte metode cunoscute și a fost testată pentru acordarea regulatorului PID al sistemului de reglare a temperaturii în extruderul imprimantei 3D. Au fost evidențiate avantajele procedurii elaborate.

Cuvinte-cheie: *sistem automat; regulator PID; model matematic; algoritm de reglare; metodă de acordare; metoda gradului maximal de stabilitate cu iterații.*

Introduction

The issue of synthesizing the automatic controller, which involves the design or selection of a control algorithm and determining its optimal parameters is one of the fundamental problems in automatic control. Obtaining the optimal parameters of the controller requires knowledge of the mathematical model of the process, so that the control objective can be achieved and maintained, and the performance requirements imposed on the automatic system are met as: system stability, sensitivity, static and dynamic properties, insensitivity, and system robustness.

In this paper, the mathematical model of the third-order control object with additional zero and time delay is used [3, 4]:

$$H(s) = \frac{e^{-\tau s} k(T_1 s+1)}{(T_2 s+1)(T_3 s+1)(T_4 s+1)} = \frac{e^{-\tau s}(b_0 s+b_1)}{a_0 s^3 + a_1 s^2 + a_2 s + a_3}, \quad (1)$$

where kk is the transfer coefficient, T_1, T_2, T_3 and T_4 are the time constants of the process, τ – time delay and $b_0 = kT_1,\ b_1 = k,\ a_0 = T_2T_3T_4, a_1 = T_2T_3 + T_2T_4 + T_3T_4,$ $a_2 = T_2 + +T_3 + T_4$ and $a_3 = 1$ are the generic coefficients.

The algorithm for tuning the PID controller refers to the method by which the parameters P (proportional), I (integral), and D (derivative) are set to ensure optimal control system performance. The controller tuning methods can be realized through experimental or analytical methods, depending on the system characteristics and application requirements. There are several methods for tuning the PID controller to the object model: Ziegler-Nichols, frequency response-based, polynomial method, etc. [1-7, 11].

The Ziegler-Nichols (ZN) method involves: setting the I and D tuning parameters of the controller to zero, and the P tuning parameter is increased until the system response exhibits sustained oscillations. The critical value of the P tuning parameter (k_c) and the period of the oscillations (T_c) are noted. These values are used to calculate the P, I, and D tuning parameters of the controller [11].

The polynomial method is also an analytical approach that leads to solving the control algorithm synthesis problem. However, it can be challenging to determine the characteristic equation of the designed system [8, 9].

In this paper, the tuning algorithm was developed using the maximum stability degree method with iterations (MSDI) [8, 9] to tune the PID controller to the control object model (1).

An example of tuning the PID controller to the object model (1) is studied, and the object parameters are varied from their nominal values, and the system's performances are analyzed.

Materials and methods

The close loop system structure with time delay in Figure 1 consists of the fixed part (FP) described by the transfer function $H_{FP}(s)$, composed of dead time $e^{-\tau s}$, the transfer function $H(s)$ of the control object and the controller described by the transfer function $H_C(s)$. The input signal $x(t) = 1(t)$ is considered a unit step, $\varepsilon(t) = x(t) - y(t)$ is the system error, $u(t)$ is the command signal, $y(t)$ is the system output, and $d(t)$ is the system disturbance.

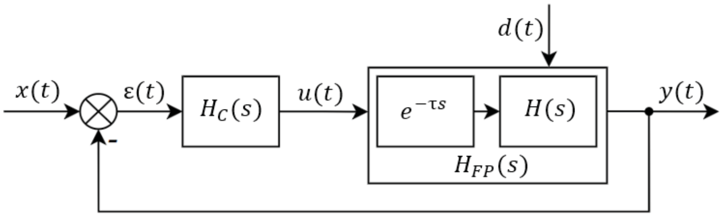


Figure 1. ***The close loop system structure***

Source: *Developed by the author*

The standard PID algorithm in parallel connection is described by the following transfer function:

$$H_{PID}(s) = k_p + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s}, \qquad (2)$$

where k_p, k_i, and k_d are the tuning parameters of the PID controller.

The transfer function of the closed-loop system with the PID controller is represented in the following form:

$$H_0(s) = \frac{H_d(s)}{1+H_d(s)} = \frac{(k_d s^2 + k_i + k_p s)e^{-\tau s}(b_0 s + b_1)}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + (k_d s^2 + k_i + k_p s)e^{-\tau s}(b_0 s + b_1)}, \qquad (3)$$

where $H_d(s) = H_{PID}(s) H_{FP}(s)$ is open-loop transfer function of system.

