ON THE INTERACTION OF WAVES WITH INHOMOGENEITIES IN THE IDEAL CONTINUOUS MEDIUM

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INTRODUCTION

Any smooth mathematical function from time can be presented as the sum of harmonic functions from time. Similarly function from spatial coordinates can be presented as the sum of harmonic functions from spatial coordinates. The first representation constitutes a basis of the radio signals analysis, and on second the holography is based. It is logical to assume, that the function from time and spatial coordinates can be presented as the sum of harmonic functions from time and spatial coordinates, it means as sum of waves. Thus, it is possible to make a deduction, that the waves have sufficient completeness for the analysis and synthesis of as much as composite objects evolving in space and time. In article [1], we have proposed to accept as waves carrier the ideal continuous medium. Then, in correspondence with offered model, both objects of observation, and the tools will represent waves in this medium. In the same article the theorem was proved: if the tools and the objects, studied with their help, represent waves in same medium, in such "world" the maximal velocity of the wave propagation will not depend from velocity of reference frame. That is in such model the principles of a special theory of relativity are completely observed. Or else, we have shown that the presence of the carrier medium of waves does not contradict the relativity theory.

In articles [2, 3] we explored, how the waves interact among themselves. Was shown that the stable standing waves can interact as a particle. And the Compton effect takes place in interaction between standing wave and progressive wave, in other words have a place the quantification of a travelling wave.

In this article a problem we put to explore, how the wave-tool will interact with fluctuations in medium. In other words, we want to elucidate, whether it is possible to discover the pressure drops and drops of velocities in medium using the waves as tools, and how it will become apparent.

1. INTERACTION OF THE WAVE-TOOL WITH A PRESSURE DROP

Let's assume that the instantaneous profile of pressure of the wave-tool, along the axis x, represents or surplus (fig. 1*a*) or deficit (fig. 1*b*) of pressures P_T, in comparison with pressure in a unperturbed continuum P_0. For brevity we name the wave figured on (a fig. 1*a*) as positive wave-tool, and wave figured on (a fig. 1*b*) - as negative wave-tool.

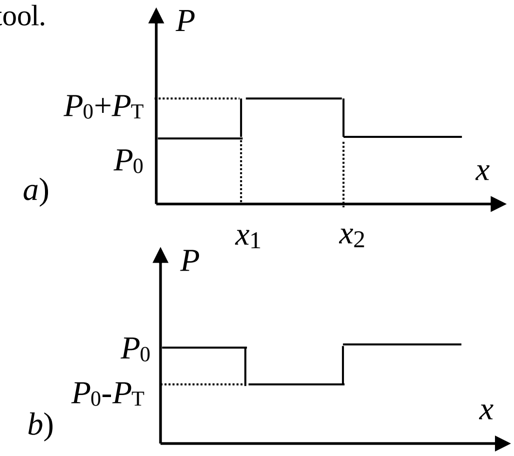

Figure 1. The profile of waves-tools

To simplify the analysis, at first we shall assume, that the allocation of redundant pressure in a wave looks like a parallelepiped figured in figure as a rectangle. The result obtained thus can be extended simply for the wave-tool of any shape by summing (integration) of such elementary volumes.

Let's determine the force, which act on such the wave-tool during the time, while the wave-tool not changes its shape. That is, in case of the positive wave-tool we are interested by instantaneous force, which act on continuum surplus, and in case of the negative wave-tool by force, which act on a continuum deficit. This situation resembles to what takes place with electrons and vacancy in semiconductors.

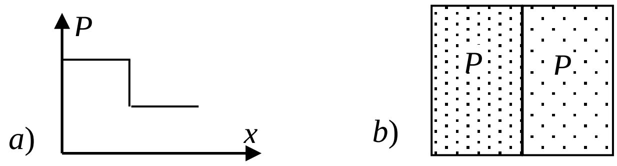

Figure 2. The representations of a pressure drop in a continuum.

We mark pressure and force, which act on surplus of a continuum as P^+ and F^+, and the pressure and force, which act on a deficit of a continuum as P^- and F^-.

