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DETERMINATION OF THE MAIN CHARACTERISTICS OF THE HEAT LOAD DURATION CURVE FOR A NEW ANALYTICAL MODEL

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Abstract. The paper deals with a new analytical model of the classical load of a heat supply system. The proposed model offers better quality approximation of actual consumption curves. The model includes two components - a power function (the Sochinsky-Rossander model) and a modified logistics function for which their integrals are, in essence, traced. Determination of T_M time allows to calculate the load factor and the annual energy consumption.

Keywords: *full load duration, load curve, load factor, logistic function, Sochinsky-Rossander model.*

Introduction

Lately the interest in modeling the load curve has increased, being determined by the fact that the integration of a significant share of variable (photovoltaic and wind) renewable energy sources into the energy systems raises the issue of flexibility of traditional systems on both demand and supply/production side [1].

In this context, knowing the energy consumption and production regime, the possibilities for their modification, as well as their analytical description becomes more and more important [2 - 3].

For heat supply systems (HSS) a new analytical model for the load duration curve was proposed. Hence, new calculation formulae for the amount of energy, full load duration and load factor are needed.

The most complete information regarding the consumption / production regime of a node could be provided by the recorded load curve; where the area under the curve represents the amount of energy consumed / produced.

Usually, it is sufficient that a load curve is characterized only by those three values – the peak q_M, average q_{med} and minimum q_{min} load. In most practical applications, produced both at energy systems' planning/design and at operational phase, instead of the average value q_{med}, another indicator of the consumption or production regime is widely used – full load duration T_M.

The T_M indicator is defined as a time period (duration) for which the amount of energy consumed during a time period (day, month, year), determined with the equation –

$Q = q_M \cdot T_M$, is equal to the real amount consumed during that period of time- $Q = \int q(\tau)dt$.
Both variables - q_{med} and T_M equally allow to evaluate the annual energy consumption as a product of $q_{med} \cdot \tau_{an}$ and $q_M \cdot T_M$, while the usefulness of T_M is net superior to q_{med}.
This is due to the fact that the duration T_M being divided by the number of hours in a year (constant value), τ_{an}= 8760 h, leads to another indicator – load factor FS, $FS = T_M / \tau_{an} = T_{M*}$, which serves as a parameter that characterizes the consumption regime of an entire category of consumers [4].

So, the parameter T_M is of great importance for calculating and optimization of energy systems.

Herewith, the emphasis is put on obtaining the calculation formula for the T_M for load duration curve (LDC) for a new analytical model of this curve.

1. Description of the new analitical model of the Ldc

Generally, a heat or electrical load duration curve (LDC) indicates in a synthetic mode how the load varies over time. The heat load duration curve of a system that supplies heat to end-users within an urban area (to the whole range of end-users: from residential consumers to industry) is going in a downwards direction, with small slope elements (SmS - CP) on concave and/or convex curves, as well as steep slope elements (StS - CA).
In order to obtain an LDC as close as possible to the real one, a new analytical model was proposed which allows creating curves that comprise all the characteristic elements of an LDC. This six-parameter model includes two functions - a capacity function and a modified sigmoidal function.

The first equation is well known as Sochinsky-Rossander equation [5-12], while the second is a sigmoid described by a classical logistic function [13-14], slightly modified in order to adapt it to the specificity of the problem.
The $q(\tau)$ shall be the function that describes the variation of heat load *q* over the time *τ*, *τ* = 0…8760 h/yr. Based on all mentioned above, the load duration curve can be described by –

$$q(\tau) = q_{SR}(\tau) - q_L(\tau), \tag{1}$$

where $q_{SR}(\tau)$ represents the Sochinsky-Rossander (SR) load function,

$$q_{SR}(\tau) = q_M \cdot \left[1 - (1 - \frac{q_{min}}{q_M}) \cdot (\frac{\tau}{\tau_{an}})^{\beta}\right] \quad (2a) \qquad q_{SR}(\tau_*) = q_M \cdot \left[1 - (1 - q_{min*}) \cdot \tau_*^{\beta}\right] \quad (2b)$$

and $q_L(\tau)$ - is a modified logistic function,

$$q_L(\tau) = q_M \cdot \left[\frac{k}{1 + e^{-m \cdot (\tau - \tau_0)}} (1 - \frac{\tau}{\tau_{an}})\right] \quad (3a) \qquad q_L(\tau_*) = q_M \cdot \left[\frac{k}{1 + e^{-m \cdot (\tau_* - \tau_{0*})}} (1 - \tau_*)\right] \quad (3b)$$

