MATHEMATICAL MODEL OF DRYING PROCESS VELOCITY FACTOR

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Abstract: One of the basic factors influencing drying process is the velocity of the drying agent. It’s well known is one of the controlled characteristics of the drying process. We made a tentative to calculate the mathematical model for the controllable velocity factor, as result one could control the velocity of the drying agent, thereby optimizing energy costs for the drying process.

Keywords: Mathematical model, velocity, drying, thermal agent, air.

One of the most important factors affecting drying process intensity is the drying agent velocity, and it’s one of the dehydration process’ controllable characteristics. The influence of this factor fully determines drying process’ intensity while constant drying velocity phase is on (first phase), its influence drops greatly while in decreasing velocity phase.

We made a tentative of modeling and calculating the drying agent velocity to control food drying process. Moisture evaporating process in constant velocity phase could be approximately counted equal to an open surface evaporating process. Dalton’s low for open surface evaporating process velocity has the form:

$$W = G(H - h)\frac{760}{b} \tag{1}$$

W – evaporated water quantity, [*kg/(m^2·h)*];
G – evaporation coefficient, [*kg/(m^2 h mmHg)*];
H – ***P$_{sat}$*** saturated steam pressure at evaporating water temperature, [*mmHg*]
h – ***P$_{par}$*** environment or flowing by air partial pressure, [*mmHg*];
b – ***P*** barometric pressure, [*mmHg*].

Since drying process velocity within its constant phase depends on steam diffusing rate through superficial film, it’s also affected by factors influencing the film’s thickness. The most notable such factor is material superficial air velocity.

According to Hinchley – Himus [1], the ***G*** coefficient depends on the thickness of the film which, as we know from heat transfer data, changes in 0,8 degree of air velocity, therefore it can be calculated in *kg/(m^2 h mmHg)* using the formula

$$G = 0{,}029(\rho_{air}\,\omega)^{0{,}8}$$

$$G = 0{,}029(1{,}29\omega)^{0{,}8} = 0{,}0035\omega^{0{,}8} \tag{2}$$

Saturated steam pressure at evaporating water temperature, [*mmHg*]

$$P_{sat} = \left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot 0{,}0075 \tag{3}$$

Environment or flowing by air partial pressure, [*mmHg*]

$$P_{par} = \frac{d\cdot P}{d + 0{,}622} \tag{4}$$

d – moisture content, [*kg/kg*];
P – steam-air mix total pressure 745 [*mmHg*].

$$P_{par} = \frac{d\cdot 745}{d + 0{,}622} \tag{5}$$

$$d = 0{,}622\cdot\frac{\varphi\cdot\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot 0{,}0075}{745 - \varphi\cdot\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot 0{,}0075} \tag{6}$$

b – barometric pressure, 745 [*mmHg*].

For a better control of drying process velocity, one could adjust air moisture, withal within constant drying velocity phase, drying process velocity remains constant as far as drying process conditions are permanent.

Substituting equations (1), (2), (3), (5) and (6) we came to:

$$W = 0{,}0035\omega^{0{,}8}\left(\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot 0{,}0075 - \right.$$

$$\left. - \frac{d\cdot 745}{d + 0{,}622}\right)\frac{760}{745} = 0{,}0035\omega^{0{,}8}\left(\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot\right.$$

$$\cdot 0{,}0075 - \frac{0{,}622\cdot\dfrac{\varphi\cdot\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot}{745 - \varphi\cdot\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot}}{0{,}622\cdot\dfrac{\varphi\cdot\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot}{745 - \varphi\cdot\left(0{,}159\cdot t^3 - 8{,}7018\cdot t^2 + 285{,}3\cdot t - 593{,}27\right)\cdot}}$$

$$\left.\frac{\dfrac{\cdot 0{,}075}{\cdot 0{,}075}\cdot 745}{\dfrac{\cdot 0{,}075}{\cdot 0{,}075} + 0{,}622}\right)1{,}02 \tag{7}$$

The same time the evaporated water quantity can be calculated with the formula [2] (*kg/s*):

$$W = \beta F \Delta d_{med}$$

Or (kg/m^2s):

$$W = \beta \Delta d_{med} \tag{8}$$

β – gas form mass-transfer coefficient, [$\dfrac{kg}{m^2 s \dfrac{kg}{kg}}$].

F – evaporation surface area, [*m*^{2}];
Δd$_{med}$ – average driving force, [*kg steam/kg dry air*].

$$\Delta d_{med} = d_{sat} - d_{int} \tag{9}$$

$$d_{sat} = 0{,}622 \cdot \frac{\varphi_{sat} \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27\right) \cdot}{745 - \varphi_{sat} \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27\right) \cdot}$$

$$\frac{\cdot 0{,}0075}{\cdot 0{,}0075} \tag{10}$$

$$d_{int} = 0{,}622 \cdot \frac{\varphi_{int} \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27\right) \cdot}{745 - \varphi \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27\right) \cdot}$$

$$\frac{\cdot 0{,}0075}{\cdot 0{,}0075} \tag{11}$$

β mass-transfer coefficient can be calculated using the criterion equation:

$$Nu = A Re^{n} Pr^{0{,}33} Gu^{0{,}135} \tag{12}$$

Nu – Nusselt criterion.

$$Nu = \frac{\beta l}{D} \tag{13}$$

D – diffusivity coefficient, *m*^{2}*/s*

$$D = 2{,}8 \cdot 10^{-5} 1\, m^2 / s$$

L – evaporation surface length, [*m*].
Re – Reynolds criterion.

$$Re = \frac{\omega l}{\nu} \tag{14}$$

ν – cinematic viscosity = $2 \cdot 10^{-5}$ [*m*^{2}*/s*].