Let's assume that in a continuum there is a pressure drop (fig. 2*a*). In figure (fig. 2*b*) it is figured as points density difference. On border the pressure drop, which act on surplus of a continuum, will be:

$$\Delta P^{+} = P_1 - P_2 . \tag{1}$$

At the same time the pressure acting on a continuum deficit will be:

$$\Delta P^{-} = -(P_1 - P_2). \tag{2}$$

That is, it is possible to tell, that the surplus of a continuum aspires to pass the border from left to right, and with the same success it is possible to tell, that the deficit of a continuum aspires to pass border from the right to the left. Thus, on border, the pressure, which acts on the surplus and on the deficit of a continuum, have equal modulo, but direction opposite.

Now let's consider the behaviour of wave-tool in the domain of a pressure drop. The fig. 3*a* and fig. 3*b* represents the positive and negative wave-tool accordingly in domain of pressure drop.

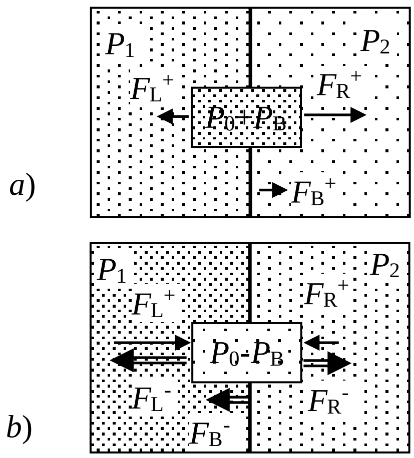

Figure 3. Positive *(a)* and negative *(b)* waves-tools in the domain of a pressure drop

In this case, the force acting on the left wall of the positive wave-tool (a Fig. 2 *a*), will be:

$$F_L = SP_L = S[P_1 - (P_0 + P_T)], \tag{3}$$

And the force acting on the right wall of the positive wave-tool:

$$F_R = SP_R = S[(P_0 + P_T) - P_2]. \tag{4}$$

The resultant force acting on the positive wave-tool:

$$\begin{aligned} F_T = F_L + F_R = S[P_1 - (P_0 + P_T)] + \\ + S[(P_0 + P_T) - P_2] = S(P_1 - P_2) \end{aligned} . \tag{5}$$

The force acting on the left wall of a negative wave-tool, on continuum surplus (Fig. 2 *b*):

$$F_L^{+} = SP_L = S[P_1 - (P_0 + P_T)], \tag{6}$$

And the force acting at the left on "a deficit of a continuum" created by the negative wave-tool will be guided to the opposite party:

$$F_L^{-} = -SP_L = -S[P_1 - (P_0 + P_T)], \tag{7}$$

Similarly, the force acting on the right on " a deficit of a continuum " created by the negative wave-tool:

$$F_R^{-} = -SP_R = -S[(P_0 + P_T) - P_2]. \tag{8}$$

So, the resultant force acting on a negative wave-tool:

$$\begin{aligned} F_T^{-} = F_L^{-} + F_R^{-} = -S[P_1 - (P_0 + P_T)] - \\ - S[(P_0 + P_T) - P_2] = -S(P_1 - P_2) \end{aligned} . \tag{9}$$

Thus, in the domain of the pressure drop on negative and positive waves act forces, having identical modulo, but opposite direction.

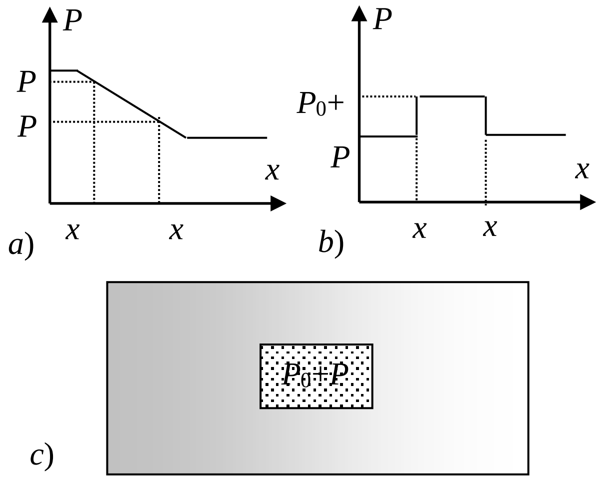

Figure 4. The wave-tool in the domain of a smoothly varying pressure drop.

