In Memoriam

Eugeniu GREBENIKOV: 20.01.1932 – 29.12.2013

December 29, 2013, marked the departure of one of us of Professor Eugeniu Grebenikov, a noted expert in applied mathematics, celestial mechanics, the theory of nonlinear oscillations, and mathematical simulation. It is amazing how a single person has managed to obtain fundamental results in all these areas and to contribute to many others associated with science management and training activities at various higher educational institutions. Despite his age, Grebenikov leads an active life. He frequently takes part in national and international scientific conferences and seminars, where he gives talks.

His contributions are devoted to pure mathematical issues in the stability theory of solutions to differential equations and to applications in celestial mechanics, computer simulation, computer programming, etc. Due to Grebenikov's achievements in these areas, he is a renowned expert in each of them. For example, specialists in celestial mechanics, where he began his scientific career, have named asteroid No. 4268 after Grebenikov (with 11 other Romanian personalities: *Mihai Eminescu (asteroid No. 9.495; George Enescu (asteroid No. 9. 493; Constantin Brîncuşi (asteroid No.6.429; Elena Văcărescu (formation from Venus planet; Nicolae Donici (asteroid No. 9.494;Victor Daimaca (comet Van Gent-Peltie-Daimaca; Herman Oberth (asteroid No. 9.253; Nicolae Sănduleac (asteroid No. 9.403; Jan Drăgescu (asteroid No. 12.498; Spiru Haret (crater from invisible face of the Moon; Constantin Dârvulescu (asteroid No. 2.331)*. In recent years, he has made important contributions to the restricted many-body theory. They were obtained via computer simulation and computer algebra languages and could not be derived by analytical methods.

Grebenikov was born on January 20, 1932, in the village of Sloboziya Mare (Romania, now the Republic of Moldova) into the family of an orthodox priest and an elementary school teacher. In 1949, he graduated with honors from the Romanian Lyceum at Kahul (Moldova) and entered the Department of Astronomy at the Faculty of Mechanics and Mathematics of Moscow State University (MSU). In 1954, he graduated with honors from the university. While a student, Grebenikov did research in celestial mechanics under notable MSU Professors N.D. Moiseev and G.N. Duboshin. After graduation, he continued his studies as a graduate student at MSU under the supervision of Moiseev, who had great influence on Grebenikov's early scientific career. In 1957, Grebenikov completed his graduate studies and defended at MSU his candidate's dissertation in astronomy entitled "*Analytical Theory of the Motion of Saturn's Eighth Moon Iapetus"*. In 1967, Grebenikov defended his doctoral dissertation "*Qualitative Studies of Differential Equations in Celestial Mechanics*", in which he was the first to substantiate the well-known Krylov–Bogolyubov method as applied to resonance multifrequency systems of differential equations with slow and fast phase variables. For such systems, he developed a general analytical perturbation theory (up to an arbitrary order with respect to a small parameter) and devised an analytical integration method for infinite systems of partial differential equations as applied to the determination of the Krylov–Bogolyubov transformation functions. For multifrequency systems, he designed an optimal choice algorithm for determining unknown functions appearing in high-order averaged systems. These methods were designed for resonance systems of differential equations, which involve small denominators—the most substantial obstacle to the application of the Krylov–Bogolyubov method.

In various years, Grebenikov held various high positions associated with his science management activities. In 1969–1978, he headed the Department of Mathematics at the Institute of Theoretical and Experimental Physics of the USSR State Committee on Atomic Energy. In 1978–1988, he was director of the MSU Research Computer Center. In 1988–1997, he was a department head at the Institute of Problems of Cybernetics of the Russian Academy of Sciences and worked as deputy director at the Institute for High-Performance Computer Systems of the Russian Academy of Sciences. After this institute was disbanded, Grebenikov, together with his team, was transferred to the Computing Center of the Russian

Academy of Sciences, where he headed the Department of Nonlinear Analysis Methods in 1997.