The characteristic equation $D(s)$ of the automatic control system (3) is:

$$D(s) = a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + (k_d s^2 + k_i + k_p s)e^{-\tau s}(b_0 s + b_1) = 0 \qquad (4)$$

The concept of the stability degree J is introduced and substituted $s = -J$ according to the maximum degree of stability algorithm, and after some transformations, the following expression is obtained:

$$D(-J) = \frac{e^{\tau J}(a_0 J^4 - a_1 J^3 + a_2 J^2 - a_3 J)}{b_1 - b_0 J} + k_d J^2 - k_p J + k_i = 0. \qquad (5)$$

By differentiating expression (5) twice with respect to the variable J, the calculation expressions for the PID controller parameters are obtained:

$$k_p = \frac{e^{-\tau J}(d_0 J^5 - d_1 J^4 + d_2 J^3 - d_3 J^2 + d_4 J - d_5)}{(-b_0 J + b_1)^2} + 2k_d J = f_p(J), \qquad (6)$$

where $d_0 = a_0 b_0 \tau$, $d_1 = 3a_0 b_0 + a_0 b_1 \tau + a_1 b_0 \tau$, $d_2 = 4a_0 b_1 + 2a_1 b_0 + a_1 b_1 \tau + a_2 b_0 \tau$, $d_3 = 3a_1 b_1 + a_2 b_0 + a_2 b_1 \tau + a_3 b_0 \tau$, $d_4 = 2a_2 b_1 + a_3 b_1 \tau$, $d_5 = a_3 b_1$.

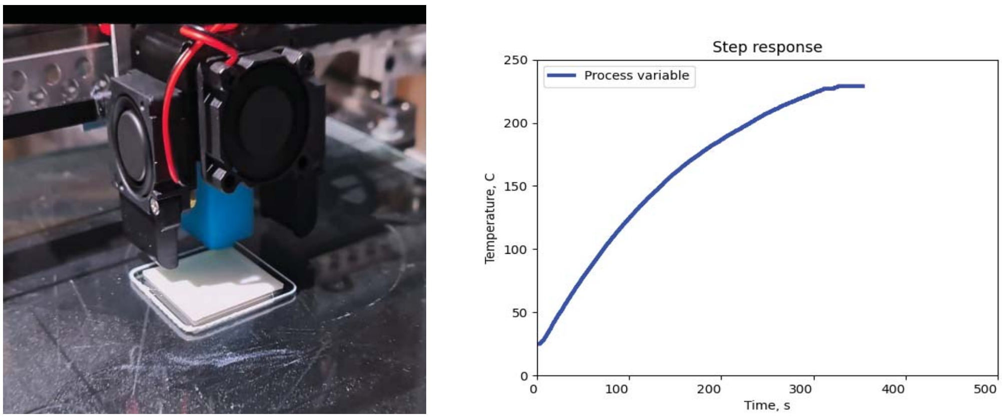
$$k_i = \frac{e^{-\tau J}(-a_0 J^4 + a_1 J^3 - a_2 J^2 + a_3 J)}{b_1 - b_0 J} - k_d J^2 + k_p J = f_i(J), \qquad (7)$$

$$k_d = \frac{e^{-\tau J}(-d_0 J^6 + d_1 J^5 - d_2 J^4 + d_3 J^3 - d_4 J^2 + d_5 J - d_6)}{2(-b_0 J + b_1)^4} = f_d(J), \qquad (8)$$

$$d_0 = a_0 b_0^2 \tau^2,$$
$$d_1 = 6a_0 b_0^2 \tau + 2a_0 b_0 b_1 \tau^2 + a_1 b_0^2 \tau^2,$$
$$d_2 = 6a_0 b_0^2 + 14a_0 b_0 b_1 \tau + a_0 b_1^2 \tau^2 + 4a_1 b_0^2 \tau + 2a_1 b_0 b_1 \tau^2 + a_2 b_0^2 \tau^2,$$
$$d_3 = 16a_0 b_0 b_1 + 8a_0 b_1^2 \tau + 2a_1 b_0^2 + 10a_1 b_0 b_1 \tau + a_1 b_1^2 \tau^2 + 2a_2 b_0^2 \tau + 2a_2 b_0 b_1 \tau^2 + a_3 b_0^2 \tau^2,$$
$$d_4 = 12a_0 b_1^2 + 6a_1 b_0 b_1 + 6a_1 b_1^2 \tau + 6a_2 b_0 b_1 \tau + a_2 b_1^2 \tau^2 + 2a_3 b_0 b_1 \tau^2,$$
$$d_5 = 6a_1 b_1^2 + 4a_2 b_1^2 \tau + 2a_3 b_0 b_1 \tau + a_3 b_1^2 \tau^2,\ d_6 = 2a_2 b_1^2 - 2a_3 b_0 b_1 + 2a_3 b_1^2 \tau.$$

