Two-dimensional Magnetoexciton-polariton in Semiconductor Microcavity

Sveatoslav A. MOSKALENKO[1], Igor V. PODLESNY[1], Michael A. LIBERMAN[2], Boris V. NOVIKOV[3], and Elena S. KISELYOVA[4]

[1]Institute of Applied Physics, ASM, Chisinau, Republic of Moldova
[2]Department of Physics, Uppsala University, Uppsala, Sweden
[3]Department of Solid State Physics, St. Petersburg State University, St. Petersburg, Russia
[4]Moldova State University, Chisinau, Republic of Moldova
exciton@phys.asm.md

***Abstract* — The Hamiltonian describing the interaction of the two-dimensional (2D) magnetoexcitons with photons propagating with arbitrary oriented wave vectors in the three-dimensional (3D) space was deduced. The magnetoexcitons are characterized by the numbers n_e and n_h of the electron and hole Landau quantizations, by circular polarization $\vec{\sigma}_M$ of the holes in the p-type valence bands and by in-plane wave vectors $\vec{k}_{\|}$. The photons are characterized by the wave vectors $\vec{k}$ with in-plane component $\vec{k}_{\|}$ and perpendicular component k_z which is quantized in the case of microresonator. The interaction is governed by the conservation law of the in-plane components $\vec{k}_{\|}$ of the magnetoexcitons and photons and by the rotational symmetry around the axis perpendicular to the layer which leads to the alignment of the magnetoexcitons under the influence of the photons with circular polarization $\vec{\sigma}_{\vec{k}}^{\pm}$ and with probability proportional to $\left|\left(\vec{\sigma}_{\vec{k}}^{\pm}\cdot\vec{\sigma}_M^{*}\right)\right|^2$.**

***Index Terms* — magnetoexciton, polariton, two-dimensional electron-hole system.**

I. INTRODUCTION

The microcavity exciton-polariton Bose-Einstein condensation (BEC) described in the Refs. [1-5] emerged in the last decades as a new promising direction of the exciton BEC in solids [6-9]. The polariton, whose notion was introduced by Hopfield [10], is a quasiparticle in solids with a half-matter and half-light composition. Their properties in microcavities were described in Ref. [11]. The cavity polaritons have integer spin and can reveal bosonic properties responsible for stimulated scattering, polariton lasing, Bose-Einstein condensation (BEC) and superfluidity [12]. In recent years much attention is attracted to the complexity of the phenomena related to the influence of the external magnetic field on the microcavity polaritons [12-17]. There is a rich variety of nonlinear effects induced by magnetic field. One of them is the Zeeman splitting of the exciton eigenstates when the magnetic field is applied in Faraday geometry [12, 13] as well as the suppression of the polariton superfluidity and spin Meissner effect [14, 16]. The evolution of the circularly polarized nonequilibrium Bose-Einstein condensates of spinor-polaritons in the excited spin state at $B < 3\text{T}$, and in the ground spin state at $B > 3\text{T}$ was studied [15]. The TE-TM splitting of the cavity modes [12] and the Faraday rotation of the plane polarization of the light passing through the microcavity was observed experimentally. The change of the relative weights of photonic and excitonic components within the given state depending on the strength of the magnetic field [17] was investigated theoretically. The combined exciton-cyclotron resonance in quantum wells (QWs) was studied in Refs. [18, 19] in a strong magnetic fields exceeding the binding energies of the 2D Wannier-Mott excitons and of magnetoexcitons.

A comprehensive review by Savona et al. [20] gives the theoretical analysis of the optical properties of semiconductor quantum wells (QWs) embedded in a planar Fabri-Perot (F-P) microcavity. F-P resonator is a simplest structure for the confinement of the electromagnetic field (EMF). The planar F-P resonator has the parallel mirrors separated by some dielectric spacer with thickness L_c and refraction index n_c, inside of which the QW with much smaller width L_{QW} may be embedded.

The high finesse semiconductor microcavities with distributed Brag reflectors and feasibility to obtain the needed information were performed. The EMF can exist inside F-P resonator in the condition of a constructive interference between the successive passes of the propagating waves.

Following the Ref. [20] we will consider the two-dimensional (2D) exciton with frequency $\omega_{ex}(\vec{k}_{\|})$ and 2D wave vector $\vec{k}_{\|}$ oriented in the plane of the QW interacting with the photon propagating in the three-dimensional (3D) space with an arbitrary oriented wave vector $\vec{k} = \vec{k}_{\|} + \vec{a}_3 k_z$, where $\vec{a}_3$ is the unit vector perpendicular to the QW plane. The photon energy in the media with refraction index n_c has the dependence $\omega(\vec{k}) = \omega(\vec{k}_{\|}, k_z) = (c / n_c)\sqrt{\vec{k}_{\|}^2 + k_z^2}$, where c is the light velocity in vacuum. Due to the translational symmetry along the QW plane the exciton-photon interaction obeys the conservation law of the in-plane wave vector $k_{\|}$.