Taking into consideration the formulae (1)-(3) the proposed model for LDC $q(\tau)$ takes the form:

in absolute units in relative units (normalized)

$$q(\tau) = q_M \cdot \left[1-(1-\frac{q_{min}}{q_M})\cdot(\frac{\tau}{\tau_{an}})^{\beta} - \frac{k}{1+e^{-m\cdot(\tau-\tau_0)}}\cdot(1-\frac{\tau}{\tau_{an}})\right] \quad (4a) \qquad q(\tau_*) = q_M \cdot \left[1-(1-q_{min*})\cdot\tau_*^{\beta} - \frac{k}{1+e^{-m\cdot(\tau_*-\tau_{0*})}}\cdot(1-\tau_*)\right] \quad (4b)$$

The graph of the function q(τ) described above is presented in the figure 1.

In the equations (1)-(4) the variables are as follows:

q_M, q_{med} and q_{min} - the maximum (calculated), average and minimum load value in the respective year;		
q_{med*} and q_{min*}	-	relative values of the average load and respectively minimum load, divided to the maximum value - $q_{med*} = q_{med} / q_M$, $q_{min*} = q_{min} / q_M$;
τ and τ_*	-	the actual absolute and relative time period, $\tau_* = \tau / \tau_{an}$
β	-	power function exponent or non-uniformity coefficient of the load duration curve, $\beta = (q_{med*} - q_{min*})/(1-q_{med*})$ or $\beta = (T_M - T_{an}\cdot q_{min*})/(T_{an} - T_M)$;
k	-	the maximum value of the logistic component of the LDC, k ≈ 0.1-0.5;
m	-	the slope of the curve q(τ) in the point $\tau = \tau_0$;
τ_0 and τ_{0*}	-	the duration of the heating season, the absolute and relative value, $\tau_{0*} = \tau_0 / \tau_{an}$;
τ_{an}	-	calendar year duration, τ_{an} = 8760 h;
T_M and T_{M*}	-	full load duration, its absolute and relative value, $T_{M*} = T_M / \tau_{an}$.
$0 \leqslant (\tau_*, \tau_{0*}, q_{min*}) \leqslant 1$; 0,01 ≤ β ≤ 10; 30 ≤ m ≤ 300.		

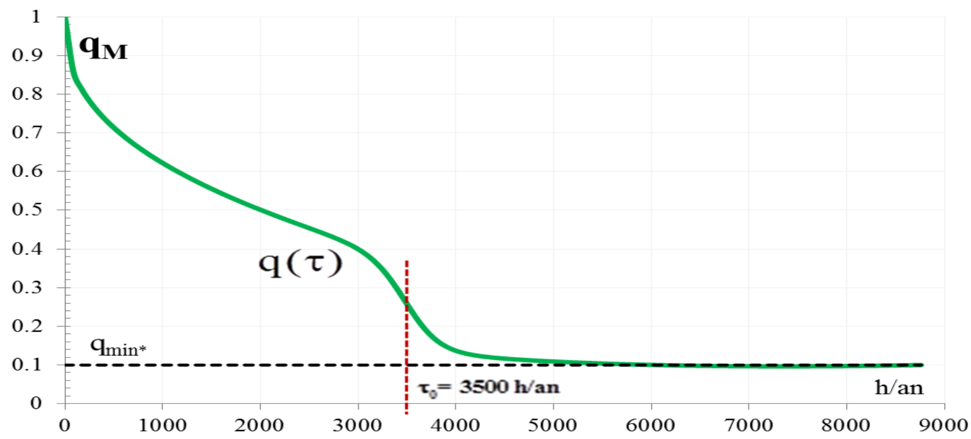

Figure 1. Load duration curve and the τ_0 time moment of the slope.

The task is to find the equation for calculating the surface area under the load duration curve $q(\tau)$ for the proposed model (4), which represents the volume of energy consumed during the period of time τ_{an}, on one hand, and the full load duration, as well as the load factor related to this curve, on the other hand.

2. Calculation of the area under the ldc

The area under the load duration curve can be calculated by integration of the function (4) that describes this curve. Since the equation (4) has two components, one could find two areas: S_{SR} - for the power-function $q_{SR}(\tau)$ and respectively S_L for the logistic function $q_L(\tau)$, and in the end the area under LDC - $S_{CC} = S_{SR} - S_L$ (figure 2).

