GENERALIZED NUMERIC ALGORITHM FOR STIFFNESS DEGRADATION OF STEEL AND CONCRETE ELEMENTS STRENGTH

Elena Sidorenco
Universitatea Tehnică a Moldovei

1. INTRODUCTION

The nonlinear calculus of the structures now represents a necessity. This difficult calculus at structural ensemble level is, nowadays, possible because of the fast development of computational resources. In current practice of design of the reinforced concrete structures, the real nonlinear behaviour from the physical point of view of this composite material can't be emphasized. This behaviour is reflected in the development of the cracks from the tensioned concrete, in the crash of the material in the compressed concrete area as well as in reinforcing yield. All these constitutive particularities determine the response of the material not to be produced as a single whole at some post – elastic loading levels. Consequently, a nonlinear analysis is approached, using the modern concept of stiffness degradation of the strength elements under the action of increasing/decreasing monotonous loading or cyclic.

The algorithm of computational incremental solving of elasto-plastic problems is based on the plastic flow theory with the application of von Mises yield criterion for concrete and reinforced concrete and considering elastic kinematic/isotropic hardening behaviour and the computing model for determining the generalized modulus of plastic displacement. The numerical integration of condition equations is made by the finite and boundary element computational method, considering the material yield theory adapted to this method. Computational iterative methods are used for the extension of the class of problems solved in case of body deformation according to the plastic limit with yield criterion.

At each iteration the objective in solving with displacement method is to observe the conditions of static equilibrium. When solving the problem the algorithm diverges in two directions depending on the chosen computational method. This method is chosen from the family of computational methods compatible with finite element method. The two directions are:

-calculus through initial stresses, also called the elastic solution method;

-calculus through analytical design method on the yield surface (the antigradient lowering of inadmissible stresses on the yield surface method).

The paper also deals with the algorithm of foreseeing optimal step of considering the loading in the crack propagation stage and with the scheme of the adjustment process. At the same time the scheme of the regularization process utilization is presented.

An often difficulty of the modelling process of the crack increase, appears and it is expressed by the next paradox: the boundary conditions depend on the variation crack parameter, which also depends on the control parameter. These conditions must be calculated before the crack increase, while they are determined in the final increase stage. This problem is motivated by the fact that this limit conditions are explicitly expressed in FEM. This difficulty can be avoided in two ways:

- the loading promoting with small steps;
- the extension of the eventual value of the characteristic parameter, depending on the crack length.

The major contribution of this work is finishing a mathematic algorithm of the proposed methodology development.

2. ITERATIVE PROCESS OF THE INITIAL DEFORMATION METHOD

The presentation of the computing algorithm will be done in solving stages, as follows:

1st stage: The incremental loading process is applied (step by step) in static /dynamic numerical computational analysis. The initial action $\{P_0\}$ is applied according to the case of loading. The linear analysis with FEM is done and consequential stress and strain fields are obtained $\{\sigma_0\}$, $\{\varepsilon_0\}$. The initial loading is conditioned so that after its application all the points of the discreet model should displace (linearly or angularly) only elastically; the reach of the boundary state of yield in some points of the model should take place when applying the next loading step, which is incremented $\{\Delta P_1\}$. This can be done by using the scale coefficient method, taking into account the fact that the loading is proportional. The analysis of the distribution of plastic areas depends on the size of the loading step $\{\Delta P\}$.

2nd stage: *1*. The increase of the loading $\{\Delta P_i\}$ is applied. The linear computational calculus with FEM is done, consequential stress increase $\{d\sigma^*\}$ and strain increase $\{d\varepsilon^*\}$, are obtained. The values of the total current deformations are calculated by adding the plastic deformation increment $\{d\varepsilon^p_{ij}\}$ to the previously obtained value. Further on the work presents the algorithm of checking the boundary areas corresponding to the functions of plastic potential, f_0, f, F (the potential function).

2. The value of the yield function (the plastic potential function) is calculated depending on the adopted constitutive law and the stress function around that point is checked to see if it falls on the boundary aria frontier.