The polaritons in microcavity were discussed in Ref. [20]. Their energy spectrum is represented in fig. 2. Recently [12-17] the investigations of the microcavity polaritons under the influence of a magnetic fields were initiated. The external magnetic field is oriented along the axis of the microcavity and it is perpendicular to the surface of the semiconductor quantum well (QW) embedded inside the resonator. Until recently magnetic fields about 2-3 T were used [15]. They are not too high, and the two-dimensional (2D) excitons did not change their electron structure remaining the Wannier-Mott excitons, but with Zeeman and diamagnetic effects. In the presence of a strong magnetic field about 10 T, the electron-hole (e-h) pairs in GaAs QW will form the magnetoexcitons with completely different structure and interaction between them. The Hamiltonian of the electron-radiation interaction in the 2D electron structures of the type GaAs QWs in a strong perpendicular magnetic field describing only the band-to-band quantum transitions was obtained in Ref. [18]. Here the electrons of the p-type valence band situated on the given Landau levels undergo quantum transitions to the s-type conduction band on another levels of Landau quantization. Such Hamiltonian may be interpreted as describing the creation and annihilation of the e-h pairs with definite numbers of Landau levels occupied by the partners of the pair. The transitions are accompanied by the creation or annihilation of the photons.

Contrary to this approach in Ref. [17] was proposed the physical model based on the semiconductor QW but with the intra-band instead of inter-band quantum transitions. In fact these two approaches are complementary each other. The aim of Ref. [17] was to determine the conditions of the Bose-Einstein condensation (BEC) of 2D magnetopolaritons in a trap. For this purpose the Hamiltonian describing the interaction of 2D electrons in the condition of intra-band excitation with the photons confined by the microcavity was derived. The mixed magnetoexciton-photon states named magnetoexciton-polaritons or magnetopolaritons were introduced. But these new states happened to be related in Ref. [17] with the intra-band and not with the inter-band excitations in QWs. It was shown that the effective polariton mass increases with the magnetic field strength B as $B^{1/2}$, whereas the critical temperature of the BEC of intra-band magnetopolaritons in a trap decreases as $T_c \sim B^{-1/4}$. It increases with the increase of the spring constant of the parabolic trap. The constant of the magnetoexciton-photon interaction, which determines the Rabi splitting of the intra-band magnetopolariton branches is proportional to $B^{-1/2}$ in graphene, while in a QW it does not depend on the magnetic field when it is strong [17]. The last result concerning the QW has to be compared with the results of Ref. [18], where the inter-band quantum transitions were considered. In the last paper it was shown and will be demonstrated below that the Rabi frequency is proportional to $B^{1/2}$.

As was mentioned above the apparent contradiction between two papers can be resolved if one takes into account that in Ref. [17] the intra-band quantum transitions in fact were considered. They are relevant for the graphene-type structures with a small band gap in comparison with the cyclotron energy. One can remember and it will be underlined below that the inter-band optical transitions are characterized in the absence of the magnetic field by the matrix element $\vec{P}_{cv}$ calculated with $\vec{A}\vec{\nabla}$ perturbation using the periodic parts of the electron Bloch functions. They determine also the band gap of the semiconductor QW. The "strong" magnetic field is strong only in comparison with the exciton binding energy but it is very weak in comparison with the band gap in GaAs-type QWs. The matrix element P_{cv} will be not changed by the "strong" magnetic field and it must be present in any theoretical calculations of the optical inter-band quantum transitions in semiconductors. Only in the case of intra-band quantum transitions the periodic parts of the same band Bloch functions without $\vec{A}\vec{P}$ perturbation give rise to normalization integral. In this case the $\vec{A}\vec{P}$ perturbation is calculated with the envelope functions determined by the Landau quantization functions with quantum numbers n and n', which differ by unity. Exactly this variant was realized in Ref. [17] and was not considered in Ref. [18]. Instead of it we consider for the first time the magnetoexciton-photon interaction related to the band-to-band optical quantum transitions.