Case 1: Without splitting the curve in intervals

<u>The area under the Sochinsky-Rossander curve</u>

For the area under the SR curve (2b) one can write (figure 2) -

$$S_{SR} = \int_0^1 q_{SR}(\tau_*)\,d\tau_* = q_M \cdot \int_0^1 \left[1-(1-q_{min*})\,\tau_*^{\beta}\right] d\tau_*$$

or $S_{SR} = q_M \cdot T_{M-SR*}$,

where T_{M-SR*} is the full load duration for the SR curve, in relative units –

$$T_{M-SR*} = \int_0^1 \left[1-(1-q_{min*})\,\tau_*^{\beta}\right] d\tau_* . \tag{5}$$

Applying to (5) the integration rule for power-function - $\int x^a dx = \dfrac{x^{a+1}}{a+1} + c$, for the duration T_{M-SR*} one can obtain:

$$T_{M-SR*} = \left[\tau_* - (1-q_{min*}) \cdot \frac{\tau_*^{\beta+1}}{\beta+1}\right]_0^1 \text{ or,}$$

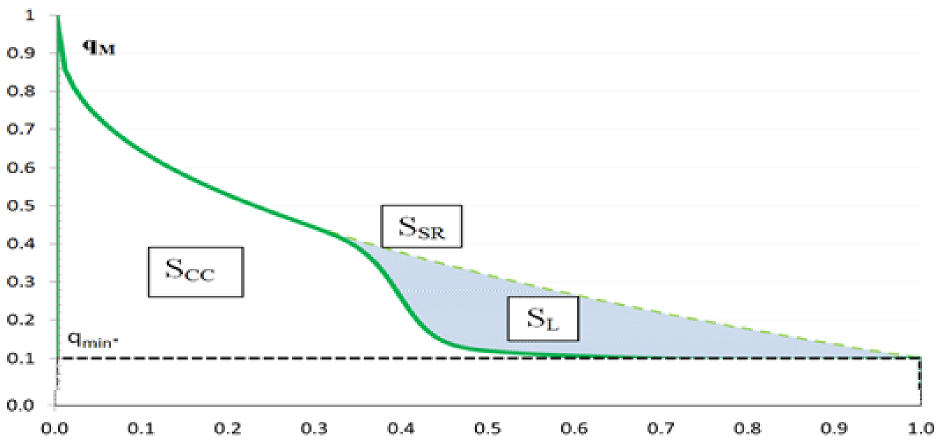

Figure 2. Areas under the SR curve, logistic curve and load duration curve.

finally, $T_{M-SR*} = \dfrac{\beta + q_{min*}}{\beta+1}$. (6)

The duration T_{M-SR}, in hours per year - $T_{M-SR} = T_{M-SR*} \cdot \tau_{an}$ or

$$\boxed{T_{M-SR} = \tau_{an} \cdot \frac{\beta + q_{min*}}{\beta+1}}.$$

So, the calculation formula for the area S_{SR} is:

$$S_{SR} = q_M \cdot T_{M-SR*} = q_M \cdot \frac{\beta + q_{min*}}{\beta+1}, \tag{7}$$

and for the annual energy consumption is –

$$Q_{an \cdot SR} = q_M \cdot T_{M-SR} = q_M \cdot \tau_{an} \cdot \frac{\beta + q_{min*}}{\beta + 1}.$$

<u>The area under the logistic curve</u>

The logistic function (3b) can be presented as the algebraic sum of two components -

$q_L(\tau_*) = q_{L1}(\tau_*) - q_{L2}(\tau_*)$, where $q_{L1}(\tau_*) = q_M \cdot \dfrac{k}{1+e^{-m \cdot (\tau_* - \tau_{0*})}}$

and $q_{L2}(\tau_*) = q_M \cdot \dfrac{k \cdot \tau_*}{1+e^{-m \cdot (\tau_* - \tau_{0*})}}$.

The area S_L under the logistic curve is calculated by integrating the function $q_L(\tau_*)$:

$$S_L = \int_0^1 q_L(\tau_*)\, d\tau_* = q_M \cdot T_{M-L*},$$

and subsequently, the full load duration for the logistic curve T_{M-L*} is –

$$T_{M-L*}(\tau_*) = \int_0^1 \left[\frac{k}{1+e^{-m \cdot (\tau_* - \tau_{0*})}} (1-\tau_*) \right] d\tau_* . \tag{8}$$

By dividing the function $q_L(\tau_*)$ in those two components $q_{L1}(\tau_*)$ and $q_{L2}(\tau_*)$, one shall operate with two areas - S_{L1} and S_{L2}, so $S_L = S_{L1} - S_{L2}$,

where $S_{L1} = \int_0^1 q_{L1}(\tau_*)\, d\tau_* = q_M \cdot T_{M-L1*}$ (9)

and $S_{L2} = \int_0^1 q_{L2}(\tau_*)\, d\tau_* = q_M \cdot T_{M-L2*}$. (10)