If $f_0(\{\sigma^*_i\})<0$, then the point corresponding to the given stress state is situated in the elastic behaviour domain (in the elastic deformation stage); but if $f_0(\{\sigma^*_i\})>0$ there results that at the considered point, a process of plastic flow of the material takes place, which corresponds to the potential function, f_0. This condition is verified for all the points from the considered discrete domain. The time history (for several steps of loading) for the deformation of the material around the point where the plastic flow condition is obtained, is examined.

If $f_0(\{\sigma^*_i\})=0$ and $f_0(\{\sigma^*_{I-1}\})>0$, then it is necessary that the inadmissible stress state should be corrected, so that one should obtain $f_0(\{\sigma_i\})=0$. This condition means that the point corresponding to the stress state falls on the limit just on the yield area.

3. The split of the part corresponding to the increase of elastic deformation in proportion to the rate of the increase of plastic deformation is done through the r coefficient. The determination of this coefficient is done by putting the condition of observing the next equation (1), which has the meaning from the above mentioned point 2:

$$f_0\left(\{\sigma_{i-1}\}+r\{d\sigma^*\}\right)=0 \tag{1}$$

The linear interpolation is made and the first approximate value of r_1 coefficient is obtained through (2):

$$r_1=-\left(f_{i-1}/\left(f\left(\{\sigma_i^*\}\right)-f\left(\{\sigma_{i-1}\}\right)\right)\right) \tag{2}$$

From several reasons (nonlinearity, nonmaintainance of the *1D* loading condition, the convexity of the yield area) first approximation of the r_1 coefficient doesn't always satisfy the incremental development strain condition:

$$f=f_0\left(\{\sigma_{i-1}\}+r_1\{d\sigma^*\}\right)\neq 0 \tag{3}$$

4. Then, the following approximation for the determination of the r coefficient is done, considering the next (4) formula:

$$r=r_1-\frac{f}{\{\partial f_0/\partial\sigma\}^T\{d\sigma^*\}} \tag{4}$$

The elastic part of the deformation is defined by the relation $r\{d\sigma^*\}$ and the remaining part corresponds to the plastic deformation characteristic curve *" σ-ε"*.

5. That is why the influence of the loading step be decreased as much as possible while the elasto-plastic deformation interval is divided into m subintervals.

$$m=f_i^*/\Delta f \tag{5}$$

6. Then calculus of the corresponding stresses for each subinterval m_I, is done as follows:

$$\{\sigma_1\}=\{\sigma_{i-1}\}+\{d\sigma^*\}-\lambda\{d\} \tag{6}$$

$$\{\sigma_i\}=\{\sigma_1\}-\left\{\frac{\partial F}{\partial\sigma}\right\}\frac{F_1}{\{\partial F/\partial\sigma\}^T\cdot\{\partial F/\partial\sigma\}} \tag{7}$$

where,

$$\lambda=\frac{\{d\}^T\{d\varepsilon\}}{A+\beta},\quad \{d\}=[D]\left\{\frac{\partial F}{\partial\sigma}\right\},$$

$$\beta=\left\{\frac{\partial F}{\partial\sigma}\right\}^T\{d\},\qquad A=H' \tag{8}$$

$H'$is the slope of the curve of material deformation. The nominator from (5) depends on the size of the loading step ΔP.

$[D]$ – is the constitutive matrix (stress – strain relationship) for a linear elastic, isotropic material and for plane strain/stress.

***3rd stage*:** If the conditions for the current point situated in the considered discrete domain are fulfilled:

$$f\left(\{\sigma_i^*\}\right)>0,\quad f\left(\{\sigma_{i-1}\}\right)=0, \tag{9}$$

then the incremental increase of the stress $\{d\sigma^*\}$ is the result of the plastic deformation of the body. In this case, the pursuit of the curve of the material deformation is done through the relations from 2nd stage; we then obtain the point where the value of the coefficient is zero.