Reynolds criterion’s value is close to 6000, which gives the following values for ***A*** and ***n***:

A = 0,347, n = 0,65 [2 Pavlov].

Pr – Prandtl criterion.

$$Pr = \frac{\nu}{D} \tag{15}$$

$$Pr = 0{,}71$$

Gu – Guhman criterion.

$$Gu = \frac{T_c - T_m}{T_c} \tag{16}$$

$$Gu = 0{,}17$$

T – temperature, [*K*].

From equation (12) we find ***β***

$$Nu = \frac{\beta l}{D} = A Re^n \, Pr^{0,33} \, Gu^{0,135}$$

$$\beta = \frac{D}{l} A Re^n \, Pr^{0,33} \, Gu^{0,135}$$

$$\beta = 0{,}347 Re^{0,65} \, Pr^{0,33} \, Gu^{0,135} \, \frac{D}{l} \tag{17}$$

Equating equations (1) and (9):

$$\begin{gathered}
\frac{1}{3600} \cdot 0{,}0035\omega^{0,8} \left(\left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27 \right) \cdot 0{,}0075 - \right. \\
- \frac{0{,}622 \cdot \dfrac{\varphi \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27 \right) \cdot 0{,}0075}{745 - \varphi \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27 \right) \cdot 0{,}0075} \cdot}{0{,}622 \cdot \dfrac{\varphi \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27 \right) \cdot 0{,}0075}{745 - \varphi \cdot \left(0{,}159 \cdot t^3 - 8{,}7018 \cdot t^2 + 285{,}3 \cdot t - 593{,}27 \right) \cdot 0{,}0075} +} \\
\left. \frac{\cdot 745}{+ 0{,}622} \right) 1{,}02 = \beta \Delta d_{med}
\end{gathered} \tag{18}$$

And considering (10), (11) and (17), we obtain the expression, which allows us to calculate the drying agent average velocity,

$$\omega^{0,8}=0{,}347Re^{0,65}\,Pr^{0,33}\,Gu^{0,135}\,\frac{D}{l}\left[\,0{,}622\cdot\frac{\varphi_{sat}\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot\right.}{745-\varphi_{sat}\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot\right.}\right.$$

$$\left.\frac{\left.\cdot t^{2}+285{,}3\cdot t-593{,}27\right)\cdot 0{,}0075}{\left.\cdot t^{2}+285{,}3\cdot t-593{,}27\right)\cdot 0{,}0075}\right]\cdot\left[1{,}6\cdot 10^{-6}\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot\right.\right.$$

$$\left.\cdot t-593{,}27\right)\cdot 0{,}0075-\frac{0{,}622\cdot\dfrac{\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right.}{745-\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right.}}{0{,}622\cdot\dfrac{\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right.}{745-\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right.}}$$

$$\left.\frac{\dfrac{\left.-593{,}27\right)\cdot 0{,}0075}{\left.-593{,}27\right)\cdot 0{,}0075}\cdot 745}{\dfrac{\left.-593{,}27\right)\cdot 0{,}0075}{\left.-593{,}27\right)\cdot 0{,}0075}+0{,}622}\right]^{-2} \tag{19}$$

By marking by ***K*** the expression:

$$K=0{,}347Re^{0,65}\,Pr^{0,33}\,Gu^{0,135}\,\frac{D}{l}\cdot\left[\,0{,}622\cdot\frac{\varphi_{sat}\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot\right.}{745-\varphi_{sat}\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot\right.}\right.$$

$$\frac{\left.\cdot t^{2}+285{,}3\cdot t-593{,}27\right)\cdot 0{,}0075}{\left.\cdot t^{2}+285{,}3\cdot t-593{,}27\right)\cdot 0{,}0075}-0{,}622\cdot\frac{\varphi_{int}\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot\right.}{745-\varphi_{int}\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot\right.}$$

$$\left.\frac{\left.\cdot t^{2}+285{,}3\cdot t-593{,}27\right)\cdot 0{,}0075}{\left.\cdot t^{2}+285{,}3\cdot t-593{,}27\right)\cdot 0{,}0075}\right]\cdot\left[1{,}6\cdot 10^{-6}\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot\right.\right.$$

$$\left.\cdot t-593{,}27\right)\cdot 0{,}0075-\frac{0{,}622\cdot\dfrac{\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right)}{745-\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right)}}{0{,}622\cdot\dfrac{\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right)}{745-\varphi\cdot\left(0{,}159\cdot t^{3}-8{,}7018\cdot t^{2}+285{,}3\cdot t-\right)}}$$

$$\left.\frac{\dfrac{\left.-593{,}27\right)\cdot 0{,}0075}{\left.-593{,}27\right)\cdot 0{,}0075}\cdot 745}{\dfrac{\left.-593{,}27\right)\cdot 0{,}0075}{\left.-593{,}27\right)\cdot 0{,}0075}+0{,}622}\right]^{-2} \tag{20}$$

We calculate the velocity using the following:

$$\omega=\sqrt[0,8]{K} \tag{21}$$

Due to manageable velocity factor this model allows us to vary drying agent velocity optimizing this way drying process energy costs.

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