II. HAMILTONIAN OF THE MAGNETOEXCITON-PHOTON INTERACTION

In the Ref. [18] the Hamiltonian of the electron-radiation interaction in the second quantization representation for the case of two-dimensional (2D) coplanar electron-hole (e-h) system in a strong perpendicular magnetic field was discussed. The s-type conduction-band electrons with spin projections $s_z = \pm 1/2$ along the magnetic field direction and the heavy holes with the total momentum projections $j_z = \pm 3/2$ in the p-type valence band were taken into account. Their orbital Bloch wave functions are similar to $(x \pm iy)$ expressions with the orbital momentum projections $M = \pm 1$ on the same selected direction. The Landau quantization of the 2D electrons and holes is described in the Landau gauge with oscillator type motion in one in-plane direction characterized by the quantum numbers n_e and n_h and with the free translational motion described by the uni-dimensional (1D) wave numbers p and q in another in-plane direction perpendicular to the previous one. The electron and hole creation and annihilation operators $a^+_{s_z,n_e,p}$, $a_{s_z,n_e,p}$, and $b^+_{j_z,n_h,q}$, $b_{j_z,n_h,q}$ were introduced correspondingly. The Zeeman effect and the Rashba spin-orbit coupling are not taken into account below.

The electrons and holes have a free orbital motion on the surface of the layer with the area S and are completely confined in $\vec{a}_3$ direction. The degeneracy of their Landau levels equals to $N = S/(2\pi l_0^2)$, where l_0 is the magnetic length. In contrast, the photons are supposed to move in any direction in the three-dimensional (3D) space with the wave vector $\vec{k}$ arbitrary oriented as regards the 2D layer as it is represented in the Fig. 1 reproduced from the Ref. [18]. There are three unit vectors $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, the first two being in-plane oriented whereas the third $\vec{a}_3$ is perpendicular to the layer. We will use the 3D and 2D wave vectors $\vec{k}$ and $\vec{k}_{\|}$ and will introduce the circular polarization vectors $\vec{\sigma}_M$ for the valence electrons, heavy holes and magnetoexcitons as follows

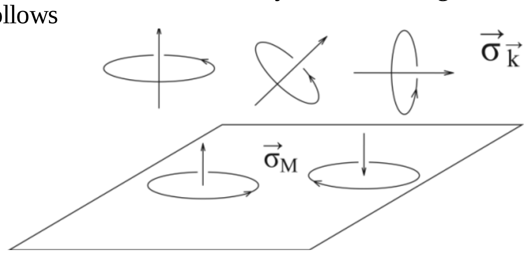

Fig. 1. The reciprocal orientations of the circularly polarized vectors $\vec{\sigma}_{\vec{k}}$ and $\vec{\sigma}_M$, reproduced from the Ref. [18].

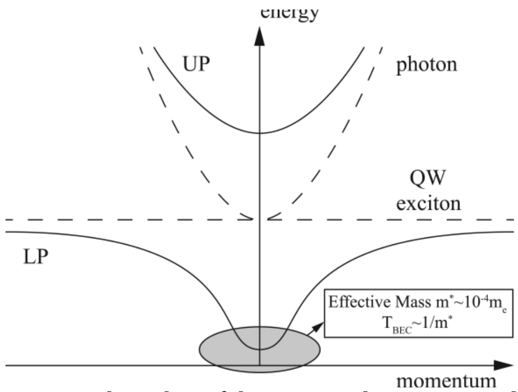

Fig. 2. Two branches of the cavity polaritons, reproduced from the lecture [21].

$$\vec{k} = \vec{k}_{\|} + \vec{a}_3 k_z; \vec{k}_{\|} = \vec{a}_1 k_x + \vec{a}_2 k_y; \vec{\sigma}_M = \frac{1}{\sqrt{2}}(\vec{a}_1 \pm i\vec{a}_2); M = \pm 1.$$

(1)

The photons are characterized by two linear vectors $\vec{e}_{k,j}$ or by two circular polarization vectors $\vec{\sigma}_{\vec{k}}^{\pm}$ obeying the transversality conditions:

$$\vec{\sigma}_{\vec{k}}^{\pm} = \frac{1}{\sqrt{2}}(\vec{e}_{\vec{k},1} \pm i\vec{e}_{\vec{k},2}); (\vec{e}_{k,j} \cdot \vec{k}) = 0; \; j = 1, 2.$$

(2)

The photon creation and annihilation operators can be introduced in two different polarizations as follows

$$C_{\vec{k},\pm}^{\dagger} = \frac{1}{\sqrt{2}}\left(C_{\vec{k},1} \pm iC_{\vec{k},2}\right); \left(C_{\vec{k},\pm}\right)^{\dagger} = \frac{1}{\sqrt{2}}\left(C_{\vec{k},1}^{\dagger} \pm iC_{\vec{k},2}^{\dagger}\right);$$

$$\sum_{j=1}^{2} \vec{e}_{\vec{k},j} C_{\vec{k},j} = C_{\vec{k},-}\sigma_{\vec{k}}^{+} + C_{\vec{k},+}\sigma_{\vec{k}}^{-};$$

$$\sum_{j=1}^{2} \vec{e}_{\vec{k},j} C_{\vec{k},j}^{\dagger} = \left(C_{\vec{k},-}\right)^{\dagger}\sigma_{\vec{k}}^{-} + \left(C_{\vec{k},+}\right)^{\dagger}\sigma_{\vec{k}}^{+}.$$