4^th^ stage: We verify the equilibrium condition by previously determining the deviation value, which defines the difference between the state created by correcting the inadmissible stresses $\{\sigma^*_i\}$ and the state corresponding to the current stresses $\{\sigma_I\}$:

$$\{d\sigma^{**}\} = \{\sigma_i^*\} - \{\sigma_i\} \tag{10}$$

$$\{\Delta P\} = \int_V \{d\sigma^{**}\}[B]^T \, dV \tag{11}$$

Next, the $\{\Delta P\}$ vector is applied as external loading (the right part of the equilibrium equation system) and the linear problem is solved. The increases of the displacements and those corresponding to $\{d\sigma^*_i\}$ stresses and to $\{d\varepsilon^*_i\}$ strains are obtained.

5^th^ stage: The process from 2nd and 3rd stages is repeated and the convergence criteria of the iterative process, defined by the next (12) relation, is verified:

$$\frac{\{dq_i\} \cdot \{dq_i\}^T}{\{q_i\} \cdot \{q_i\}^T} \le \varepsilon \tag{12}$$

where, ε - is the allowable variation of the accuracy.

6^th^ stage: For verifying the accomplishment of the convergence criterion, the next loading step with exterior forces $\{\Delta P\}$ is activated. Linear analysis is accomplished. The observance of the boundary conditions is analyzed in the potential curve neighborhood in every point of the discrete domain, according to the stages presented above. The iterative incremental analysis continues till the stipulate application of the exterior force is obtained (corresponding to the increase of the crack length) or the calculus steps in case the iterative process doesn't converge, which means that the convergence condition is not observed after (10-20) iterations. Physically, this means that the pass of stable increase into an instable increase of the crack, therefor the breakage of the body; from the mathematic point of view this signifies the fact that the structure matrix degenerates.

Table 1.

I	II		III		IV	
n	0 or 1	1, 2, …	0 or 1	1, 2, …	0 or 1	1, 2, …

The identification is introduced for the „*n*" point (*n* indicates the point's number). In table 1 above, number II - corresponds to the initial yield ($f_0 = 0$), number III - indicates the current loading surface ($f = 0$), number IV - refers to the limit yield surface ($F = 0$). In every cell two digits are indicated: 0 and 1. 1 - indicates that when loading, in that point, the initial fracture surface is achieved, and if the stress state around that point didn't achieve the initial fracture surface, *0* will be written. The rest of the are needed to reach the considered stress surface.

3. THE ALGORITHM OF THE ELASTO-PLASTIC DEFORMATION MODELLING BY ANALYTICAL PROJECTION ON THE YIELD SURFACE

According to the arguments shown above, the application of the projection on the yield surface method simplifies the algorithm of elasto-plastic problem, essentially. The algorithm is made in stages, as follows:

I. The P_0 force is applied. The stress and the strain states components $\{\sigma\},\{\varepsilon\}$ are calculated around current point.

II. The numerical value of the following yield function, *F* (the plastic potential function) is calculated for every discrete point of the domain and the next expressions are verified.

- If the value of the yield function, $F \ge 0$, we pass to IV.
- If $F<0$, an inadmissible stress/strain state takes place and we pass to III.

III. An element which has the discrete point of the domain is actioned by an unloading force (the unloading phenomena by which the return on the yield surface is made, takes place) through the mentioned analytical expressions, according to the next sequence:

- the α (angle of open crack), *f, d,* γ parameters are calculated;
- the plasticity matrix components are calculated;
- the FEM computational process is pursuing the observance of the convergence condition.

IV. We verify the continuing criterion of the problem.

V. We action with the next loading step $\{\Delta P\}$. The persistence of the loading is made by passing to II. The process described at II and III is repeated.

4. CONCLUSIONS

1. We made an analysis of the modern constitutive models which renders the concrete and

reinforced concrete elements behavior, elements solicited in any possible way (monotonous, increasingly /decreasingly, or certain solicitation). The models were elaborated by using computational simplifying theories and by proper mathematics for continuous spectrum and deformed mechanics and for nonlinear breakage mechanics.

2. We promote an original computational model in which three functional areas are considered: - the initial yield of the material, the flowing area of the material and the damage surface of the body. With such mathematical - mechanical approaching the variables for each case are argued by experimental physical determinations; Uniaxial, bi-triaxial tests are used. We prefer those computational models in which experimental data are confirmed by analytical solutions or by some special elaborated tests according to the existent engineering computational standards.