For the indicator T_{M-L*} similarly, one shall operate with two components - T_{M-L1*} and T_{M-L2*} for which it yields:

$$T_{M-L1*}(\tau_*) = k \int_0^1 \frac{1}{1+e^{-m \cdot (\tau_* - \tau_{0*})}} d\tau_* \quad \text{and} \tag{11}$$

$$T_{M-L1*}(\tau_*) = k \int_0^1 \frac{\tau_*}{1+e^{-m \cdot (\tau_* - \tau_{0*})}} d\tau_* \ . \tag{12}$$

Obviously, $$T_{M-L*} = T_{M-L1*} - T_{M-L2*}. \tag{13}$$

So, in order to calculate the area under the logistic curve it is necessary to solve the integrals (11) and (12).

As a result of integration of the function (11) for the duration T_{M-L1*} it is obtain (see Annex 1):

$T_{M-L1*} = \dfrac{k}{m} \cdot \ln\left|1+e^{m \cdot (1-\tau_0)}\right|$ or

$$\boxed{T_{M-L1*} \approx k \cdot (1\text{-}\tau_{0*})}, \tag{14}$$

and in this case, according to formula (9) one can calculate also the area

$$S_{L1} = q_M \cdot k \cdot (1\text{-}\tau_{0*}) \tag{15}$$

As for the integral (12) – solving it raises problems; for this reason to determine S_{L2} we will move from integration to the planimetric curve analysis $q_{L1}(\tau_*)$ and $q_{L2}(\tau_*)$ (fig.3).

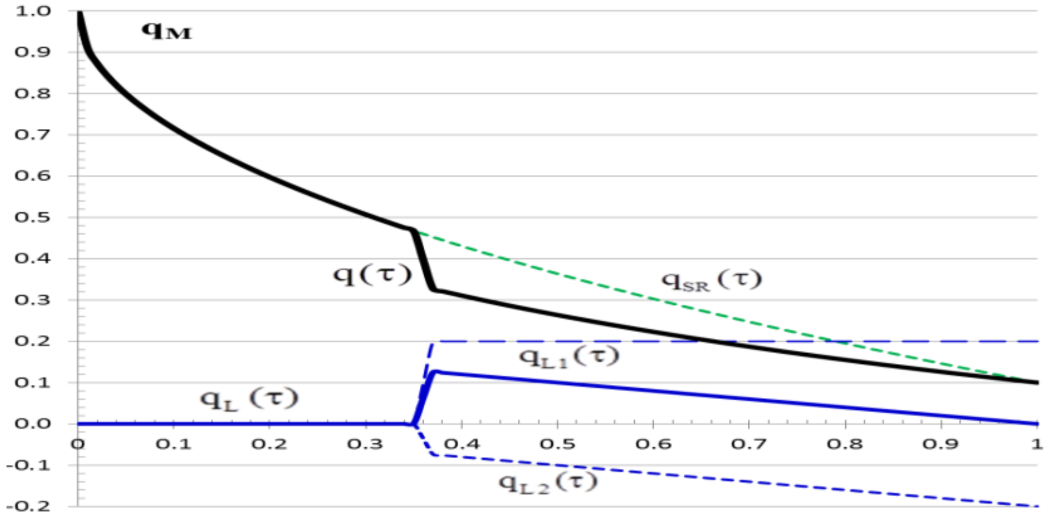

Figure 3. Load duration curve q(τ) and functions $q_{SR}(\tau)$ and $q_L(\tau)$

The analysis of these curves allows establishing the existence of the next relationship:

$$S_{L2} = S_{L1} - \delta S = q_M \cdot 0.5k \cdot (1 - \tau_{0*}^2), \tag{16}$$

$$\delta S = q_M \cdot 0.5k \cdot (1\text{-}\tau_{0*})^2 \;. \tag{17}$$

Once the S_{L2} is known, we can also find the duration T_{M-L2*} -

$$\boxed{T_{M-L2*} = 0.5k \cdot (1\text{-}\tau_{0*}^2)} \tag{18}$$

So, for the area S_L a of the logistic curve, finally, one can get:

$$S_L = S_{L1} - S_{L2} = \delta S = q_M \cdot 0.5k \cdot (1\text{-}\tau_{0*})^2, \tag{19}$$

and considering the equation $S_L = q_M \cdot T_{M-L*}$ for the duration T_{M-L*} it is obtained:

$$T_{M-L*} = 0.5k \cdot (1\text{-}\tau_{0*})^2 \tag{20}$$

or in hours/year -

$$\boxed{T_{M-L} = \tau_{an} \cdot 0.5k \cdot (1\text{-}\tau_{0*})^2}. \tag{21}$$