(3)

The reciprocal orientations of the circular polarizations $\vec{\sigma}_{\vec{k}}^{\pm}$ and $\vec{\sigma}_M$ will determine the values of the scalar products $(\vec{\sigma}_{\vec{k}}^{\pm} \cdot \vec{\sigma}_M^{*})$. The electron-radiation interaction describing only the band-to-band quantum transitions with the participation of the e-h pairs in the presence of a strong perpendicular magnetic field was obtained in Ref. [18] and can be used as initial expression for obtaining the interaction of 2D magnetoexcitons with the electromagnetic field. Following the formula (12) of Ref. [18] we have

$$\hat{H}_{e-rad} = \left(-\frac{e}{m_0}\right) \sum_{\vec{k}(k_x,k_y,k_z)} \sqrt{\frac{2\pi\hbar}{V\omega_k}} \sum_{n_e,n_h} \sum_{p}$$

$$\Big\{ P_{cv}(k_y, p)\Phi\left(n_e, p; n_h, p-k_x; k_y\right)$$

$$\times\Big(\Big[C_{\vec{k},-}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_1\right) + C_{\vec{k},+}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_1\right)\Big] a_{1/2,n_e,p}^{\dagger} b_{-3/2,n_h,k_x-p}^{\dagger} +$$

$$+\Big[C_{\vec{k},-}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_{-1}\right) + C_{\vec{k},+}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_{-1}\right)\Big] a_{-1/2,n_e,p}^{\dagger} b_{3/2,n_h,k_x-p}^{\dagger}\Big) +$$

$$+ P_{cv}^{*}(-k_y, p)\Phi^{*}\left(n_e, p; n_h, p+k_x; -k_y\right)$$

$$\times\Big(\Big[C_{\vec{k},-}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_{-1}\right) + C_{\vec{k},+}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_{-1}\right)\Big] b_{-3/2,n_h,-p-k_x} a_{1/2,n_e,p} +$$

$$+\Big[C_{\vec{k},-}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_1\right) + C_{\vec{k},+}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_1\right)\Big] b_{3/2,n_h,-p-k_x} a_{-1/2,n_e,p}\Big) +$$

$$+ P_{cv}(-k_y, p)\Phi\left(n_e, p; n_h, p+k_x; -k_y\right)$$

$$\times\Big(\Big[\left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_1\right) + \left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_1\right)\Big] a_{1/2,n_e,p}^{\dagger} b_{-3/2,n_h,-p-k_x}^{\dagger} +$$

$$+\Big[\left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_{-1}\right) + \left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_{-1}\right)\Big]$$

$$\times a_{-1/2,n_e,p}^{\dagger} b_{3/2,n_h,-p-k_x}^{\dagger}\Big) +$$

$$+ P_{cv}^{*}(k_y, p)\Phi^{*}\left(n_e, p; n_h, p-k_x; k_y\right)$$

$$\times\Big(\Big[\left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_{-1}\right) + \left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_{-1}\right)\Big] b_{-3/2,n_h,k_x-p} a_{1/2,n_e,p} +$$

$$+\Big[\left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{+}\cdot\vec{\sigma}_1\right) + \left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}_{\vec{k}}^{-}\cdot\vec{\sigma}_1\right)\Big]$$

$$\times b_{3/2,n_h,k_x-p} a_{-1/2,n_e,p}\Big)\Big\}.$$

(4)

Here volume V of the 3D space can be represented as the product $V = SL_z$, where L_z is the size of the 3D space in the direction $\vec{a}_3$. In the case of microcavity $L_z = L_c$. The matrix elements P_{cv} of the band-to-band quantum transition are determined by the formula (A7) of Ref. [18]. In the case of conduction and valence bands of different parities it is assumed to be of allowed type according to the classification of Elliott [22, 23] and does not depend on the wave vectors. The functions $\Phi\left(n_e, p; n_h, p-k_x; k_y\right)$ were determined by the formula (A11) of Ref. [18] and are also listed below

$$P_{cv}(\vec{k}_{\|}, g) = \frac{1}{v_0} \int_{v_0} d\vec{\rho}\, U_{c,s,g}^{*}(\vec{\rho}) e^{ik_y\rho_y} (-i\hbar\frac{\partial}{\partial\rho}) U_{v,p,x,g-k_x}(\vec{\rho});$$

$$\Phi\left(n_e, p; n_h, p-k_x; k_y\right) = e^{ik_y p l_0^2}\Phi\left(n_e, n_h; \vec{k}_{\|}\right);$$

$$\Phi\left(n_e, n_h; \vec{k}_{\|}\right) = \int_{-\infty}^{+\infty} dy \varphi_{n_e}^{*}(y)\varphi_{n_h}\left(y + k_x l_0^2\right) e^{ik_y y},$$