Now we shall consider the case, when the wave-tool is located in the domain of a smoothly varying pressure drop (Fig. 4*a*). For simplicity we shall be restricted to a situation, when density in area interesting for us, varies under the linear law. The deductions are simple for extending on arbitrary allocation of density by piecewise linear approximation and passage to the limit, when the sites become infinitesimal, and their number will increase ad infinitum. In other words, we can pass from the sum to integral.

So, we suppose that density in area, where there is a wave-tool, varies under the linear law (Fig. 4*a*).

As well as in the previous case, force acting on the left wall of a positive wave-tool:

$$F_L^{+} = SP_L^{+} = S[P_1 - (P_0 + P_T)], \tag{9}$$

And the force acting on right wall of wave-tool:

$$F_R^{+} = SP_R^{+} = S[(P_0 + P_T) - P_2]. \tag{10}$$

The resultant force acting on the positive wave-tool:

$$\begin{aligned} F_T^{+} = F_L^{+} + F_R^{+} = S[P_1 - (P_0 + P_T)] + \\ + S[(P_0 + P_T) - P_2] = S(P_1 - P_2) \end{aligned} . \tag{11}$$

The forces acting from the left and from the right on the each intermediate plane between *x1* and *x2* will be compensated reciprocally.

If to compare expressions (5) and (11) with (9), we can see, that the force acting on the negative wave-tool in similar conditions will be:

$$F_T^{\,-} = F_L^{\,-} + F_R^{\,-} = -S[P_1 - (P_0 + P_T)] - \\ - S[(P_0 + P_T) - P_2] = -S(P_1 - P_2) \quad (12)$$

Let's designate: $E = \dfrac{P_1 - P_2}{x_1 - x_2} = \dfrac{\Delta P}{\Delta x}$. (13)

$$q^{\pm} = \pm S(x_1 - x_2) = \pm S\Delta x = \pm V. \quad (14)$$

E is a parameter of a pressure drop of a continuum, i.e. the parameter of a field, while q - is own parameter of wave-tool. q^+ characterise the positive wave-tool, and q^- negative.

Taking into account (12) and (13) the expression (11) will be copied:

$$F_T = qE \quad (14)$$

If to pass to the limit,

$$E = \lim_{\Delta x \to 0} \frac{\Delta P}{\Delta x} = \frac{\partial P}{\partial x} \quad (15)$$

Generally at arbitrary orientation of axes in relation to pressure drop

$$\mathbf{E} = \mathrm{grad}P \quad (16)$$

And $\mathbf{F}_T = q\mathbf{E}$ (17)

If to compare the formulas (14), (15) and (16) to the similar formulas of an electrodynamics, it is visible, that q - corresponds to the charge, E - to the electric field, and P - to the scalar potential.

Transformations of *E* and *q* at change of the reference frame. Further we need to view values E and q from various reference frames, therefore we shall spot, how they will be conversed at change of a reference frame. The inhomogeneity can be viewed as a part of a wave. In article [1] we have shown, that the waves are conversing according to Lorentz transformation laws. But if the wave is bodily conversed according to Lorentz transformation laws, it is naturally to expect, as its parts will be converted also according to Lorentz transformation laws. The problem represents interest, how the value q and E will vary at transition from own reference frame to other reference frame. We will take into account, that in our model the unique tools, which the observer possesses, are the waves in the same continuum.

The transformation of *q*. The observer can judge about transformation of q, measuring the change of force acting on q in the area of a pressure drop at transition of the observer from one system in another. As in this case the relativistic reductions of wave-tool volume and drop pressure area are identical, the force acting on q, will be invariant concerning a transformation of coordinates.