Simultaneously with his intensive scientific activities, Grebenikov participates in training young scientists at different levels. In his student and postgraduate years, he taught mathematics at evening schools in the Krasnopresnenskij District of Moscow. Starting in 1957, after completing his graduate studies, he taught at the Faculty of Mechanics and Mathematics and at the Faculty of Physics of MSU. Later, he headed the Department of Mathematical Analysis at the Peoples' Friendship University of Russia, worked as a professor of the Department of Cybernetics at the Moscow State Institute of Electronics and Mathematics (Technical University), headed the Department of Algebra and Analysis at the Moscow State Aviation Institute (Technical University), served as a professor of the Department of Higher Mathematics at the Engineering University of the Republic of Moldova, and headed the Department of Mathematical Analysis at the University of Podlasie (Siedlce, Poland) (1996-2003). Over 50 years of his activities in science and science management, Grebenikov has trained numerous highly skilled professionals. Over the last five years, he supervised four candidate's and two doctoral dissertations. His students do research and teach in Armenia, Byelorussia, Israel, Kazakhstan, Mexico, the Republic of Moldova, Poland, Romania, and the USA.

For his great achievements, Grebenikov won the State Prize of the USSR in 1971. He was also awarded the Prize of the USSR Council of Ministers in 1983 and the Academician Krylov Prize of the Ukrainian Academy of Sciences in 1999. Professor Grebenikov was a full member of the Academy of Nonlinear Sciences, an honorary member of the Academy of Sciences of the Republic of Moldova, and an Doctor Honoris Causa of several foreign universities: Doctor Honoris Causa al Universităţii „*Babeş - Bolyai*" from Cluj-Napoca, Romania (1993); Doctor Honoris Causa of State University from Atârau, Kazahstan (2001); Doctor Honoris Causa of Technical University of Moldova, Chishinau, Republic of Moldova (2003); Doctor Honoris Causa of Technical University „*Gh. Asachi*", Iassy, Romania; Doctor Honoris Causa of University „*V. Alecsandri*", Bacau, Romania; Doctor Honoris Causa of University „*B.P.Hasdeu'*"", Cahul, Republic of Moldova.

Due to his active participation in MSU seminars on celestial mechanics and the theory of ordinary differential equations, Grebenikov's major research interests in those years were focused on the analytical and qualitative theory of ordinary differential equations and their applications to nonlinear mechanics, in particular, to celestial space dynamics and the theory of nonlinear oscillations. In his first publications, Grebenikov demonstrated the effectiveness of Hill's analytical method as applied to the dynamics of natural and artificial satellites orbiting with large inclinations relative to the ecliptic plane and the planets' equator plane.

Grebenikov proposed and justified the new idea of developing asymptotic methods that minimize the deviations of solutions to averaged equations from those to the original equations. For this purpose, he used the stepwise correction of initial conditions combined with the principle of nonlinear multifrequency systems given on multidimensional tori. Following this approach, Grebenikov and his students examined new dynamic aspects of well-known problems, such as the restricted three(and more)-body problem with various resonances; resonance Hamiltonian systems; the motion of a geostationary satellite; problems in highenergy physics (the dynamics of charged beams in accelerators); and mathematical modelling in biology, geology, and other disciplines.

During the last decade, Grebenikov and his followers have been successfully developing a mathematical area that can be called Lagrange–Wintner homographic dynamics. Due to new information technologies, in particular, new computer algebra systems (such as Mathematica, Maple, etc.), they found new multiparameter classes of exact solutions to the Newtonian many-body problem such as exact Lagrangian and Euler solutions to the differential equations in the Newtonian three-body problem. The objects of study are new gravitation models in celestial mechanics and space dynamics with complete and incomplete geometric and dynamic symmetries.

Continuing Lagrange and Wintner's studies in the theory of homographic three-body solutions, Grebenikov formulated necessary and sufficient conditions for the existence of homographic solutions to the Newtonian many-body problem with an arbitrary finite number of bodies. Due to these results, he concluded that the Trapezium cluster in the Orion Nebula is not a homographic solution to the Newtonian four-body problem; hence, its trapezoidal shape cannot be conserved. Grebenikov proposed a

new dynamic model of the restricted many-body problem (n> 3) that studies the motion of a passive mass in the gravitational field generated by a large number of bodies whose trajectories are homographic curves. For the restricted many-body problem, he proved a theorem on the existence of a first integral similar to the Jacobi first integral in the restricted three-body problem. Together with his students, Grebenikov developed effective computer methods for linearizing Hamiltonian systems in the neighbourhood of any stationary solution. Mathematica-based software packages were developed for the symbol (not numerical) normalization of Hamiltonians in the neighborhood of any stationary solution. In other words, effective software tools were designed for one of the most complicated problems in qualitative celestial mechanics and space dynamics – the Lyapunov stability of stationary solutions to the restricted manybody problem. These studies were based on results obtained in KAM theory (the theory of the existence of conventional periodic solutions to multidimensional Hamiltonian systems on multidimensional tori named after Kolmogorov, Arnold, and Moser).

