Collective Elementary Excitations of 2D Magnetoexcitons Taking Into Account Excited Landau levels.

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***Abstract* – The collective elementary excitations of the two-dimensional magnetoexcitons in a state of Bose-Einstein condensation (BEC) with wave vector $\vec{k}=0$ were investigated in the frame of the Bogoliubov theory of quasiaverages. The starting Hamiltonian of the electrons and holes lying on the lowest Landau levels (LLLs) contains the supplementary interactions due to the virtual quantum transitions of the particles to the excited Landau levels (ELLs) and return back. As a result the interaction between the magnetoexcitons with $\vec{k}=0$ does not vanish and their BEC becomes stable as regards the collapse. The energy spectrum of the collective elementary excitations consists from two exciton-type branches (energy and quasienergy branches) each of them with energy gap and roton-type section, from the gapless optical plasmon branch and from the acoustical plasmon branch, which reveals the absolute instability in the range of small wave vectors.**

***Index Terms* – Bose-Einstein Condensation, elementary excitations, magnetoexciton, plasmon**

I. INTRODUCTION

Properties of atoms and excitons are dramatically changed in strong magnetic fields, such that the distance between Landau levels $\hbar\omega_c$, exceeds the corresponding Rydberg energies R_y and the magnetic length $l=\sqrt{\hbar c/eH}$ is small compared to their Bohr radii [1,2]. Even more interesting phenomena are exhibited in the case of two-dimensional (2D) electron systems due to the quenching of the kinetic energy at high magnetic fields, with the representative example being integer and fractional Quantum Hall effects [3-5]. The discovery of the FQHE [6-8] changed fundamentally the established concepts about charged elementary excitations in solids [5]. The notion of the incompressible quantum liquid (IQL) was introduced in Ref.[7] as a homogeneous phase with the quantized densities $v=p/q$, where p is an integer and $q\neq 1$ is odd having charged elementary excitations with a fractional charge $e^*=\pm e/q$. These quasiparticles were named as anyons. A classification for free anyons and their hierarchy were studied in [9,10]. An alternative concept to hierarchical scheme was proposed in [11], where the notion of composite fermions (CF) was introduced. The CF consists from the electron bound to an even number of flux quanta. In the frame of this concept the FQHE of electrons can be physically understood as a manifestation of the IQHE of CFs [11]. The statistics of anyons was determined in [10,12]. It was established that the wave function of the system changes by a complex phase factor $\exp[i\pi\alpha]$, when the quasiparticles are interchanged. For bosons $\alpha=0$, for fermions $\alpha=1$ and for anyons with $e^*=-e/3$ their statistical charge is $\alpha=-1/3$. As was shown in Ref.[13], there were no soft branches of neutral excitations in IQL. The energy gap Δ for formation of a quasielectron-quasihole pair has the scale of Coulomb energy $E_Q=e^2/\varepsilon l$, where ε is the dielectric constant of the background. However delta was found to be small $\Delta \approx 0.1E_Q$. The lowest branch was called as magnetoroton [13] and can be modelled as a quasiexciton [5]. As was mentioned in [5] the traditional methods and concepts based either on the neglecting of the electron-electron interaction or on self-consistent approximation are inapplicable to IQL. In a strong magnetic field the binding energy of an exciton increases from R_y to I_l.

There are two small parameters of the theory. One of them determines how strong the magnetic field strength H is, and it verifies whether the starting supposition of a strong magnetic field is fulfilled. This parameter is expressed by the ratio $I_l/\hbar\omega_c<1$. Here I_l is the magnetoexciton ionization potential, ω_c is the cyclotron frequency $eH/\mu c$ calculated with the reduced mass μ and the magnetic length l. Another small parameter has a completely different origin and is related with the concentration of the electron-holes(e-h) pairs. In our case it can be expressed as a product of the filling factor $v=\mathrm{v}^2$ and of another factor $(1-\mathrm{v}^2)$ which reflects the Pauli exclusion principle and the phase-space filing(PSF) effect. This compound parameter $\mathrm{v}^2(1-\mathrm{v}^2)$ in the case of Bose-Einstein condensed excitons can take the form $u^2\mathrm{v}^2$, where u, v are Bogoliubov transformation coefficients and $u^2=(1-\mathrm{v}^2)$. The both small parameters will be used below. But in the case of FQHE the filling factor $v=\mathrm{v}^2$ basically determines the underlying physics and it can not be changed arbitrarily. Instead of the perturbation theory on the filling factor ν the exact numerical diagonalization for a few number of particles $N \le 10$ proved to be the most powerful tool in studies of such systems [5]. The spherical geometry for these calculations was proposed [10, 14], considering a few number of particles on the surface of a sphere with the radius $R = \sqrt{S}l$, so as the density of the particles on the sphere to be equal with the filling factor of 2DEG. The magnetic monopole in the center of the sphere creates a magnetic flux through the sphere $2S\phi_0$, which is multiple to the flux quantum $\phi_0 = 2\pi\hbar c/e$. The angular momentum L of a quantum state on the sphere and the quasimomentum k of the FQHE state on the plane obey the relation $L = Rk$. Spherical model is characterized by continuous rotational group, which is analogous with the continuous translational symmetry in the plane.