The annual energy consumption, which corresponds to the *logistic curve* –

$$Q_{an \cdot L} = q_M \cdot T_{M-L} = q_M \cdot \tau_{an} \cdot 0.5k \cdot (1\text{-}\tau_{0*})^2 \;.$$

<u>Parameters of the load duration curve</u>

For the *load duration curve*, finally, it yields the following equations for calculating the main parameters:

- the area under the LDC (fig. 2) - $S_{CC} = S_{SR} - S_L$,
  and taking into consideration the equations (7) and (19) it yields:
  $$S_{CC} = Q_{an \cdot CC} = q_M \cdot \left[\frac{\beta + q_{min*}}{\beta + 1} - 0.5 \cdot k \cdot (1\text{-}\tau_{0*})^2 \right]$$
- full load duration, in relative units - $T_{M-CC*} = T_{M-SR*} - T_{M-L*}$
  and taking into consideration the equation (6) and (20) it yields -

$$T_{M-CC*} = \frac{\beta + q_{min*}}{\beta+1} - 0.5\cdot k\cdot(1\text{-}\tau_{0*})^2 \quad , \tag{22}$$

or in hours/year -

$$\boxed{T_{M-CC} = T_{M-CC*}\cdot\tau_{an} = \tau_{an}\cdot\left[\frac{\beta+q_{min*}}{\beta+1} - 0.5\cdot k\cdot(1\text{-}\tau_{0*})^2\right]}. \tag{23}$$

The load factor of the LDC: $FS_{CC} = T_{M-CC} / \tau_{an} = T_{M-CC*}$.

The annual energy consumption, which corresponds to LDC, described through the equation (4), can be easily determined with the formula:

$$Q_{an\cdot CC} = q_M \cdot T_{M-CC},$$

where q_M represents the peak annual load, while T_{M-CC} is the full load duration, calculated with the formula (23).

Case 2: By dividing the load duration curve in intervals

Sochinsky-Rossander curve

In case of division of the time axis, τ_*, τ_* = 0…1, in intervals, be it three: 0-τ_1, τ_1-τ_2 and τ_2-1, for the area $S_{\tau_1-\tau_2}$ under the SR curve in the interval $\tau_1 - \tau_2$, it yields –

$$S_{SR,\tau_1-\tau_2} = \int_{\tau_1}^{\tau_2} q_{SR}(\tau_*)\, d\tau_* = q_M\cdot\left[\tau_* - (1-q_{min*})\cdot\frac{\tau_*^{\beta+1}}{\beta+1}\right]_{\tau_1}^{\tau_2}. \tag{24}$$

If we present this area as the product -

$S_{\tau_1-\tau_2} = q_M\cdot T_{M,\tau_1-\tau_2}$, for the equivalent duration $T_{M,\tau_1-\tau_2}$ it yields –

$$T_{M,SR,\tau_1-\tau_2} = \tau_{an}\left[(\tau_2-\tau_1) - (1-q_{min*})\cdot\frac{\tau_2^{\beta+1}-\tau_1^{\beta+1}}{\beta+1}\right]. \tag{25}$$

Obviously, for the considered case the following relation is valid –

$$T_{M-SR} = T_{M,0-\tau_1} + T_{M,\tau_1-\tau_2} + T_{M,\tau_2-1} = \tau_{an}\cdot\frac{\beta+q_{min*}}{\beta+1} \quad .$$

Logistic curve

Through the interval τ = 0… τ_0 the logistic curve is unfolding on its inferior asymptote, so that for this case it yields: $S_{L,0-\tau_0} = T_{M,L,0-\tau_0} = 0$.

For any interval τ_1-τ_2 that is after the point τ_0 it yields: for the duration $T_{M,L,\tau_1-\tau_2}$,

$T_{M,L,\tau_1-\tau_2} = 0.5\cdot k\cdot(1\text{-}\tau_{0*})(\tau_2-\tau_1)$ and for the area $S_{L,\tau_1-\tau_2}$ - $S_{L,\tau_1-\tau_2} = \int_{\tau_1}^{\tau_2} q_L(\tau)\, d\tau = q_M\cdot T_{M,\tau_1-\tau_2}$.