(5)

where $\varphi_{n_e}(y)$ and $\varphi_{n_h}(y)$ are the Landau quantization functions, whereas the functions $U_{c,s,g}(\rho)$ and $U_{v,p,x,g-k_x}(\rho)$ are the periodic parts of the electron Bloch functions in the conduction and valence bands. The last integral in the case $\vec{k}_{\|} = 0$ is the normalization or orthogonality integral. The dipole active transitions ($\vec{k}_{\|} = 0$) take place only in the case when $n_e = n_h$. It means that the 2D magnetoexciton can be created in the dipole-active transition only if it is constructed by the electron and hole on the Landau levels with the same quantum numbers $n_e = n_h$. In other words, the valence electron from the Landau level of quantization with a given number n_v can be excited by light in the conduction band only on the level of Landau quantization with the same number $n_c = n_v$. It is true only for the dipole-active transitions. In the case of quadrupole-active transitions when the amplitudes of the quantum transitions (5) are proportional to the projections of the wave vector $\vec{k}_{\|}$ the selection rules are $n_e = n_h \pm 1$. Instead of e-h pair representation (4) we have introduced the magnetoexciton creation operator depending on the wave vector $\vec{k}_{\|}$, on the orbital momentum projection M and on the Landau quantization numbers n and m [24, 25] as follows

$$\Psi^{\dagger}_{ex}(\vec{k}_{\|}, M, n, m) = \frac{1}{\sqrt{N}} \sum_t e^{ik_y t l_0^2} a^{\dagger}_{s_z, n, \frac{k_x}{2}+t} b^{\dagger}_{j_z, m, \frac{k_x}{2}-t}; \; s_z + j_z = M.$$

(6)

The obtained Hamiltonian in the magnetoexciton representation looks as

$$\hat{H}_{magex-ph} = \left(-\frac{e}{m_0 l_0}\right) \sum_{\vec{k}(\vec{k}_{\|}, k_z)} \sum_{M=\pm 1} \sum_{n,m=0}^{\infty} \sqrt{\frac{\hbar}{L_z \omega_{\vec{k}}}}$$
$$\times\{P_{cv}(\vec{k}_{\|}) \Phi\left(n, m, \vec{k}_{\|}\right) \mathrm{Exp}[ik_x k_y l_0^2 / 2]$$
$$\times\left[C_{\vec{k},-}\left(\vec{\sigma}^{+}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right) + C_{\vec{k},+}\left(\vec{\sigma}^{-}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)\right]\hat{\Psi}^{\dagger}_{ex}(\vec{k}_{\|}, M, n, m)$$
$$+ P^{*}_{cv}(\vec{k}_{\|}) \Phi^{*}\left(n, m, \vec{k}_{\|}\right) \mathrm{Exp}[-ik_x k_y l_0^2 / 2]$$
$$\times\left[\left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}^{+}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)^{*} + \left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}^{-}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)^{*}\right]\hat{\Psi}_{ex}(\vec{k}_{\|}, M, n, m)$$
$$+ P_{cv}(-\vec{k}_{\|}) \Phi\left(n, m, -\vec{k}_{\|}\right) \mathrm{Exp}[ik_x k_y l_0^2 / 2]$$
$$\times\left[\left(C_{\vec{k},-}\right)^{\dagger}\left(\vec{\sigma}^{+}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)^{*} + \left(C_{\vec{k},+}\right)^{\dagger}\left(\vec{\sigma}^{-}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)^{*}\right]\hat{\Psi}^{\dagger}_{ex}(-\vec{k}_{\|}, -M, n, m)$$
$$+ P^{*}_{cv}(-\vec{k}_{\|}) \Phi^{*}\left(n, m, -\vec{k}_{\|}\right) \mathrm{Exp}[-ik_x k_y l_0^2 / 2]$$
$$\times\left[C_{\vec{k},-}\left(\vec{\sigma}^{+}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right) + C_{\vec{k},+}\left(\vec{\sigma}^{-}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)\right]\hat{\Psi}_{ex}(-\vec{k}_{\|}, -M, n, m)\}.$$

(7)