Properties of the symmetric 2D electron-hole (e-h) system (i.e. $h = 0$), with equal concentrations for both components, with coincident matrix elements of Coulomb electron-electron, hole-hole and electron-hole interactions in a strong perpendicular magnetic field also attracted a great attention during last two decades [15-22]. A hidden symmetry and the multiplicative states were discussed in many papers [19, 23, 24]. The collective states such as the Bose-Einstein condensation (BEC) of two-dimensional magnetoexcitons and the formation of metallic-type electron-hole liquid (EHL) were investigated in [15-22]. The search for Bose-Einstein condensates has became a milestone in the condensed matter physics [25]. The remarkable properties of super fluids and superconductors are intimately related to the existence of a bosonic condensate of composite particles consisting of an even number of fermions. In highly excited semiconductors the role of such composite bosons is taken on by excitons, which are bound states of electrons and holes. Furthermore, the excitonic system has been viewed as a keystone system for exploration of the BEC phenomena, since it allows to control particle densities and interactions *in situ*. Promising candidates for experimental realization of such system are semiconductor quantum wells (QWs) [26], which have a number of advantages compared to the bulk systems. The coherent pairing of electrons and holes occupying only the lowest Landau levels (LLLs) was studied using the Keldysh-Kozlov-Kopaev method and the generalized random-phase approximation [20, 27]. The importance of the excited Landau levels (ELLs) and their influence on the ground states of the systems was first noticed by the authors of the papers [16-19]. The influence of the excited Landau levels (ELLs) of electrons and holes was discussed in details in paper [21, 22]. The indirect attraction between electrons (e-e), between holes (h-h) and between electrons and holes (e-h) due to the virtual simultaneous quantum transitions of the interacting charges from LLLs to the ELLs is a result of their Coulomb scattering. The first step of the scattering and the return back to the initial states were described in the second order of the perturbation theory.

II. HAMILTONIAN OF THE SUPPLEMENTARY INTERACTION

The Hamiltonian of the Coulomb interaction of the electrons and holes in the frame of lowest Landau levels(LLLs) has the form:

$$\hat{H} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}\left[\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \hat{N}_e - \hat{N}_h\right] - \mu_e\hat{N}_e - \mu_h\hat{N}_h + \hat{H}_{\text{suppl}} \quad (1)$$

where $W_{\vec{Q}}$ is the Fourier transform of the Coulomb interaction in the frame of LLLs, $\hat{N}_e$ and $\hat{N}_h$ are the operators of the numbers of electrons and holes on the LLLs. They are determined below. $\hat{H}_{\text{suppl}}$ is the supplementary indirect attractive interaction between the particle lying on the lowest Landau levels(LLLs) in view of their virtual transitions on the excited Landau levels(ELLs) and their return back [22]:

$$H_{\text{suppl}} = -\frac{1}{2}\sum_{p,q,s} \phi_{e-e}(p,q;s)a_p^\dagger a_q^\dagger a_{q+s}a_{p-s} - \frac{1}{2}\sum_{p,q,s} \phi_{h-h}(p,q;s)b_p^\dagger b_q^\dagger b_{q+s}b_{p-s} - \sum_{p,q,s} \phi_{e-h}(p,q;s)a_p^\dagger b_q^\dagger b_{q+s}a_{p-s} \quad (2)$$

Here the creation and annihilation operators $a_p^\dagger, a_p$ for electrons and $b_q^\dagger, b_q$ for holes were introduced. The matrix elements of indirect interaction $\phi_{i-j}(p,q,z)$ are described by the common expressions [22]

$$\phi_{i-j}(p,q,s) = \sum_{n,m}\frac{\phi_{i-j}(p,q,z;n,m)}{n\hbar\omega_{ci} + m\hbar\omega_{cj}} \quad (3)$$