Results and discussion

Subsequently, the results obtained from tuning the PID algorithm for the extruder of a 3D printer are presented. The external appearance of the extruder is shown in Figure 2, *a*. Before performing the controller calculation, it is necessary to identify its mathematical model. For this purpose, the experimental curve of the extruder heating process was recorded, as shown in Figure 2, *b*.



a) *b*)

Figure 2. *Extruder and experimental curve of heating process*

Source: *Developed by the author*

The mathematical model (1) of the control object is identified using the System Identification Toolbox from MATLAB software which is represented by the following transfer function:

$$H_{FP}(s) = \frac{e^{-s} 0.72653(163.4092s+1)}{(634.1044s+1)(61.5524s+1)(0.50895s+1)} = \frac{e^{-s}(118.72s+0.72)}{19864.64s^3+39384.7s^2+696.16s+1}. \quad (9)$$

For the calculation of the PID controller parameters, expressions (6)-(8) were used, after substituting the generic coefficients, the expressions take the form:

$$k_p = \frac{e^{-J}(167.205J^5 - 834.15J^4 + 675J^3 - 11.99J^2 + 0.071J - 5.1*10^{-5})}{(-118.72J+0.72)^2} + 2k_d J = f_p(J), \quad (10)$$

$$k_i = \frac{e^{-J}(19864.64J^4 - 39384.7J^3 + 696.16J^2 - J)}{0.72 - 118.72J} - k_d J^2 + k_p J = f_i(J), \quad (11)$$

$$k_d = \frac{e^{-\tau J}(-83.68J^6 + 669.018J^5 - 1177.859J^4 + 356.103J^3 - 6.274J^2 + 0.037J - 1.68*10^4)}{2(-118.72J+0.72)^4} = f_d(J). \quad (12)$$

The stability degree variable J is varied from 0.01 to 1.5, and the graph of dependencies k_p, k_i, k_d as functions of J are plotted. According to the original method, the optimal value of the stability degree J of the designed system corresponds to the first maximum of the dependencies of the controller's tuning parameters obtained by varying $J = 0 \dots \infty$.

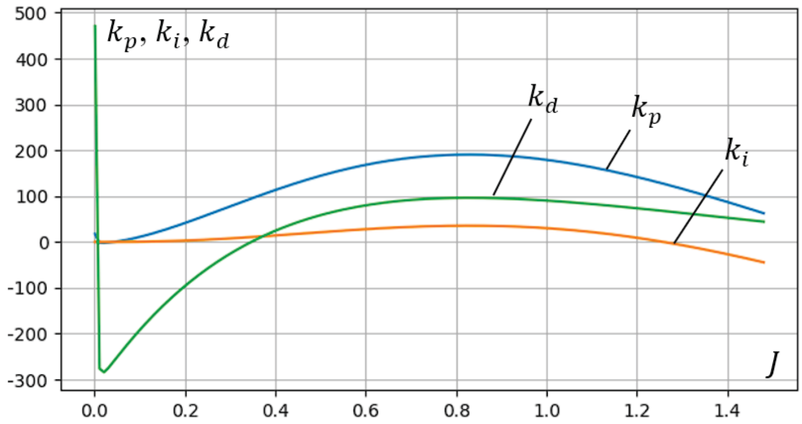


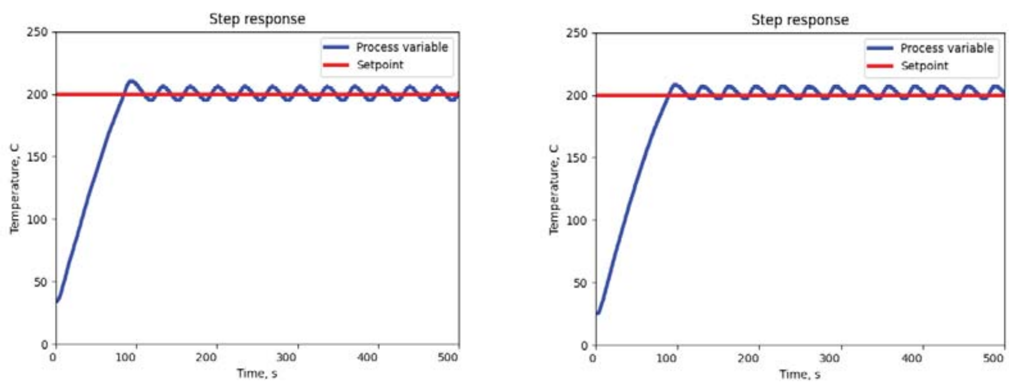
Figure 3. ***Dependences*** $k_p = f(J)$, $k_i = f(J)$ ***şi*** $k_d = f(J)$