The transformation of *E*. In this case the observer uses q as the tool for measuring E. Passing from system in system he keep with himself the tool, therefore q does not support the relativistic contraction.

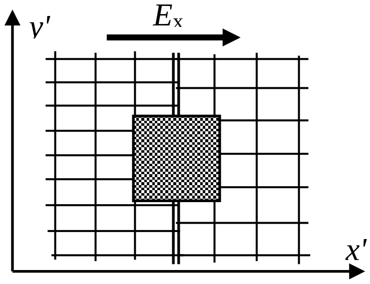

Figure 5 *a*. The component E_x of a field from the point of view of a laboratory reference frame.

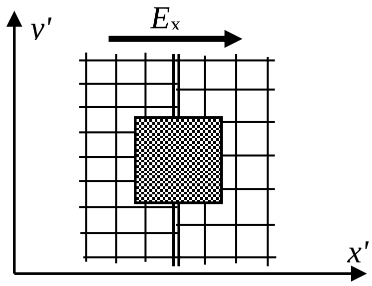

Figure 5 *b*. The component Ex' of a field from the point of view of system moving along the axis

Let's considers a situation given in figure 3. Following the idea expressed in article [1] we use the coordinate net, which consists from waves and represents standing waves having reciprocally perpendicular wave vectors. In this case as coordinates will serve the wave vectors, as standard of length will be the wave length, and as times standards - the period. 5 *a*. Let's consider, that everyone brick contains the same quantity of a continuum.

If the observer will transfer in the reference frame moving concerning laboratory system, from his point of view will take place the contraction of scale i.e. contraction of net of laboratory frame (fig. 5 *b*). However area S, perpendicular to axis x, on which the interaction of the wave-tool with a continuum takes place remains constant. Therefore the component Ex remains constant at transition from a laboratory reference frame to system of the moving observer. That is $E_x{}' = E_x$.

Now we shall assume, that the pressure drop along an axis y (fig. 6 *a*) takes place. If observer pass from laboratory system to moving system, there will be contraction of laboratory system scale along the axis x in γ times (fig. 6 *b*). Therefore, the wave-tool will adjoin already to greater number of bricks. It is equivalent to magnification in γ times of effective area S of the wave-tool. Thus, at augmentation of velocity of the observer, the transverse component E_y will increase in γ times (fig. 6 *b*). Hence, and the transverse force F_y, which act on a wave-tool, also will increase in γ times more. $E_y{}' = \gamma E_y$

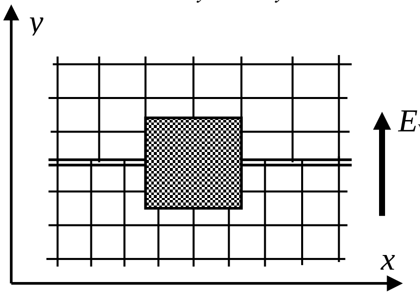

Figure 6 *a*. The component of a field *Ey* from the point of view of laboratory system.

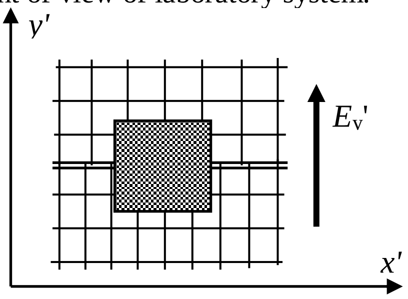

Figure 6 *b*. The component of a field *Ey* from the point of view of system moving along an

2. INTERACTION OF THE WAVE-TOOL WITH A FLUX

As well as in the previous paragraph we shall consider, that the wave-tool represents either surplus, or deficit of a continuum, that is density, in comparison with an equilibrium state. Let's consider now, how the wave-tool with a flux will interact.

As was shown in the quoted article [1], the wave-tool "can not detect" the absolute motion relatively to medium, in which it exist, as all inertial systems for it are equivalent. For wave all the same there it is located in a flux or in "fixed" medium. Therefore it is sense to view reaction of a wave only on difference or drop of velocity of medium. In other words, it is possible to expect, that the wave is capable to detect the border of flux, or, or else, difference of velocity.