In the case of electron-electron and hole-hole interaction the expression (3) has the form[22]:

$$\phi_{i-i}(p,q,z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,k}W_{z-t,\sigma}\exp\left(i\kappa(p-q-t)l^2\right)\times \exp\left(i\sigma(p-q-t-z)l^2\right)(t+i\kappa)^{n+m}(t-z+i\sigma)^{n+m} \quad (4)$$

but in the case of electron-hole interaction is:

$$\phi_{e-h}(p,q,z;n,m) \cong \sum_{t,\kappa,\sigma} W_{t,k}W_{z-t,\sigma}\exp\left(i(\kappa+\sigma)(p+q)l^2\right)\times (t+i\kappa)^n(t-i\kappa)^m(t-z+i\sigma)^n(t-z-i\sigma)^m, \quad (5)$$

where

$$W_{s,\kappa} = \frac{2\pi e^2}{\varepsilon_0 S\sqrt{s^2+k^2}}e^{-\frac{\left(s^2+k^2\right)l^2}{2}}, \quad W_{s,k} = W_{-s,-k} = W_{-s,k} = W_{s,-k} \quad (6)$$

The Hamiltonian of supplementary indirect attractive interaction (2) has the form:

$$H_{\text{suppl}} = \frac{1}{2}B_{i-i}N - \frac{1}{2N}\sum_{s,\sigma}\psi_{i-i}(s,\sigma)\times \left[\rho_e(-s,-\sigma)\rho_e(s,\sigma) + \rho_h(-s,-\sigma)\rho_h(s,\sigma)\right] - \frac{1}{N}\sum_{s,\sigma}\psi_{e-h}(s,\sigma)\rho_e(-s,-\sigma)\rho_h(-s,-\sigma) \quad (7)$$

Instead of density operators for electrons and holes we can introduce their in-phase and in opposite-phase linear combinations

$$\hat{\rho}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) - \hat{\rho}_h(-\vec{Q});$$
$$\hat{D}(\vec{Q}) = \hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q});$$
$$\hat{\rho}_e(\vec{Q}) = \frac{1}{2}\left[\hat{\rho}(\vec{Q}) + \hat{D}(\vec{Q})\right]; \qquad (8)$$
$$\hat{\rho}_h(\vec{Q}) = \frac{1}{2}\left[\hat{D}(-\vec{Q}) - \hat{\rho}(-\vec{Q})\right]$$

They lead to the following relations

$$\hat{\rho}_e(-\vec{Q})\hat{\rho}_e(\vec{Q}) + \hat{\rho}_h(-\vec{Q})\hat{\rho}_h(\vec{Q}) =$$
$$= \frac{1}{2}\left[\hat{\rho}(-\vec{Q})\hat{\rho}(\vec{Q}) + \hat{D}(-\vec{Q})\hat{D}(\vec{Q})\right];$$
$$\sum_Q \psi_{e-h}(Q)\left[\hat{\rho}(-\vec{Q})\hat{D}(\vec{Q}) - \hat{D}(-\vec{Q})\hat{\rho}(\vec{Q})\right] =$$
$$= \sum_Q \psi_{e-h}(Q)\left[\hat{\rho}(-\vec{Q})\hat{D}(\vec{Q}) - \hat{D}(\vec{Q})\hat{\rho}(-\vec{Q})\right] = 0;$$

and to the final expression

$$H_{\text{suppl}} = \frac{1}{2}B_{i-i}N - \frac{1}{4N}\sum_Q V(Q)\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q})$$
$$-\frac{1}{4N}\sum_Q U(Q)\hat{D}(\vec{Q})\hat{D}(-\vec{Q}) \qquad (9)$$

where

$$U(Q) = \psi_{i-i}(Q) + \psi_{e-h}(Q);$$
$$V(Q) = \psi_{i-i}(Q) - \psi_{e-h}(Q); \qquad (10)$$
$$\psi_{i-j}(s,\sigma) = \sum_\kappa \phi_{i-j}(s,\kappa)\exp(i\kappa\sigma l^2)$$

The estimations show that

$$U(0) = 2A_{i-i};\ \ V(0) = 0;$$
$$\frac{1}{N}\sum_{\vec{Q}} U(\vec{Q}) = B_{i-i} + \Delta(0)$$

It means that one can suppose the dependences

$$U(\vec{Q}) \cong U(0)e^{-\frac{Q^2l^2}{2}};\quad V(\vec{Q}) \cong V(0) = 0 \qquad (11)$$

III. BOSE-EINSTEIN CONDENSATION OF MAGNETOEXCITONS IN TWO ALTERNATIVE DESCRIPTIONS

Bose-Einstein condensation(BEC) of 2D magnetoexcitons was considered in Ref.[20, 21] in the frame of Keldysh-Kozlov-Kopaev method [27], when the influence of the ELLs was neglected. The main results of this description will be remembered below.