The interaction constant in the case of dipole-active transitions is proportional to $1/l_0$ and increases as $\sqrt{B}$ when the magnetic field strength B increases. In the case of quadrupole-active transitions it does not depend on B, but is proportional to $|\vec{k}_{\|}|$. The first two resonance terms describe the annihilation of the photon with circular polarization $\vec{\sigma}^{\pm}_{\vec{k}}$ and the creation of the magnetoexciton with the circular polarization $\vec{\sigma}_M$ and vice versa. The abilities of the photon to effectuate these transformations are determined by the scalar products $\left(\vec{\sigma}^{\pm}_{\vec{k}} \cdot \vec{\sigma}^{*}_{M}\right)$. The next two addenda are the anti-resonance terms describing the simultaneous creation or annihilation of the both partners namely of the photon and of the magnetoexciton with opposite sign 2D wave vectors $\vec{k}_{\|}$ and $-\vec{k}_{\|}$, and with opposite sign orbital momentum projections M and $-M$.

Side by side with the electron-radiation interaction of the type $\sum_i \vec{A}(\vec{r}_i) \cdot \vec{\nabla}_i$ taken into account above there is also another interaction term proportional to the square of the vector potential $\vec{A}(\vec{r}_i)$ of the electromagnetic field in the form $\sum_i \vec{A}^2(\vec{r}_i)$. It gives rise to a supplementary quadratic form in the photon operators containing the resonance and anti-resonance terms [23], which were neglected.

The interaction Hamiltonian must be supplemented by the Hamiltonian H_0 of the free magnetoexcitons and photons

$$H_0 = \sum_{\vec{k}_{\|}} \sum_{M} \sum_{n,m} E_{ex}(\vec{k}_{\|}, M, n, m) \hat{\Psi}^{\dagger}_{ex}(\vec{k}_{\|}, M, n, m) \hat{\Psi}_{ex}(\vec{k}_{\|}, M, n, m)$$
$$+ \sum_{\vec{k}(\vec{k}_{\|}, k_z)} \hbar\omega_{\vec{k}} \left[\left(C_{\vec{k},+}\right)^{\dagger} C_{\vec{k},+} + \left(C_{\vec{k},-}\right)^{\dagger} C_{\vec{k},-}\right], \qquad (8)$$

where $E_{ex}(\vec{k}_{\|}, M, n, m) = \hbar\omega_{ex}(\vec{k}_{\|}, M, n, m)$ is the energy of the 2D magnetoexciton. It contains the contributions of the cyclotron energies $n\hbar\omega_{ce} + m\hbar\omega_{ch}$ of the e-h pair forming the magnetoexciton and of the Coulomb e-h interaction in the presence of a strong magnetic field. The cyclotron frequencies ω_{ce} and ω_{ch} increase linearly as a function of B, whereas the Coulomb energy increases as a $\sqrt{B}$ in the same way as the constant of the magnetoexciton-photon interaction. We supposed that the Coulomb e-h interaction leading to the formation of the magnetoexciton is greater than the magnetoexciton-photon interaction leading to the formation of the magnetopolariton. It means that the ionization potential of the magnetoexciton $I_l = (e^2/\varepsilon l_0)\sqrt{\pi/2}$, where ε is the dielectric constant, is greater than the Rabi energy $\hbar|\omega_R|$ introduced in the next section. The magnetoexciton energy does not depend on M when the Zeeman effect is not taken into account. The photon frequency depends on the 3D wave vector $\omega_{\vec{k}} = (c/n)\sqrt{\vec{k}_{\|}^2 + k_z^2}$. The full Hamiltonian describing the magnetoexciton-polariton is

$$H = H_0 + H_{magex-ph}. \qquad (9)$$

III. MAGNETOEXCITON-POLARITON IN MICROCAVITY

The dispersion law of the magnetoexciton polariton in a rotating way approximation and in the case of dipole-active transitions can be obtained neglecting by the anti-resonance terms and all corrections proportional to $\vec{k}_{\|}$ in the case $n_e = n_h = 0$. When k_z and L_z have the well defined values as in the case of microcavity namely $L_z = L_c$ and $k_z = \pi / L_c$ the Rabi frequency in the case of dipole transition is:

$$|\omega_R| = \frac{e}{m_0 l_0} \sqrt{\frac{1}{L_c \hbar \omega_{\vec{k}}}} |P_{cv}(0)|.$$

(10)

In the case of quadrupole transitions a supplementary factor $|\vec{k}_{\|}| l_0$ appears and ω_R is proportional to $|\vec{k}_{\|}|$ but do not depend on magnetic field strength.