The resulted load duration curve

Finally, for the LDC we have:

- for any time interval τ_1 - τ_2 before the point in time τ_0, the duration $T_{M,CC}$ -

  $$T_{M,CC} = T_{M,SR,\tau_1-\tau_2} = (\tau_2-\tau_1) - (1-q_{min*})\cdot\frac{\tau_2^{\beta+1}-\tau_1^{\beta+1}}{\beta+1}$$

  and the area S_{CC}: $\quad S_{CC} = S_{SR} = q_M\cdot T_{M,SR,\tau_1-\tau_2}$;

- for any time interval after the point τ_0 , $T_{M,CC} = T_{M,SR,\tau_1-\tau_2} - T_{M,L,\tau_1-\tau_2}$

or $$\boxed{T_{M,CC} = (\tau_2-\tau_1)-(1-q_{min*})\cdot\frac{\tau_2^{\beta+1}-\tau_1^{\beta+1}}{\beta+1}-0.5\cdot k\cdot(1\text{-}\tau_{0*})(\tau_2-\tau_1)}$$

and $$S_{CC} = S_{SR} - S_L = q_M \cdot T_{M,CC,\tau_1-\tau_2} .$$

3. Numerical example

For the diversity of curves shown in fig. 4, with their known parameters (table 1), the full load durations $T_{M\text{-}CC}$ were calculated for those two cases.

Case 1: Without dividing the LDC in time intervals

For the curves a) - h) from figure 4 there were calculated the full load durations $T_{M\text{-}SR}$, $T_{M\text{-}L}$ and $T_{M\text{-}CC}$. Below, for a load duration curve – the curve c), the calculation of the three durations is exemplified:

$T_{M\text{-}CC} = T_{M\text{-}SR} - T_{M\text{-}L}$ = 3129 – 631= 2498 h/an,

where: $T_{M-SR} = \tau_{an}\cdot(\beta+q_{min*})/(\beta+1)$ = 8760·(0,4+0,1)/(0,4+1) = 8760·0,357 = 3129 h/yr

and $T_{M-L} = \tau_{an}\cdot 0.5\cdot k\cdot(1-\tau_{0*})^2$ = 8760·0,5·0.4·(1-0,4)^2 = 8760·0,08268 = 631 h/yr.

The results of these calculations are presented in table 1.

Case 3: By dividing the LDC in two time intervals

While making the calculations aiming at optimization and analysis of options to meet the load by many energy sources, often the LDC is divided horizontally and/or vertically.

The results of calculations made for the case of division of the annual duration in two time intervals are presented in table 2 and 3: the first covers the heating season (the interval 0 - τ_0), and the second – the other part of the year (interval τ_0 - τ_{an}).

Table 1. The descriptive parameters of load duration curves and their full load durations $T_{M\text{-}CC}$

Figure	q_{min*}	β	k	m	$\tau_{0,}$	$T_{M\text{-}SR}$	$T_{M\text{-}L}$	$T_{M\text{-}CC}$
-	r.u.	r.u.	r.u.	r.u.	h/yr	h/yr	h/yr	h/yr
a	0,1	0,08	0,1	30	2628	1460	215	1245
b	0,1	0,2	0,22	100	3066	2190	407	1783
c	0,1	0,4	0,4	300	3504	3129	631	2498
d	0,1	0,5	0,45	300	4380	3504	493	3011
e	0,1	0,6	0,5	300	5256	3833	350	3483
f	0,35	0,7	0,4	50	6132	5411	158	5253
g	0,35	1,4	0,8	50	7008	6388	140	6248
h	0,35	5,0	3,0	50	7884	7811	131	7680