The creation $d^\dagger(\vec{P})$ and annihilation $d(\vec{P})$ operators of the 2D magnetoexciton have the form:

$$d^\dagger(\vec{P}) = \frac{1}{\sqrt{N}}\sum_t e^{-iP_y t l^2} a^\dagger_{t+\frac{P_x}{2}} b^\dagger_{-t+\frac{P_x}{2}};$$
$$d(\vec{P}) = \frac{1}{\sqrt{N}}\sum_t e^{iP_y t l^2} b_{-t+\frac{P_x}{2}} a_{t+\frac{P_x}{2}}; \qquad (12)$$

The energy of the two-dimensional magnetoexciton $E_{ex}(P)$ depends on the two-dimensional wave vector $\vec{P}$ and forms a band with the dependence

$$E_{ex}(\vec{P}) = -I_{ex}(\vec{P}) = -I_l + E(\vec{P});$$
$$I_{ex}(\vec{P}) = I_l e^{-\frac{P^2l^2}{4}} I_0\left(\frac{P^2l^2}{4}\right);\qquad I_l = \frac{e^2}{\varepsilon_0 l}\sqrt{\frac{\pi}{2}};\ \sum_{\vec{Q}} W_{\vec{Q}} = I_l \qquad (13)$$

To introduce the phenomenon of Bose-Einstein condensation (BEC) of excitons the gauge symmetry of the initial Hamiltonian was broken by the help of the unitary transformation $\hat{D}(\sqrt{N_{ex}})$ following the Keldysh-Kozlov-Kopaev method [27]. We can shortly remember the main outlines of the Keldysh-Kozlov-Kopaev method [27], [33] as it was realized in the papers [20, 21]. The unitary transformation $\hat{D}(\sqrt{N_{ex}})$ was determined by the formula (8) [20]. Here N_{ex} is the number of condensed excitons. It transforms the operators a_p, b_p to another ones α_p, β_p, as is shown in the formulas (13), (14) [20], and gives rise to the BCS-type wave function $|\psi_g(\bar{k})\rangle$ of the new coherent macroscopic state represented by the expression (10) [20]. These results are summarized below

$$\hat{D}(\sqrt{N_{ex}}) = \exp[\sqrt{N_{ex}}(d^\dagger(\bar{k}) - d(\bar{k}))]$$
$$|\psi_g(\bar{k})\rangle = \hat{D}(\sqrt{N_{ex}})|0\rangle$$
$$\alpha_p = \hat{D}a_p\hat{D}^\dagger = ua_p - \mathrm{v}(p - \frac{k_x}{2})b^\dagger_{k_x-p}$$
$$\beta_p = \hat{D}b_p\hat{D}^\dagger = ub_p + \mathrm{v}(\frac{k_x}{2} - p)a^\dagger_{k_x-p} \qquad (14)$$
$$a_p = u\alpha_p + \mathrm{v}(p - \frac{k_x}{2})\beta^\dagger_{k_x-p}$$
$$b_p = u\beta_p - \mathrm{v}(\frac{k_x}{2} - p)\alpha^\dagger_{k_x-p}$$

$$a_p|0\rangle = b_p|0\rangle = 0;$$
$$\alpha_p|\psi_g(\bar{k})\rangle = \beta_p|\psi_g(\bar{k})\rangle = 0$$
$$u = \cos g;\ \ \mathrm{v} = \sin g;\qquad \mathrm{v}(t) = \mathrm{v}e^{-ik_y t l^2} \qquad (15)$$
$$g = \sqrt{2\pi l^2 n_{ex}};\ \ n_{ex} = \frac{N_{ex}}{S} = \frac{\mathrm{v}^2}{2\pi l^2}\ \ g = \mathrm{v};\ \ \mathrm{v=Sinv};$$

The developed theory [20, 21] is true in the limit $\mathrm{v}^2 \approx Sin^2\mathrm{v}$, what means the restriction $\mathrm{v}^2 < 1$. In the frame of this approach the collective elementary excitations can be studied constructing the Green's functions on the base of operators α_p, β_p and having deal with the transformed cumbersome Hamiltonian $\hat{\hat{H}} = D(\sqrt{N_{ex}})\hat{H}D^\dagger(\sqrt{N_{ex}})$.