At first we shall consider, that border between a flux and "fixed" continuum is sharp. Let's mark considered area by a undular net, as shown in figure 7*a*. We shall suppose, that in laboratory system the flux pressure is equal to pressure in continuum, that is $p_F = p_0$ or $\Delta p_F = p_F - p_0 = 0$. From the point of view of the wave-tool, resting in laboratory system on border, the image will be such, as shown on the fig. 2.1 *a*. If the pressure on both parties of border is identical, and bricks are identical, each brick will contain the same quantity of continuum. Therefore the forces, which act on the wave-tool on the part of a continuum and on the part of a flux, will be identical. Hence, the resulting force acting on a wave-tool will be equal to zero.

$$F_R = F_F - F_C = 0$$

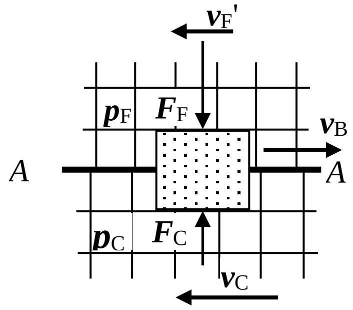

Figure 7 *b*. The image from the point of view of wave-tool, moving in laboratory system with velocity v_B.

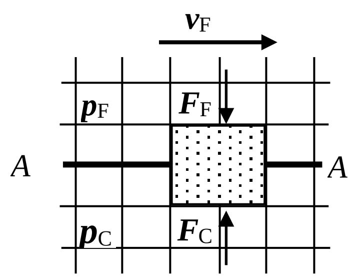

Figure 7 *a*. The image from the point of view of wave-tool, resting in laboratory system.

Let's assume now, that the velocity of the flux in laboratory frame is v_F, and the wave-tool moving along border of a flux with velocity v_B (fig. 2.1b). In the article [1] we have shown, that, in the model, of the world of waves, offered by us, the transition between frames of reference is accomplish in correspondence with Lorentz transformations. The transformation of velocities is executed in correspondence with Lorentz transformation also. In system of the wave-tool, the velocity of a flux v_F' will be result of a relativistic velocity addition of the flux v_F and wave-tool v_T velocities, measured in laboratory frame. Therefore **relativistic contraction of net, bounded with a flux, will differ from contraction of net located in the environmental continuum**. Hence, numbers of "bricks", which adjoin to a moving body on the part of a flux and from the party "fixed" of continuum, will be various.

As the contraction is caused by transition of the observer in system of the wave-tool, but not to any action on continuum or flux, the quantity of continuum contained in everyone brick remains constant. Thus, from "point of view" of the moving wave-tool, the pressure in a flux will not be equal to pressure in an environmental continuum. Or else, in frame of the moving wave-tool take place a pressure drop. As was shown above, the pressure drop in medium cause a resulting force, which act on the wave-tool. In this case force will be guided perpendicularly to border of a flux. If the wave-tool is positive and direction of flux velocity coincides with a direction of wave-tool velocity, the force will be directed to the flux. If the velocities of the wave-tool and flux have opposite directions, the force will have direction from the flux. Let's found the numerical value of this force. We formulate a problem as follows: **Let's spot value and direction of resulting force acting on the wave-tool situated on the border of flux in laboratory system, as function from flux velocity v_F and wave-tool velocity v_B.**

In this problem there are three reference frames:

- the laboratory system;
- the flux system;
- the system of the wave-tool.

Let's mark the following circumstances.

1. The laboratory system is picked so that density of a flux was equal to density of an environmental continuum. It allows excluding influence of static difference of density, which we have considered in the first section.
2. We know the transformation rules for scale in case of transition from a own frame in other system. Therefore, if the moving object is observed by two observers *A* and *B*, for transformation of scale from system *A* in system *B* it is necessary at first to pass from system *A* in own system, and then from own system in system *B*.
3. Two forces can be compared, only if they are conversed to the same frame of reference.

