PARAMETRIC EXCITON–POLARITON OSCILLATOR IN A MICROCAVITY

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Abstract: *We study the dynamics of parametric oscillations of polaritons in a microcavity that consists of a periodic conversion of a pair of pump polaritons into polaritons of signal and idle modes and vice versa. The period and amplitude of oscillations considerably depend on the initial polariton density, the initial phase difference, and the resonance detuning. We show that there is a possibility of phase controlling the polariton dynamics in the microcavity.*

Key words: *exciton–polariton, microcavity, phase control.*

I. Introduction

Mixed exciton–photon states in planar semiconductor microcavities with quantum wells in the active layer belong to a new class of quasi two dimensional states with unique properties [1–10]. They arise due to a strong coupling of excitons with eigenmodes of electromagnetic radiation of a microcavity, as a result of which upper and lower exciton–polariton microcavity modes are formed. Large interest is drawn to polariton–polariton scattering, due to which the exciton–polariton system demonstrates strongly nonlinear properties. These nonlinearities were revealed in luminescence spectra of microcavities upon resonant excitation of the lower polariton branch, and they are explained by parametric scattering of photoexcited pump polaritons into signal and idle modes. Using the pump–probe method, the authors of [8, 9] were first to observe parametric amplification in a microcavity, while the authors of [9] were first to observe the parametric oscillator regime upon pumping of the lower polariton branch.

II. Statement of the problem and basic equations

The objective of this work is to study exciton–polariton dynamics in the parametric oscillator regime. It was shown in [4, 5] that, upon excitation of exciton–polaritons on the lower branch of the dispersion law, a parametric scattering of two pump polaritons (p) into polaritons of signal (s) and idle (i) modes,which are described by the interaction Hamiltonian of the form

$$H_{\text{int}} = \hbar m\left(\hat{a}_p \hat{a}_p \hat{a}_s^+ \hat{a}_i^+ + \hat{a}_s \hat{a}_i \hat{a}_p^+ \hat{a}_p^+\right), \quad (1)$$

where m is the constant of the parametric polariton–polariton conversion and $\hat{a}_p$, $\hat{a}_s$ and $\hat{a}_i$ are the annihilation operators of polaritons of the corresponding modes (p, s, i). Using (1), we can obtain a system of Heisenberg equations for these operators. Averaging this system and applying the mean_field approximation [11] yield a system of nonlinear evolution equations for the complex amplitudes of polaritons $a_{p,s,i} = \left\langle \hat{a}_{p,s,i} \right\rangle$. Then, we will introduce the polariton densities $n_{p,s,i} = a^*_{p,s,i} a_{p,s,i}$ and two polarization components $Q = i\left(a_p a_p a_s^* a_i^* - a_s a_i a_p^* a_p^*\right)$ and $R = a_p a_p a_s^* a_i^* + a_s a_i a_p^* a_p^*$. As a result, we arrive at the following system of nonlinear differential equations:

$$\dot{n}_p = 2mQ\,,\ \dot{n}_s = \dot{n}_i = -mQ\,,$$
$$\dot{Q} = \Delta R + 2m\left(4n_p n_s n_i - n_p^2 n_s - n_p^2 n_i\right),\ \dot{R} = -\Delta Q\,, \qquad (2)$$

where $\Delta = 2w_p - w_s - w_i$ is the resonance detuning and $w_{p,s,i}$ are the eigenfrequencies of polaritons of the pump (p), signal (s), and idle (i) modes. The initial conditions for the new functions can be represented as

$$n_{p|t=0} = \left|a_{p0}\right|^2 = n_{p0}\,,\ n_{s|t=0} = \left|a_{s0}\right|^2 = n_{s0}\,,\ n_{i|t=0} = \left|a_{i0}\right|^2 = n_{i0}\,,$$
$$Q_{|t=0} \equiv Q_0 = 2n_{p0}\sqrt{n_{s0}n_{i0}}\sin q_0\,,\ R_{|t=0} \equiv R_0 = 2n_{p0}\sqrt{n_{s0}n_{i0}}\cos q_0\,, \qquad (3)$$

where $q_0 = j_{s0} + j_{i0} - 2j_{p0}$ is the initial phase difference and j_{p0}, j_{s0}, j_{i0} are the initial phases of the corresponding complex polariton amplitudes.