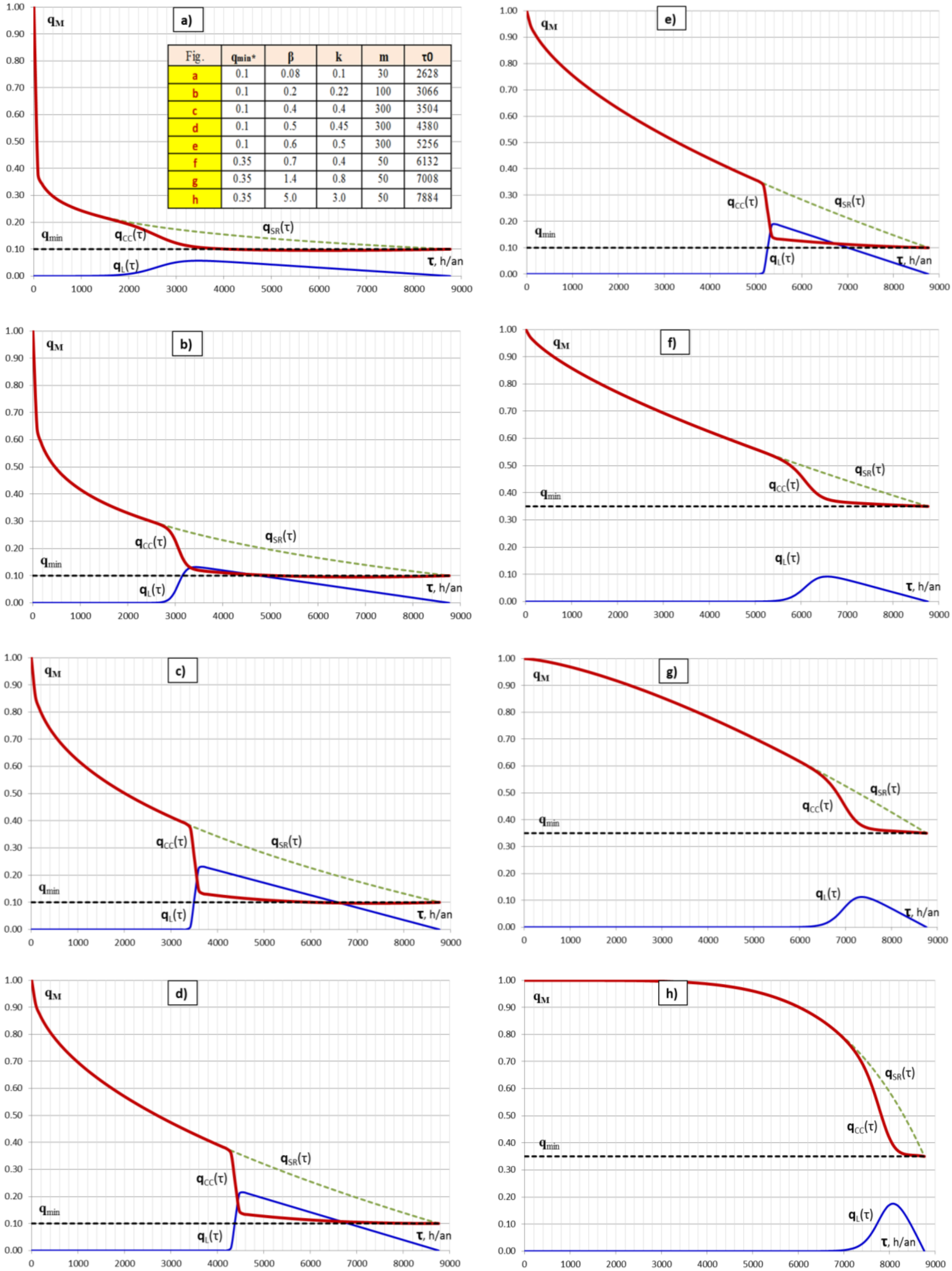

Fig.	q_{min*}	β	k	m	τ0
a	0.1	0.08	0.1	30	2628
b	0.1	0.2	0.22	100	3066
c	0.1	0.4	0.4	300	3504
d	0.1	0.5	0.45	300	4380
e	0.1	0.6	0.5	300	5256
f	0.35	0.7	0.4	50	6132
g	0.35	1.4	0.8	50	7008
h	0.35	5.0	3.0	50	7884

Figure 4. A range of load duration curves, which characterize a large diversity of possible real situations.

Table 2. Full load durations T_{M-SR}, T_{M-L} and T_{M-CC} determined for those two intervals, in r.u.

Fig. 4	$τ_{0*}$	Interval 0 - $τ_o$			Interval $τ_o$ - $τ_{an}$			Total annually		
		T_{M-SR}	T_{M-L}	T_{M-CC}	T_{M-SR}	T_{M-L}	T_{M-CC}	T_{M-SR}	T_{M-L}	T_{M-CC}
a	0,3	0,073	0	0,073	0,094	0,025	0,069	0,167	0,025	0,142
b	0,35	0,137	0	0,137	0,113	0,046	0,066	0,250	0,046	0,204
c	0,4	0,222	0	0,222	0,135	0,072	0,063	0,357	0,072	0,285
d	0,5	0,288	0	0,288	0,112	0,056	0,056	0,400	0,056	0,344
e	0,6	0,352	0	0,352	0,086	0,040	0,046	0,438	0,040	0,398
f	0,7	0,491	0	0,491	0,126	0,018	0,108	0,618	0,018	0,600
g	0,8	0,641	0	0,641	0,088	0,016	0,072	0,729	0,016	0,713
h	0,9	0,842	0	0,842	0,049	0,015	0,034	0,892	0,015	0,877

Table 3. Full load durations T_{M-SR}, T_{M-L} and T_{M-CC} determined for those two intervals, in h/yr