Source: *Developed by the author*

Next, from the k_p, k_i, k_d dependencies in Figure 3, several sets of values (Table 1) were chosen for the PID controller of the automatic temperature control system in the 3D printer's extruder. The controller parameters were set, and the system's step response was plotted, as shown in Figure 4. The results of tuning the controller using the MSDI method were compared with the Ziegler-Nichols method (a method for tuning the PID controller without knowing the mathematical model of the object), as shown in Figure 4, *d*. The experimental validation of the data was carried out on the thermal process in the extruder of a 3D printer (Figure 2 a and Figure 4).

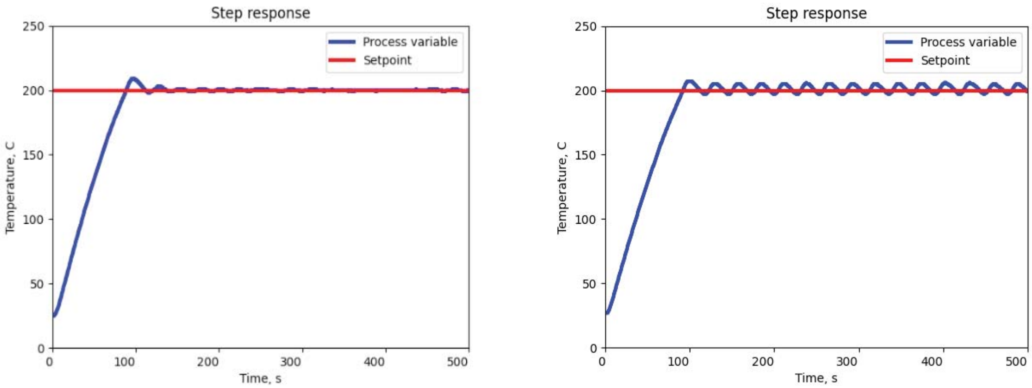
The oscillations present in Figures 4, *a* and 4, *b* are due to the high values of the tuning parameter of the integrative component k_i, which can accumulate a large integral error and lead to overshoot and sustained oscillations.

In the case of the system with a PID controller tuned using the Ziegler-Nichols method (Figure 4, *d*), oscillations are present due to the essence of the method, as the processes are obtained with oscillations.



a)

b)



c) *d*)

Figure 4. ***Step responses of the system with the PID controller***

Source: *Developed by the author*

In Table 1, the performances of the automatic temperature control system in the extruder with the PID controller for the object model identified from (9) are presented.

Table 1

Parameters of the controller and performance of the system with the PID controller

No. iter.	Tuning method	Parameters of the controller					Performance of the system			
		J	k_p	k_i	T_i, s	k_d, s	t_r, s	σ, %	t_s, s	n
1	MSDI, fig.4, a	0.36	96.13	10.848	0.092	2.16	82	5.5	103	1
2	MSDI, fig. 4, b	0.78	189.15	35.02	0.028	95.42	82	5.0	102	1
3	**MSDI, fig. 4, c**	**1.24**	**134.2**	**4.6**	**0.21**	**70.47**	**74**	**4.5**	**82**	**0**
4	ZN, fig. 4, d	-	336.6	34.22	0.029	841.5	80	4.0	92	0

Source: *Developed by the author*

In Table 1, the following notations are used: t_r is the rise time, σ is the overshoot, t_s is the settling time, and n is the number of deviations from the set value. Typically, the deviation is considered to be the exceedance of the output value of the automatic system by ± 5 % from the set value $y(t)$.

The best performance of the automatic temperature control system in the extruder was achieved with the PID controller tuned using the MSDI method (Figure 4, *c*), iteration No. 3, Table 1, having the shortest settling time, minimal overshoot, and no oscillations.

Conclusions

Based on the results obtained, it can be asserted that the maximum stability degree method with iteration provides good results for the case of tuning the PID controllers for the third-order models with additional zero and time delay.

The extruder of a 3D printer was used as the control object. As a result of synthesizing the PID control algorithm to the object model discussed in this paper using the MSDI method offers good system performances. Among them, the best result had a settling time $t_s = 82$ seconds and an overshoot $\sigma = 4.5$ %. The other results exhibited sustained oscillations.

For a better view of the MSDI method, a comparison was made with the Ziegler-Nichols tuning method. The results of tuning the controller, for the type of object model discussed, using ZN are satisfactory.

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