IV. EQUATIONS OF MOTION FOR THE TWO-PARTICLE OPERATORS AND FOR THE CORRESPONDING GREEN'S FUCTIONS

The starting Hamiltonian in the quasiaverages theory approximation has the form

$$\hat{H} = \frac{1}{2}\sum_{\vec{Q}} W_{\vec{Q}}\left[\rho(\vec{Q})\rho(-\vec{Q}) - \hat{N}_e - \hat{N}_h\right] - \mu_e\hat{N}_e - \mu_h\hat{N}_h -$$
$$-\tilde{\eta}\sqrt{N}\left(e^{i\varphi}d^\dagger(k) + e^{-i\varphi}d(k)\right) + \frac{1}{2}B_{i-i}N - \qquad (16)$$
$$-\frac{1}{4N}\sum_Q V(Q)\hat{\rho}(\vec{Q})\hat{\rho}(-\vec{Q}) - \frac{1}{4N}\sum_Q U(Q)\hat{D}(\vec{Q})\hat{D}(-\vec{Q})$$

The density fluctuation operators (24) with different wave vectors P and Q do not commute, which is related with the helicity or spirality accompanying the presence of the strong magnetic field [18]. They are expressed by the phase factors in the structure of operators (6) and by the vector-product of two 2D wave vectors P and Q and its projection on the direction of the magnetic field. These properties considerably influence structure of the equations of motion for the operators and determine new aspects of the 2D electron-hole (e-h) physics.

The equation of motion for the creation and annihilation operators $d^{+}(\vec{P}), d(\vec{P})$ (12) and for the density fluctuation operators (8) will be deduced, when the BEC takes place on the state $k=0$. They are:

$$i\hbar\frac{d}{dt}d(\vec{P})=[d(\vec{P}),\hat{H}]=(-\bar{\mu}+E(\vec{P})-\Delta(\vec{P}))d(\vec{P})-$$
$$2i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\hat{\rho}(\vec{Q})d(\vec{P}-\vec{Q})-$$
$$-\frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)D(\vec{Q})d(\vec{P}-\vec{Q})$$
$$-\tilde{\eta}\sqrt{N}e^{i\varphi}\delta_{kr}(\vec{P},0)+\tilde{\eta}e^{i\varphi}\frac{D(\vec{P})}{\sqrt{N}}; \qquad (17)$$
$$i\hbar\frac{d}{dt}d^{\dagger}(-\vec{P})=[d^{\dagger}(-\vec{P}),\hat{H}]=(\bar{\mu}-E(-\vec{P})+\Delta(-\vec{P}))d^{\dagger}(-\vec{P})+$$
$$+2i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^{\dagger}(-\vec{P}-\vec{Q})\hat{\rho}(-\vec{Q})+$$
$$+\frac{1}{N}\sum_{\vec{Q}}U(\vec{Q})Cos\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)d^{\dagger}(-\vec{P}-\vec{Q})D(-\vec{Q})$$
$$+\tilde{\eta}\sqrt{N}e^{-i\varphi}\delta_{kr}(\vec{P},0)-\tilde{\eta}e^{-i\varphi}\frac{D(\vec{P})}{\sqrt{N}};$$
$$i\hbar\frac{d}{dt}\hat{\rho}(\vec{P})=[\hat{\rho}(\vec{P}),\hat{H}]=$$
$$=-i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{\rho}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})+\hat{\rho}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})]+$$
$$+\frac{i}{2N}\sum_{\vec{Q}}U(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)\left[D(\vec{P}-\vec{Q})D(\vec{Q})+D(\vec{Q})D(\vec{P}-\vec{Q})\right];$$
$$i\hbar\frac{d}{dt}\hat{D}(\vec{P})=[\hat{D}(\vec{P}),\hat{H}]=$$
$$-i\sum_{\vec{Q}}\tilde{W}(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{\rho}(\vec{Q})\hat{D}(\vec{P}-\vec{Q})+\hat{D}(\vec{P}-\vec{Q})\hat{\rho}(\vec{Q})]+$$
$$+\frac{i}{2N}\sum_{\vec{Q}}U(\vec{Q})Sin\left(\frac{[\vec{P}\times\vec{Q}]_z l^2}{2}\right)[\hat{D}(\vec{Q})\hat{\rho}(\vec{P}-\vec{Q})+\hat{\rho}(\vec{P}-\vec{Q})\hat{D}(\vec{Q})]$$
$$+2\tilde{\eta}\sqrt{N}\left[e^{-i\varphi}d(\vec{P})-e^{i\varphi}d^{\dagger}(-\vec{P})\right];$$