He has participated in many scientific national and international conferences. He was a member of the Program Committee of the annual International Workshop on Computer Algebra in Scientific Computing (CASC), which is held in different countries.

Together with his students and colleagues, Grebenikov has published **28** monographs and more than **200** scientific papers. The bibliography of scientific works by Grebenikov given below is reduced. Therefore, I had to pick from them a small number that are especially known.

- *Handbook of Mathematics for Entrants to Engineering and Physics University* (MTILP, Moscow, 1960) [in Russian].
- *Integral Calculus: Textbook* (Ross. Univ. Druzhby Narodov, Moscow, 1964) [in Russian] (with K. G. Danilov and P. V. Shcheglov).
- *New Qualitative Methods in Celestial Mechanics* (Nauka, Moscow, 1971) [in Russian] (with Yu. A. Ryabov).
- *Handbook of Celestial Mechanics and Astrodynamics* (Nauka, Moscow, 1971) [in Russian] (with V.K. Abalkin, etc.).
- *Handbook of Celestial Mechanics and Astrodynamics,* 2nd (revised and extended) edition (Nauka, Moscow, 1976) [in Russian] (with V.K. Abalkin, etc.).
- *Resonances and Small Denominators in Celestial Mechanics.* (Nauka, Moscow, 1978) [in Russian] (with Yu. A. Ryabov).
- *Constructive Methods for Analysis of Nonlinear Systems* (Nauka, Moscow, 1979) [in Russian] (with Yu. A. Ryabov).
- *Metoda Usrednienia w Meshanice Nielinioej.* (PNR, Warsaw, 1982) [in Polish] (with Yu. Ryabov).
- *Nicolaus Copernicus*, 2nd. ed. (Nauka, Moscow, 1973) [in Russian].
- *Constructive Methods in the Analysis of Nonlinear Systems.* (Mir, Moscow, 1983) (with Yu. Ryabov).
- *Ordinary Differential Equations: Notes on the Development of Mathematics in the USSR.* (Naukova Dumka, Kiev, 1983) [in Russian].
- *Mathematical Modeling in Nonlinear Mechanics* (Nauka, Moscow, 1984) [in Russian].
- *Search for and Discovery of Planets.* , 2nd. ed. (Nauka, Moscow, 1975) [in Russian] (with Yu. A. Ryabov).
- *The Three-Body Problem in Celestial Mechanics.* (Mosk. State Univ., Moscow, 1985) [in Russian] (with V. G. Golubev).
- *Numerical-Analytical Research Methods for Regularly Perturbed Multifrequency Systems.* (Mosk. State Univ., Moscow, 1986) [in Russian] (with M. N. Kiosa and S. V. Mironov).
- *Averaging Method in Applications.* (Nauka, Moscow, 1986) [in Russian].
- *Introduction to the Theory of Resonance Systems.* (Mosk. State Univ., Moscow, 1987) [in Russian].
- *Averaging Method in the Study of Resonance Systems of Differential Equations.* (Nauka, Moscow, 1992) [in Russian] (with Yu. A. Mitropolsky).
- *Introduction to Resonance Analytical Dynamics.* (Yanus-K, Moscow, 1999) [in Russian] (with Yu. A. Mitropolsky and Yu. A. Ryabov).
- *Computer Algebra Methods as Applied the Many-Body Problem.* (Ross. Univ. Druzhby Narodov, Moscow, 2001; 2002, 2nd ed.) [in Russian] (with D. Kozak-Skovorodkina and M. Jakubiak).
- *Asymptotic Methods in Resonance Analytical Dynamics.* (Boca Raton, London, 2004) (with Yu. A. Mitropolsky and Yu. A. Ryabov).

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