# Quantum computing for multi-qubit systems using Schwinger's paired bosons representation of angular momentum

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**Abstract.** The fundamental difference between calculations performed using quantum and classical computers is that, unlike a bit, a quantum bit (qubit) is a linear combination of quantum states $|1/2, 1/2>$ and $|1/2, -1/2>$ of effective spin S = 1/2. The states of an N-qubit system can be characterized by spin wave functions $|S, M_S>$ ($M_S = S$, S - 1, S - 2, …, 2 - S, 1 - S, - S) of effective spin S = $2^{N-1}$ – 1/2 [1]. These 2S + 1 spin wave functions, given traditionally in the spinor representation, can also be written using the paired bosons representation proposed by J. Schwinger [2]:

$$|S, M_S> = [(S + M_S)!\,(S - M_S)!]^{-\frac{1}{2}} (a_1^+)^{S+M_S} (a_2^+)^{S-M_S} |0> = |S + M_S>_1 \cdot \cdot |S - M_S>_2, \quad (1)$$

where $a_i^+$ and $a_i$ ( $i$ = 1, 2) denote the operators of creation and annihilation of bosons related to quantum oscillators 1 and 2, $|0> = |0>_1 \cdot |0>_2$, $|0>_1$ and $|0>_2$ are the vacuum states of the quantum harmonic oscillators 1 and 2. According to (1), the 2S+1 spin wave functions related to the effective spin S are expressed in the Schwinger paired bosons representation through boson wave functions corresponding to the lowest 2S+1 energy levels of each of the harmonic oscillators 1 and 2. The spin projection operators $S_x$, $S_y$ and $S_z$ in the Schwinger paired bosons representation have the form:

$$S_x = \frac{1}{2}(a_1^+ a_2 + a_2^+ a_1),\ S_y = \frac{1}{2i}(a_1^+ a_2 - a_2^+ a_1),\ S_z = \frac{1}{2}(a_1^+ a_1 - a_2^+ a_2). \quad (2)$$

The explicit form of operators $S_x$, $S_y$ and $S_z$ from (2) does not depend on the spin S value, in contrast to the spinor representation, when the dimensions and forms of the matrices of these operators depend on the value of S. Using formulas (2), all logical elements of quantum circuits of a N-qubit system can be expressed in the Schwinger representation of paired bosons by means of operators $a_i^+$ and $a_i$ ($i$ = 1, 2). With an increase in the number of qubits N, there is a sharp increase in the number of boson states of each of the quantum harmonic oscillators 1 and 2 participating in the two-boson representation of 2S+1 spin states $|S, M_S>$ ($M_S$ = S, …, - S). Particularly, for N = 70 we obtain 2S+1 = $2^{70}$ = 1.2 x $10^{21}$. Therefore, at N $\geq$ 70 the number of boson states 2S+1 $\simeq 10^{21}$ of each of the quantum harmonic oscillators 1 and 2 participating to the paired bosons representation of spin states can be approximately considered equal to infinity. In this case, the methods of quantum field theory [3] can be used to perform quantum computations. This is another advantage of the two-boson representation of effective spin states when performing quantum computations in the case of multi-qubit systems.

**References**

[1] Ion Geru, Dieter Suter, *Resonance Effects of Excitons and Electrons: Basics and Applications*, Springer-Verlag Berlin, Heidelberg, 2013, 283 pp. (p. 201); DOI 10.1007/978-3-642-35807-4.

[2] J. Schwinger, On Angular Momentum, in: *Quantum Theory of the Angular Momentum*, ed. by L. C. Biedenharn, H. Van Dam (Academic Press, New York, 1965), pp. 229-279.

[3] A. Zee, *Quantum Field Theory, as Simple as Possible*. Publisher: Princeton University Press, 2023, 392 pp.