Following the equations of motion (49) we will introduce four interconnected retarded Green's functions at $T=0$ [28, 29]

$$G_{11}(\vec{P},t)=\left\langle\left\langle d(\vec{P},t);\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle;$$
$$G_{12}(\vec{P},t)=\left\langle\left\langle d^{\dagger}(-\vec{P},t);\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle;$$
$$G_{13}(\vec{P},t)=\left\langle\left\langle \frac{\hat{\rho}(\vec{P},t)}{\sqrt{N}};\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle; \qquad (18)$$
$$G_{14}(\vec{P},t)=\left\langle\left\langle \frac{\hat{D}(\vec{P},t)}{\sqrt{N}};\hat{X}^{\dagger}(\vec{P},0)\right\rangle\right\rangle;$$

The average $\langle\ \rangle$ will be calculated at $T=0$ in HFB approximation using the ground state wave function $\left|\psi_g(k)\right\rangle$ (14).

The Green's functions (18) will be named as one-operator Green's functions because they contain in the left hand side of the vertical line only one summary operator of the types $d(P)$, $d^{\dagger}(P)$, $\hat{\rho}(P)$ and $\hat{D}(P)$. At the same time these Green's functions are two-particle Green's functions, because the summary operators are expressed through the products of two Fermi operators. In this sense the Green's functions (18) are equivalent with the two-particle Green's functions introduced by Keldysh and Kozlov in their fundamental paper [27], forming the base of the theory of high density excitons in the electron-hole description. But in difference on [27] we are using the summary operators, which represent integrals on the wave vectors of relative motions.

The equations of motion for the Green's functions in a special case, when the BEC of magnetoexcitons takes place on the state with $k=0$, are:

$$(\hbar\omega+i\delta+\bar{\mu}-E(P)+\Delta(P))G_1(P,\omega)=C$$
$$-2i\sum_{Q}\tilde{W}(Q)Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle\rho(Q)d(P-Q)\middle|X\right\rangle\right\rangle_{\omega}-$$
$$-\frac{1}{N}\sum_{Q}U(Q)Cos\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle D(Q)d(P-Q)\middle|X\right\rangle\right\rangle_{\omega}+\tilde{\eta}G_4(P,\omega)e^{i\varphi};$$
$$(\hbar\omega+i\delta-\bar{\mu}+E(-P)-\Delta(-P))G_2(P,\omega)=C$$
$$+2i\sum_{Q}\tilde{W}(Q)Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle d^{\dagger}(-P-Q)\rho(-Q)\middle|X\right\rangle\right\rangle_{\omega}+$$
$$+\frac{1}{N}\left\langle\left\langle d^{\dagger}(-P-Q)D(-Q)\middle|X\right\rangle\right\rangle_{\omega}-\tilde{\eta}G_4(P,\omega)e^{-i\varphi};$$
$$(\hbar\omega+i\delta)G_3(P,\omega)=C-i\sum_{Q}\tilde{W}(Q)Sin\left(\frac{[P\times Q]_z l^2}{2}\right)$$
$$\times\left\langle\left\langle\frac{\rho(P-Q)\rho(Q)}{\sqrt{N}}+\frac{\rho(Q)\rho(P-Q)}{\sqrt{N}}\middle|X\right\rangle\right\rangle_{\omega}+$$
$$+\frac{i}{2N}\sum_{Q}U(Q)Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle\frac{D(P-Q)D(Q)}{\sqrt{N}}+\frac{D(Q)D(P-Q)}{\sqrt{N}}\middle|X\right\rangle\right\rangle_{\omega};$$
$$(\hbar\omega+i\delta)G_4(P,\omega)=C-i\sum_{Q}\tilde{W}(Q)Sin\left(\frac{[P\times Q]_z l^2}{2}\right)$$
$$\times\left\langle\left\langle\frac{D(Q)\rho(P-Q)}{\sqrt{N}}+\frac{D(P-Q)\rho(Q)}{\sqrt{N}}\middle|X\right\rangle\right\rangle_{\omega}+$$
$$+\frac{i}{2N}\sum_{Q}U(Q)Sin\left(\frac{[P\times Q]_z l^2}{2}\right)\left\langle\left\langle\frac{D(Q)\rho(P-Q)}{\sqrt{N}}+\frac{\rho(P-Q)D(Q)}{\sqrt{N}}\middle|X\right\rangle\right\rangle_{\omega}$$
$$+2\tilde{\eta}\left[e^{-i\varphi}G_1(P,\omega)-e^{i\varphi}G_2(P,\omega)\right];$$