From (2), the following integrals of motion can be easily obtained:

$$n_p + 2n_s = n_{p0} + 2n_{s0}\,,\ n_p + 2n_i = n_{p0} + 2n_{i0}\,,$$
$$Q^2 + R^2 = 4n_p^2 n_i n_s\,,\ R = R_0 + \frac{\Delta}{2m}\left(n_{p0} - n_p\right). \qquad (4)$$

It can be easily seen from these expressions that evolution of the system can occur only if at least two of the initial densities of particles are nonzero.

It is convenient to perform further consideration for normalized quantities

$$y = n_p / n_{p0}\,,\ \bar{n}_{s0} = n_{s0} / n_{p0}\,,\ \bar{n}_{i0} = n_{i0} / n_{p0}\,,\ a = \frac{\Delta}{2mn_{p0}}\,,\ t_0^{-1} = mn_{p0}\,,\ t = tt_0\,. \qquad (5)$$

Then, system of equations (4) can be reduced to a single nonlinear differential equation for the normalized density y of pump polaritons,

$$\frac{dy}{dt} = \pm 2q\,,\ q^2 = y^2\left(1 + 2\bar{n}_{s0} - y\right)\left(1 + 2\bar{n}_{i0} - y\right) - \left(2\sqrt{\bar{n}_{s0}\bar{n}_{i0}}\cos q_0 + a\left(1 - y\right)\right)^2. \qquad (6)$$

The quantity $W = -q^2$ is the potential energy of an equivalent nonlinear oscillator at a zero total energy. Qualitatively, the behavior of the function $y(t)$ can be determined by studying the dependence of the potential energy W on y at different values of the parameters $\bar{n}_{s0}$, $\bar{n}_{i0}$, a, and q_0. The evolution of the oscillator can be nontrivial in the range of values of the function $y(t)$ where $W(y) < 0$.

At arbitrary values of the parameters, the solution of Eq. (6) is determined by the values and by the order of arrangement of the roots of the equation $q^2(y) = 0$. This equation has four real roots, which we will arrange in decreasing order as follows: $y_1 > y_M > y_m > y_4$ where y_m and y_M are the minimal and maximal values of the function y, which it acquires in the course of its evolution. They correspond to the minimal and maximal density of pump polaritons in the course of the evolution. The evolution of the system consists of periodic oscillations of the function $y(t)$ in the limits from y_m to y_M. The plus and minus signs in the right hand side of (6) are determined by the direction of variation of the derivative $\dot{y}(t)_{|t=0} = \dot{y}_0$ (the direction of the velocity). For the plus sign, immediately after the initial moment of time, the function will increase with respect to its initial value $y_{|t=0} = y_0 = 1$, with all other conditions being equal. Conversely, for the minus sign, this function will decrease. The function $y(t)$ at $\dot{y} < 0$ differs from the function $y(t)$ at $\dot{y} > 0$ only by a constant phase that depends on the parameters $\bar{n}_{s0}$, $\bar{n}_{i0}$, a and q_0.

In the case $q_0 = 0$, one of the roots of the equation $q^2 = 0$ coincides with the initial condition $y = y_0 = 1$. Therefore, the solution of Eq. (6) will not contain any phase shift. Moreover, if the relation $4\bar{n}_{i0}\bar{n}_{s0} + 2a\sqrt{\bar{n}_{i0}\bar{n}_{s0}} = \bar{n}_{i0} + \bar{n}_{s0}$ holds, the roots y_M and y_m degenerate and oscillations are absent. Therefore, in the case in which the roots of the equation $q^2 = 0$ are ordered such that $y_1 > y_0 = 1 > y_m > y_4$, the density of pump polaritons varies in the limits from y_m to $y_0 = 1$, i.e.,under the background with the density $y = 1$. If the roots are ordered such that $y_1 > y_M > y_0 = 1 > y_4$, the density of pump polaritons oscillates from $y = 1$ to $y = y_m$, i.e., over the background with the same density $y = 1$. In the former case, the solution has the form