Figure 4	$τ_{0*}$	Interval 0 - $τ_o$			Interval $τ_o$ - $τ_{an}$			Total annually		
		T_{M-SR}	T_{M-L}	T_{M-CC}	T_{M-SR}	T_{M-L}	T_{M-CC}	T_{M-SR}	T_{M-L}	T_{M-CC}
a	2628	639	0	639	821	215	606	1460	215	1245
b	3066	1202	0	1202	988	407	581	2190	407	1783
c	3504	1943	0	1943	1186	631	555	3129	631	2498
d	4380	2522	0	2522	982	493	489	3504	493	3011
e	5256	3080	0	3080	753	350	403	3833	350	3483
f	6132	4305	0	4305	1105	158	947	5411	158	5253
g	7008	5619	0	5619	768	140	628	6388	140	6248
h	7884	7380	0	7380	431	131	300	7811	131	7680

Conclusions

1. The load curve of a consumption or production node lays at the basis for calculations, analysis and optimizations of operating regimes of energy systems – both, at the technical design/planning phase and the operating one.
2. The full load duration for a load duration curve is a key parameter of the energy consumption regime; it allows finding also other important parameters for optimal scaling of the elements of energy systems.
3. The problem of determining the full load duration for a new analytical model of the heat load duration curve of a heating system is solved: with and without dividing the load duration curve in time intervals.

Annex

Integration of the modified logistic function

The initial logistic function will be presented as the sum of two functions:

$$\boxed{f(x) = \frac{k\cdot(1-x)}{1+e^{-m\cdot(x-x_0)}} = f_1(x) + f_2(x)}, \tag{A1}$$

where

$$f_1(x) = k\cdot\frac{1}{1+e^{-m\cdot(x-x_0)}} \tag{A2}$$

and

$$f_2(x) = k\cdot\frac{x}{1+e^{-m\cdot(x-x_0)}}\,. \tag{A3}$$

First we shall find the primitive $F_1(x)$ of the function $f_1(x)$ -

$$F_1(x) = \int_0^1 f_1(x)dx \;=\; k\int_0^1 \frac{1}{1+e^{-m(x-x_0)}}dx \tag{A4}$$

For this purpose we shall apply the substitution method:

let $u = e^{-m(x-x_0)}$, then $ln\,u = -m\cdot x + mx_0$, $-mdx = \frac{du}{u}$, whence $dx = -\frac{du}{mu}$.

We revert back to (A4) by changing the variable –

$$F_1(u) = k\int\frac{1}{1+u}\cdot\frac{-du}{mu} = -\frac{k}{m}\int\frac{1}{1+u}\cdot\frac{du}{u} \tag{A5}$$

or

$$F_1(u) = -\frac{k}{m}\int\frac{1+u-u}{1+u}\cdot\frac{du}{u} = -\frac{k}{m}\cdot\left[\int\frac{1+u}{1+u}\cdot\frac{du}{u} - \int\frac{u}{1+u}\cdot\frac{du}{u}\right] \tag{A6}$$

Taking into consideration the classical equations –

$$\int\frac{du}{u} = ln|u| + c \quad\text{and}\quad \int\frac{du}{1+u} = ln|1+u| + c,$$

the formula (A6) transforms into –

$$F_1(u) = \frac{k}{m}\cdot\left[-ln|u| + ln|1+u|\right] \quad\text{or}\quad F_1(u) = \frac{k}{m}\cdot ln\left|\frac{1+u}{u}\right| = \frac{k}{m}\cdot ln\left|1+u^{-1}\right|,$$

and, finally, by replacing the *u* it yields –

$$F_1(x) = \left[\frac{k}{m}\cdot ln\,\left|1+e^{m(x-x_0)}\right|\right]_0^1 = \frac{k}{m}\cdot ln\,\left|1+e^{m(1-x_0)}\right|\cdot \tag{A7}$$

Based on the fact that the constants m and x_0 are positive measures with intervals of values - $0 \leqslant x_0 \leqslant 1$ and $30 \leqslant m \leqslant 300$, and the component $e^{m\cdot(1-x_0)}$ has two big values, much higher than 1, it allows us to transform the equation (A7) –

$$F_1 = \frac{k}{m}\cdot ln\,\left|1+e^{m(1-x_0)}\right| \approx \frac{k}{m}\cdot ln\,\left|e^{m(1-x_0)}\right| = \frac{k}{m}\cdot m\,(1-x_0) \tag{A8}$$

or, finally -

$$\boxed{F_1 = k\cdot(1-x_0)}. \tag{A9}$$

For the integral of the function $f_2(x)$ it yields –

$$\boxed{F_2 = 0.5k\cdot(1-x_0^{\,2})}, \tag{A10}$$

and for the integral of the initial function $f(x)$ it yields -

$$\boxed{F_L = F_1 - F_1 = 0.5k\cdot(1-x_0)^2}. \qquad \text{(A11}$$

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