Proceeding from these notes, the algorithm for definition of force acting on the wave-tool situated on border of a flux, will be the following:

- the pressures of the continuum and flux in laboratory system is considered p_0;
- the pressure of the flux in it own system is determined;
- the pressures of a continuum and flux in system of the wave-tool are determined, thus the resulting force is determined which act on the wave-tool in its own system.
- the resulting force acting on the wave-tool, is recalculated in laboratory system.

The coefficients, used in this section, will have the following significance:

- *T* a wave-tool;
- *F* - flux;
- *C* - the continuum - concerns to laboratory system;
- absence of the stroke designate - that the measuring is executed in laboratory system.
- The stroke designates - that the measuring is executed in a flux system;
- Two strokes designate - that the measuring is executed in wave-tool frame;

By a coefficient 0 we shall designate an equilibrium value of pressure. In other words we mark with 0 the values (for example densities or pressures), which take place in any frame of reference at absence of any opportunity of comparison, i.e. at absence in medium of inhomogeneities.

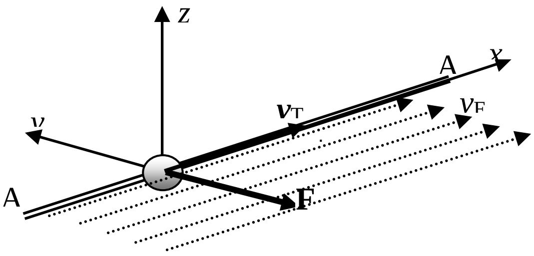

Figure 8. Wave-tool goes along border A-A of a flux.

As well as in the first section, we shall consider that the wave-tool represents either surplus, or deficit of a continuum, i.e. density, in comparison with an equilibrium state. Let the wave-tool goes along border of the flux, and the direction of the wave-tool velocity coincides with a direction of velocity of a flux (fig. 8).

If from the point of view of laboratory system the flux pressure is equilibrated by environmental continuum, i.e. $p_F = p_0$, hence, $\Delta p_F = p_F - p_0 = 0$. Therefore resulting pressure acting on a wave-tool, located on border of a flux and resting in laboratory system, will be equal to zero.

Pressure of a flux in own frame:

$$p_F' = \frac{p_F}{\gamma_F} = p_F\sqrt{1-\beta_F^2} = p_0\sqrt{1-\beta_F^2} \tag{18}$$

Where $\beta_F = \frac{v_F^2}{c^2}$ and $\gamma_F = \frac{1}{\sqrt{1-(\beta_F)^2}}$ (19)

Let wave-tool goes along flux border with velocity v_T. We pass in a frame of a wave-tool. Velocity of a continuum in wave-tool frame:

$$v_C' = -v_T\,. \tag{20}$$

Velocity of a flux in system of a wave-tool

$$v_F'' = \frac{v_F - v_T}{1-\frac{v_B v_T}{c^2}}, \tag{21}$$

or

$$\beta_F'' = \frac{\beta_F - \beta_T}{1-\beta_F\beta_T} \tag{22}$$

The pressure of a flux in system of a wave-tool is more than pressure of a flux in own system, thus coefficient of contraction of length, and, hence, and augmentation of pressure, will be:

$$\gamma_F'' = \frac{1}{\sqrt{1-(\beta_F'')^2}}. \qquad (23)$$

We converse (23) considering the (22):

$$\gamma_F'' = \frac{1}{\sqrt{1-\left(\frac{\beta_F-\beta_T}{1-\beta_F\beta_T}\right)^2}} = \frac{1-\beta_F\beta_T}{\sqrt{(1-\beta_F\beta_T)^2-(\beta_F-\beta_T)^2}} \qquad (24)$$

Thus, flux pressure in system of a wave-tool:

$$p_F'' = \gamma_F'' p_F'$$

Or, considering (18):

$$p_F'' = \gamma_F'' \frac{p_F}{\gamma_F} = p_0 \frac{\gamma_F''}{\gamma_F}. \qquad (25)$$