$$y = \frac{1 - \dfrac{y_1(1-y_m)}{y_1 - y_m} sn^2\sqrt{(y_1 - y_m)(1-y_4)}t}{1 - \dfrac{1-y_m}{y_1 - y_m} sn^2\sqrt{(y_1 - y_m)(1-y_4)}t} \qquad (7)$$

where the modulus *k* of the elliptic function, the amplitude *A*, and the period *T* of oscillations are given by

$$k^2 = \frac{(y_1 - y_4)(1-y_m)}{(y_1 - y_m)(1-y_4)},\ A = 1 - y_m,\ T = 2K(k)/\sqrt{(y_1 - y_m)(1-y_4)}. \qquad (8)$$

In the latter case, we have

$$y = \frac{1 - \dfrac{y_4(y_M - 1)}{y_M - y_4} sn^2\sqrt{(y_1 - 1)(y_M - y_4)}t}{1 - \dfrac{y_M - 1}{y_M - y_4} sn^2\sqrt{(y_1 - 1)(y_M - y_4)}t}, \qquad (9)$$

where

$$k^2 = \frac{(y_1 - y_4)(y_M - 1)}{(y_1 - 1)(y_M - y_4)},\ A = y_M - 1,\ T = 2K(k)/\sqrt{(y_1 - 1)(y_M - y_4)}. \qquad (10)$$

If we set $y_m = 1$ in (7) or $y_M = 1$ in (9), we will obtain the solution $y(t) = 1 = const$.

The time evolution of the density of pump polaritons and the dependences of the amplitude *A* and the period *T* of solutions on the parameter a at $q_0 = 0$ are given in Fig. 1. From (7) and (9) and from Fig. 1, it can be seen that the density of pump polaritons varies periodically and the oscillation amplitude and period substantially depend on the parameters of the system. The amplitude of oscillations initially decrease with increasing parameter ; then, turns to zero; and, after that, increases and rapidly attains a saturation.

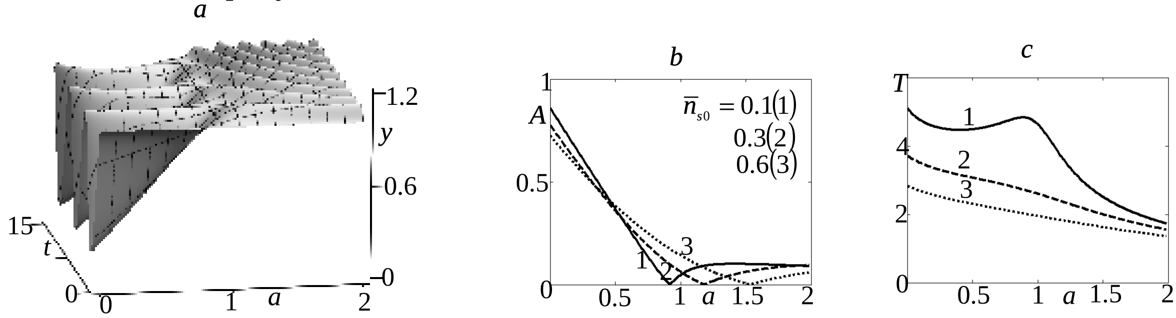

Fig. 1. (a) Time evolution of a normalized density of pump polaritons at $\bar{n}_{s0} = 0.1$, $\bar{n}_{i0} = 0.05$ and and different values of the parameter a ; dependences of the (b) amplitude *A* and (c) period *T* of oscillations on the parameter a at $q_0 = 0$.