In the Faraday geometry, when the wave vector $\vec{k}$ is oriented along the axis of microcavity, $\vec{k} = \vec{a}_3(\pi / L_c)N$, the light with circular polarization $\vec{\sigma}^{+}_{\vec{k}}$ ($\vec{\sigma}^{-}_{\vec{k}}$) excites only he magnetoexcitons with the orbital quantum number $M = 1$ ($M = -1$). This alignment of the magnetoexcitons is the manifestation of the optical orientation phenomena. In the case of slight deviation of the light wave vector $\vec{k}$ from the Faraday geometry with $|\vec{k}_{\|}| \ll \pi / L_c$, the mentioned above orbital selection rule is only approximately true. A second magnetoexciton state will be also excited, but with a much smaller amplitude, so that it will be neglected below.

The terms of the Hamiltonian (7) with the given wave vector $\vec{k}_{\parallel}$ may be separated. To simplify them we use the following denotations:

$$\hat{\Psi}_{ex}(\vec{k}_{\parallel}, M, 0, 0) = \hat{\Psi}_{ex}(\vec{k}_{\parallel}); C_{\vec{k},\vec{\sigma}} = C_{\pi/L_c,\vec{k}_{\parallel},\vec{\sigma}} = C(\vec{k}_{\parallel});$$

$$E_{ex}(\vec{k}_{\parallel}, M, 0, 0) = \hbar\omega_{ex}(\vec{k}_{\parallel}); \hbar\omega_{\vec{k}} = \hbar\omega_{\pi/L_c,\vec{k}_{\parallel}} = \hbar\omega_{ph}(\vec{k}_{\parallel}).$$

(11)

The equations of motion in this approximation are

$$i\hbar\frac{d\hat{\Psi}_{ex}(\vec{k}_{\parallel})}{dt} = \hbar\omega_{ex}(\vec{k}_{\parallel})\Psi_{ex}(\vec{k}_{\parallel}) + \hbar\omega_R C(\vec{k}_{\parallel})$$

$$i\hbar\frac{dC(\vec{k}_{\parallel})}{dt} = \hbar\omega_{ph}(\vec{k}_{\parallel})C(\vec{k}_{\parallel}) + \hbar\omega_R^*\Psi_{ex}(\vec{k}_{\parallel}).$$

(12)

For the stationary conditions they are

$$(\omega_{ex}(\vec{k}_{\parallel}) - \omega)\hat{\Psi}_{ex}(\vec{k}_{\parallel}) + \omega_R C(\vec{k}_{\parallel}) = 0$$

$$(\omega_{ph}(\vec{k}_{\parallel}) - \omega)C(\vec{k}_{\parallel}) + \omega_R^*\Psi_{ex}(\vec{k}_{\parallel}) = 0.$$

(13)

Their solutions give rise to magnetoexciton-polariton dispersion branches

$$\omega = \frac{\omega_{ex}(\vec{k}_{\parallel}) + \omega_{ph}(\vec{k}_{\parallel})}{2} \pm \sqrt{|\omega_R|^2 + (\frac{\omega_{ex}(\vec{k}_{\parallel}) - \omega_{ph}(\vec{k}_{\parallel})}{2})^2},$$

which are fully consistent with Refs. [5, 20].

The matrix element of the band-to-band quantum transition $|\vec{P}_{cv}|$ may be expressed through the oscillator strength f_{ex} of the optical quantum transition from the ground state of the bulk crystal to the 3D Wannier-Mott exciton state using the formula $f_{ex} = (2/m_0 E_g)|\vec{P}_{cv}(0)|^2|\Psi_{ex}(0)|^2 v_0; |\Psi_{ex}(0)|^2 = (1/\pi a_{ex}^3); v_0 = a_0^3$, where E_g is the semiconductor energy gap, v_0 is the volume of the lattice cell and $\Psi_{ex}(0)$ is the wave function of the relative e-h motion with the Bohr radius a_{ex}. If one supposes the parameters of the GaAs-type crystal $E_g \sim 1.5\,\text{eV}$, $a_{ex} \sim 10^{-6}\,\text{cm}$, $a_0 \sim 2\times10^{-8}\,\text{cm}$ and $f_{ex} \sim 10^{-6}$, the value $|\vec{P}_{cv}(0)| \approx 2\times10^{-20}\,\text{g cm/sec}$ will be found. Together with the parameters of the light $\hbar\omega_k \sim E_g$, resonator $L_c \approx 4\times10^{-5}\,\text{cm}$ and magnetic field strength $l_0 \sim 10^{-6}\,\text{cm}$ the Rabi frequency of the order of magnitude $\omega_R \sim 10^{12}\,\text{sec}^{-1}$ was calculated.