3. A nonlinear numerical incremental analysis was made by using the difference equations of mathematical physics until the reaching of the boundary flowing state of the reinforced and cracking/crushing of the concrete. We continued the analysis till the final state of the material damage around a point from the reach of degradation state on the section. The concept of damage development of material performance.

4. A generalized algorithm for determining the limit deterioration state of the construction element was done by making some constitutive models specific to the real state of loading, into an incremental analysis frame, for random iteration. We used the initial deformation method for this incremental analysis.

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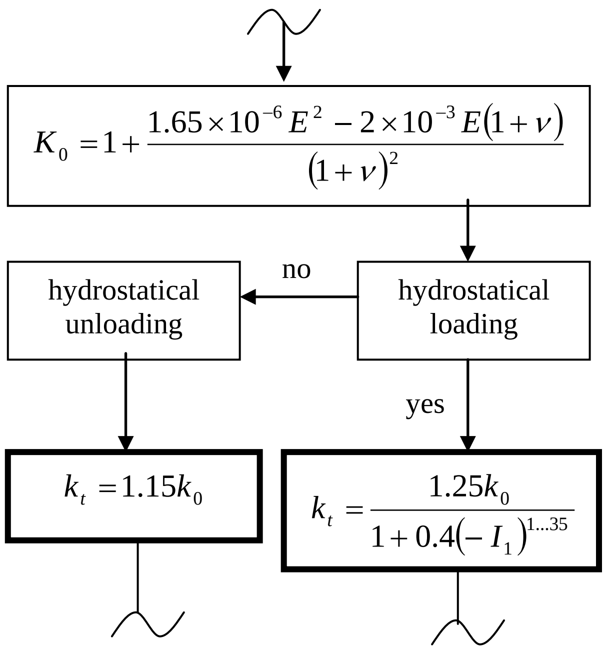
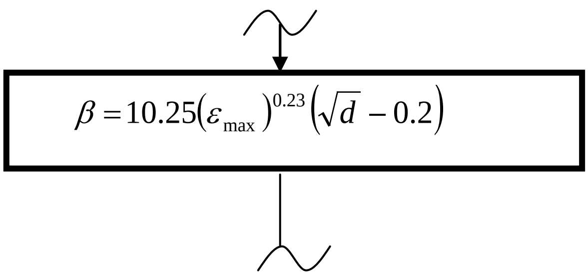
Algorithm calculus of modulus k_t



Algorithm calculus of contract - dilatation modulus



The algorithm of plastic strains calculus

The load at „i” time (iteration) $\{P\}_i = \{P\}_{i-1} + \{\Delta P\}_i$

↓

The stresses and strains at „*i*” time (iteration)

$$\{\sigma\}_i = \{\sigma\}_{i-1} + \{d\sigma\}_i$$

$$\{\varepsilon\}_i = \{\varepsilon\}_{i-1} + \{d\varepsilon\}_i$$

↓

Volumetric stress

$$\sigma_0 = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$[T_V] = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

↓

Deviatoric stress

$$[S] = [T_\sigma] - [T_V]$$

$$[S] = \begin{bmatrix} \sigma_x - \sigma_0 & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \sigma_0 & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \sigma_0 \end{bmatrix}$$

↓

Von Mises equivalent stress (yield criterion)

$$\sigma_e = \frac{\sqrt{2}}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)}$$

↓

Invariants stress state

$$I_1 = \sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x - \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)$$

↓

Calculus of material modulus $H^p; k_t; \beta$

↓

The incremental **plastic strains**

$$d\varepsilon_{ij}^p = \frac{S_{kl}}{3H^p\tau_0}\left(\frac{S_{ij}}{\tau_0} + \delta_{ij}\frac{\beta}{3}\right)d\sigma_{kl} + \delta_{ij}\frac{1}{9k_t}d\sigma_{kk}$$

The algorithm of computational calculus of generalized modulus of plastic displacement H^p