Figure 2 presents the evolution of the density of pump polaritons at $q_0 = p$ and different values of a .In this case, the solution of Eq. (6) is expressed by the formula

$$y = \frac{y_M - \dfrac{y_1(y_M - y_m)}{y_1 - y_m} sn^2\left(\sqrt{(y_1 - y_m)(y_M - y_4)}t \pm f(j_0, k)\right)}{1 - \dfrac{y_M - y_m}{y_1 - y_m} sn^2\left(\sqrt{(y_1 - y_m)(y_M - y_4)}t \pm f(j_0, k)\right)} \qquad (11)$$

It can be seen from Fig. 2 that, at $q_0 = p$, as a increases, the periodic evolution regime changes to the aperiodic regime and then becomes periodic again.

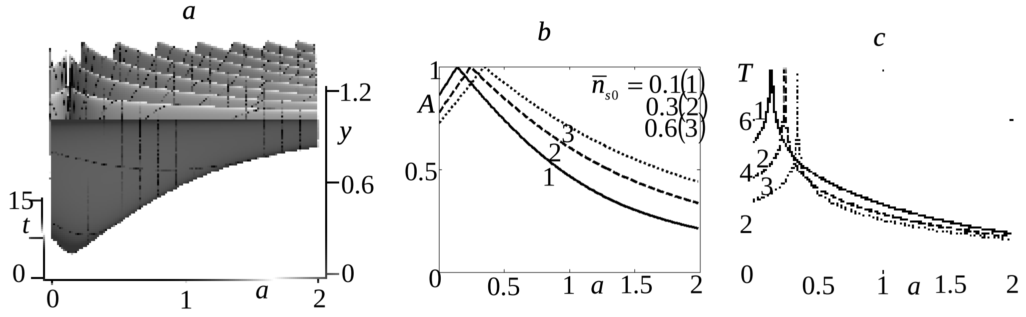

Fig. 2. The same as in Fig. 1, but for $q_0 = p$

III. Conclusions

We found that, in the regime of a parametric oscillator, the dynamics of polaritons is a periodic conversion of a pair of pump polaritons into polaritons of the signal and idle modes and vice versa. The period and amplitude of these oscillations significantly depend on the initial polariton density, the initial phase difference, and the resonance detuning. At a certain relation between the parameters, the evolution of the system can also be aperiodic, as a result of which a part of pump polaritons convert into polaritons of the signal and idle modes, thus completing the evolution. The significant dependences of the period and amplitude of oscillations on the initial phase difference indicate that it is possible to perform the phase control of the dynamics of the system. A similar effect was previously predicted for the process of atomic–molecular conversion under the conditions of the Bose–Einstein condensation of atoms and molecules [12, 13].

IV. References

1. A. V. Kavokin and G. Malpuech, *Thin Films and Nanostructures. Cavity Polaritons*, Ed. by V. M. Agranovich and D. Taylor (Elsevier, Amsterdam, 2003).
2. P. Schwendimann, C. Ciuti, and A. Quattropani, Phys. Rev. B **68**, 165324 (2003).
3. P. G. Savvidis, J. J. Baumberg, D. Porras, et al., Phys. Rev. B **65**, 073309 (2002).
4. M. M. Glazov and K. V. Kavokin, Phys. Rev. B **73**,245317 (2006).
5. I. A. Shelykh, R. Johne, D. D. Solnyshkov, et al., Phys.Rev. B **76**, 155308 (2007).
6. D. M. Whittaker, Phys. Rev. B **63**, 193305 (2001).
7. C. Ciuti, Phys. Rev. B **69**, 245304 (2004).
8. P. G. Savvidis, J. J. Baumberg, R. M. Stevenson, et al.,Phys. Rev. Lett. **1547**, 84 (2000).
9. J. J. Baumberg, P. G. Savvidis, R. M. Stevenson, et al., Phys. Rev. B **62** (2000).
10. I. A. Shelykh, A. V. Kavokin, and G. Malpuech, Phys. Stat. Sol. B **242**, 2271 (2005).
11. L. P. Pitaevskii, Usp. Fiz. Nauk **168**, 641 (1998).
12. P. I. Khadzhi and D. V. Tkachenko, JETP Lett. **83** (3), 91 (2006); JETP **104** (3), 379 (2007).
13. P. I. Khadzhi and D. V. Tkachenko, J. Nanoelectr. Optoelectr. **4**, 101 (2009).