V. DYSON EQUATION AND SELF-ENERGY PARTS

Using Zubarev's procedure [29] for the Green's function we obtain a closed system of Dyson equation for the Green's functions in the forms:

$$\sum_{j=1}^{4}G_{1j}(\vec{P},\omega)\Sigma_{jk}(\vec{P},\omega)=C_{1k}; \quad k=1,2,3,4 \qquad (20)$$

The self-energy parts $\Sigma_{jk}(\vec{P},\omega)$ contain the different average values of the two-operator products. They were calculated using the ground state wave function $\left|\psi_g(0)\right\rangle$ taken with $k=0$ and have the expressions:

$$\left\langle D(\vec{Q})D(-\vec{Q})\right\rangle=4u^2v^2N;$$
$$\bar{\mu}=-\Delta(0)+2v^2(B_{i-i}-2A_{i-i}+\Delta(0));$$
$$\left\langle D(\vec{Q})d(-\vec{Q})\sqrt{N}\right\rangle=\left\langle d^{\dagger}(\vec{Q})D(-\vec{Q})\sqrt{N}\right\rangle=-2uv^3N; \qquad (21)$$
$$\left\langle d(0)\right\rangle=\left\langle d^{\dagger}(0)\right\rangle=uv\sqrt{N}; \quad \tilde{\eta}=-(\Delta(0)+\bar{\mu})v$$

All these averages are extensive values proportional to N or $\sqrt{N}$, they essentially depend on the small parameters of the types $u^2 v^2$ or uv^3, or uv.

The cumbersome dispersion equation is expressed in general form by the determinant equation:

$$\det\left|\Sigma_{ij}(\vec{P},\omega)\right| = 0; \qquad (22)$$

We introduced some simple approximations which allow calculating our complicate equation (22). They are $\Delta(\vec{P}) \approx \Delta(0);\ U(\vec{P}) \cong U(0)e^{-\frac{P^2 l^2}{2}},\ U(0) = 2A_{i-i}$. Following these transformations we obtained results that are shown in the Figures 1, 2, 3. It is spectrum of collective elementary excitations. Three of them are energy branches, whereas another three are quasienergy branches representing the mirror reflection of the energy branches. Between three energy branches two of them are excitonic branches and one of them is the acoustic plasmon branch. One-exciton energy branch has an energy gap due to the attractive Hartree-type interaction terms, which it is needed to be got over during the excitation as well as a roton-type section in the range of the intermediary values of the wave vectors. At higher values of the wave vector its dispersion law tends to saturation. Another two-exciton energy branch is interpreted by us as being the previous one-exciton energy branch accompanied by the excitation of an condensate exciton with wave vector k=0, the extraction of which from the Hartree-type attractive environment needs also energy.

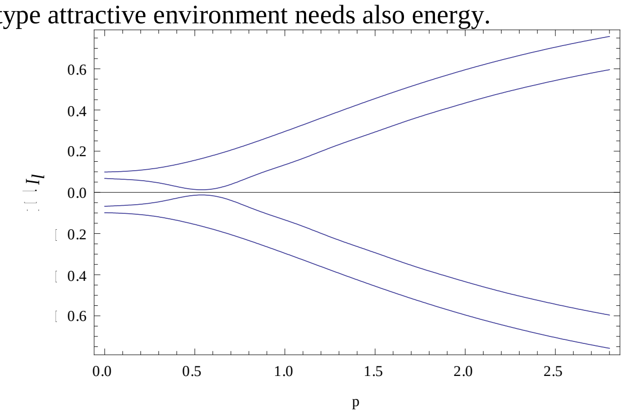

Fig. 1 The exciton branches of the energy spectrum of collective elementary excitations of the Bose-Einstein condensed magnetoexcitons on the wave vector $\vec{k} = 0$, calculated in HFBA, the filling factor $v^2 = 0{,}1$.