IV. CONCLUSION

The Hamiltonian describing the interaction of the two-dimensional magnetoexcitons with the photons propagating without confinement in an arbitrary direction of the three-dimensional space was deduced. Only the inter-band optical quantum transitions were taken into account. In this case the electron lying on the Landau level with quantum number n_h in the p-type valence band is transferred under the influence of light on the Landau level with number n_e in the s-type conduction band of the GaAs-type 2D layer and vice versa. The 2D electron-hole pairs and magnetoexcitons arising in these conditions are characterized by the numbers n_e and n_h of the Landau quantizations, by the orbital momentum projection M of the hole in the frame of p-type valence band and by the resultant 2D wave vector $\vec{k}_{\parallel}$ of the electron-hole pair and magnetoexciton. The light is characterized by two components of the wave vector $\vec{k} = \vec{k}_{\parallel} + \vec{a}_3 k_z$, where one is in-plane component $\vec{k}_{\parallel}$ and another one is oriented perpendicularly to the layer plane. When the semiconductor layer is embedded into the microresonator the k_z component of the light wave vector becomes quantized $k_z = (\pi / L_c)N$, where L_c is the length of the resonator and $N = \pm1, \pm2\ldots$.

The Rabi frequency which characterizes the magnetoexciton-photon interaction is proportional to the matrix element $P_{cv}(0)$ in the case of allowed-type band-to-band quantum transitions. The selection rules are $n_e = n_h$ and $n_e = n_h \pm 1$ for the dipole and quadrupole quantum transitions correspondingly. The Rabi constant is proportional to $\sqrt{B}$ in the case of dipole transitions and does not depend on B in the case of quadrupole transitions.

The dispersion laws of the magnetopolaritons in microcavity were deduced. The estimations of the relevant parameters were presented.

REFERENCES

[1] J. Kasprzak et al., *Nature*, vol. 443, pp. 409, 2006.
[2] D.W. Snoke, *Science*, vol. 298, pp. 1368, 2002.
[3] K.G. Lagoudakis et al., *Nature physics*, vol. 4, pp. 706, 2008.
[4] D.W. Snoke et al., *Physics Today*, vol. 63 8, pp. 42, 2010.
[5] H. Deng et al., *Rev. Mod. Phys*. vol. 82, pp. 1489, 2010.
[6] S.A. Moskalenko, *Sov. Phys. Solid State*, vol. 4, pp. 199, 1962.
[7] J.M. Blatt et al., *Phys. Rev*. vol. 126, pp. 1691, 1962.
[8] L.V. Keldysh et al., *Sov. Phys. JETP*, vol. 27, pp. 521, 1968.
[9] S.A. Moskalenko, D.W. Snoke, *Bose–Einstein Condensation of Excitons and Biexcitons and Coherent Nonlinear Optics with Excitons*, Cambridge University Press, Cambridge, 2000.
[10] J.J. Hopfield, *Phys. Rev*., vol. 112, pp. 1555, 1958.
[11] A. Kavokin and G. Malpuech, *Cavity polaritons*, Elsevier, Amsterdam, 2003.
[12] D.D. Solnyshkov et al., *Phys. Rev. B*, vol. 78, pp. 165323, 2008.
[13] I.A. Shelykh et al., *Phys. Status Solidi B*, vol. 242, pp. 2271, 2005.
[14] Yu.G. Rubo et al., *Phys. Lett. A*, vol. 358, pp. 227, 2006.
[15] A.V. Larionov et al., *Phys. Rev. Lett.*, vol. 105, pp. 256401, 2010.
[16] P. Walker et al., *Phys. Rev. Lett*., vol. 106, pp. 257401, 2011.
[17] O.L. Berman et al., *Phys. Rev. B*, vol. 80, pp. 115302, 2009.
[18] S.A. Moskalenko et al., *Phys. Rev. B*, vol. 79, pp. 125425, 2009.
[19] V.P. Kochereshko et al., in *Optical Properties of 2D Systems with Interacting Electrons, Nato Science Series II* vol. 119, 2003.
[20] V. Savona et al., *Phase Transitions*, vol. 68, 1, pp. 169, 1999.
[21] D. Snoke, *Cambridge-ITAP Workshop*, Turkey, 2009, http://www.amop.phy.cam.ac.uk/amop-ma/itap/Snoke.pdf.
[22] R.J. Elliott, *Phys. Rev*., vol. 108, pp. 1384, 1957. Birefringence in Anisotropic Crystals $CuGa(Al)S_2$ and Based on Them Optical Filters
[23] R.S. Knox, *Theory of Excitons*, Academic Press, NY, 1963.
[24] D. Paquet et al., *Phys. Rev. B*, vol. 32, pp. 5208, 1985.
[25] S.A. Moskalenko et al., *Phys. Rev. B*, vol. 66, pp. 245316, 2002