Pressure of a continuum in the wave-tool system:

$$p_C'' = \gamma_C'' p_C = \gamma_C'' p_0, \qquad (26)$$

where: $\gamma_C'' = \frac{1}{\sqrt{1-(\beta_C'')^2}}$,

but as: $\beta_C'' = \frac{v_C}{c} = -\frac{v_B}{c} = -\beta_T$,

it is possible to note: $\gamma_C'' = \frac{1}{\sqrt{1-(\beta_T)^2}} = \gamma_T$. (27)

The difference of pressures in wave-tool system:

$$p_F'' - p_C'' = p_0\left(\frac{\gamma_F''}{\gamma_F} - \gamma_C''\right) = p_0\left(\frac{\gamma_F''}{\gamma_F} - \gamma_T\right). \qquad (28)$$

It means that if the direction of velocity of the wave-tool coincides with a direction of flux velocity, from the point of view of the wave-tool the continuum will be more squeezed, and the flux less squeezed, than in laboratory system.

Let's substitute in (28) values of coefficients of scale transformation according to expressions (19), (24) and (27). We receive:

$$p_F'' - p_C'' = p_0\left(\frac{\gamma_F''}{\gamma_F} - \gamma_T\right) =$$

$$= p_0 \frac{(1-\beta_F\beta_T)\sqrt{1-\beta_F^2}}{\sqrt{1-2\beta_F\beta_T+\beta_F^2\beta_T^2-\beta_F^2+2\beta_F\beta_T-\beta_T^2}} -$$

$$- p_0 \frac{1}{\sqrt{1-\beta_T^2}} = -p_0 \frac{\beta_F\beta_T}{\sqrt{1-\beta_T^2}}$$

Finally: $p_F'' - p_C'' = -p_0\gamma_T\beta_F\beta_T$ (29)

The expression (29) gives the difference of the pressures acting from the flux and continuum on section S of the wave-tool and measured in system of the wave-tool. S coincides with border of a flux. Or else, this is resulting pressure in wave-tool system. The resulting force in the wave-tool system in this case: $F'' = S\Delta p = -S\, p_0\gamma_B\beta_F\beta_T$.

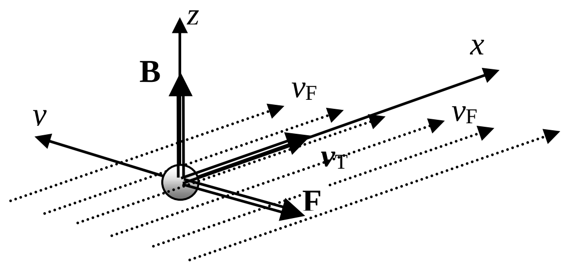

Figure 9. The reciprocal arrangement of vectors $\boldsymbol{v}_T$, **B** and **F.**

In laboratory system the same force will be:

$$F = \frac{F''}{\gamma_B} = -Sp_0\beta_F\beta_T. \qquad (30)$$

Now we assume, that the velocity of a flux varies linearly along the axis y (fig. 9).

Let's designate: $B = \frac{p_0}{c^2}\frac{v_1-v_2}{y_1-y_2} = \frac{p_0}{c^2}\frac{\Delta v}{\Delta y}$ (31)

We use parameter, which we have entered in the first part: $q^{\pm} = \pm S(y_1-y_2) = \pm S\Delta y = \pm V$ (32)

In view of expressions (31) and (32) formula (30) for force acting on the wave-tool will take a view:

$$F = v_T B. \qquad (33)$$

The vector $\boldsymbol{B}$ must be directed on the axis z. In this case: $\mathbf{F} = q(\mathbf{v}_T \times \mathbf{B}) = q[\mathbf{v}_T\mathbf{B}]$. (34)

The sign of q will determine the direction of force in relation to axis y.

CONCLUSIONS

1. The wave-tool interacts with a pressure drop as the electric charge interacts with a free electric field.

2. The interaction of the wave-tool with flux is described by the same formula as interaction of the electrical charge with a magnetic field if to assume, that the value of a vector $\boldsymbol{B}$ is proportional to derivative velocity on spatial coordinate.

The deduction for a case of a motion of the wave-tool under an arbitrary angle relatively to border of flux will be given in the following article.

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