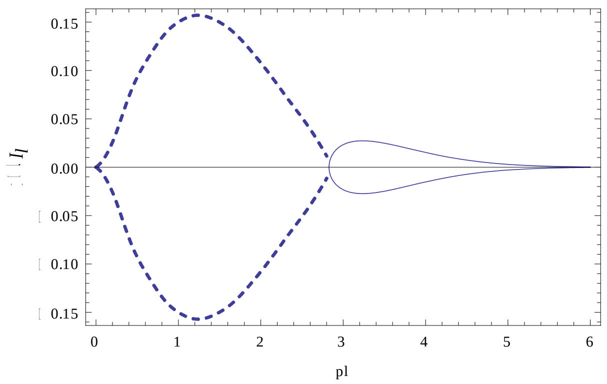

Fig. 2. The dispersion law of acoustical plasmon branch in the presence of the BEC of magnetoexcitons on the wave vector $\vec{k} = 0$, calculated in HFBA, filling factor $v^2 = 0{,}1$.

The third energy branch taking part in this set is the acoustical plasmon branch. It reveals the absolute instability of the spectrum in the range of small and intermediary values of the wave vector k and has a very small real values tending to zero in the limiting case k→∞. The optical plasmon dispersion law is gapless with quadratic dependence in the range of small wave vectors and with saturation-type dependence in the remaining part of the spectrum.

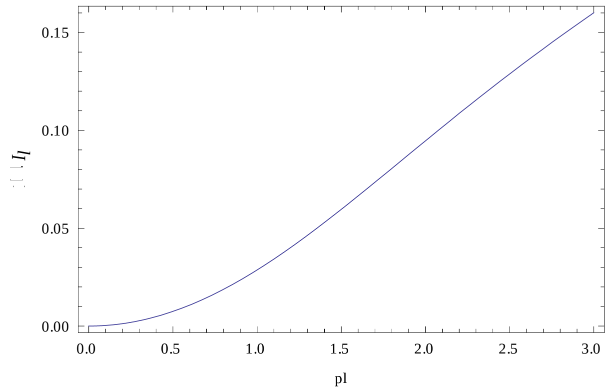

Fig. 3. The dispersion law of optical plasmon branch in the presence of the BEC of magnetoexcitons on the wave vector $\vec{k} = 0$, calculated in HFBA, the filling factor $v^2 = 0{,}1$.

VI. CONCLUSION

The energy spectrum of the collective elementary excitations of a 2D electron-hole (e-h) system situated in a strong perpendicular magnetic field in a state of Bose-Einstein condensation (BEC) with wave vector $\vec{k} = 0$ was investigated in the frame of Bogoliubov theory of quasiaveraes. The starting Hamiltonian describing the e-h system contains not only the Coulomb interaction between the particles lying on the lowest Landau levels, but also the supplementary interaction due to their virtual quantum transitions from the LLLs to the excited Landau levels and return back. This supplementary interaction generates after the averaging on the ground state BCS-type wave function the direct Hartree-type terms with attractive character, the exchange Fock-type terms giving rise to repulsion as well as the similar terms arising after the Bogoliubov $u-v$ transformation. The interplay of these three parameters gives rise to the resulting different from zero interaction between the magnetoexcitons with wave vector $\vec{k} = 0$ and to stability of their BEC as regards the collapse. It influences also on the energy spectrum as well as on the collective elementary excitations. It consists from four branches. Two of them are excitonic-type branches, one of them being the usual energy branch whereas the second one is the quasienergy branch representing the mirror reflection of the energy branch, which will be described below. Another two branches are the optical and acoustical plasmon branches. The exciton energy branch has an energy gap due to the attractive interaction terms, which is needed to be got over during the excitation as well as a roton-type section in the range of intermediary values of the wave vectors. At higher values of wave vector its dispersion law tends to saturation. The optical plasmon dispersion law is gapless with quadratic dependence in the range of small wave vectors and with saturation-type dependence in the remaining part of the spectrum. The acoustical plasmon branch reveals the absolute instability of the spectrum in the range of small and intermediary values of the wave vectors. In the remaining range of the wave vectors the acoustical plasmon branch has a very small real value of the energy spectrum tending to zero in the limiting case of great wave vectors.

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