STUDY OF THE REACTION FORCES IN A PACKING MECHANISM

V. Ciauşu, PhD, E. Merticaru, PhD, prof.
Technical University "Gh. Asachi" of Iaşi

1. INTRODUCTION

In the paper there are calculated the reaction forces, considering the redundant constraints through the effort method from the materials resistance. There is considered a mechanism in the structure of a butter-packing machine. The reactions are calculated and the influence of the redundant constraints on the reactions is analyzed.

The presence of the redundant constraints in mechanisms can have important negative effects: wear of the kinematic couplings, reduction of the mechanical efficiency, vibrations or blocking of the mechanism [2, 4, 7, and 8]. The quantitative evaluation of these effects is difficult because the mechanisms with redundant constraints are undetermined systems from the point of view of the reactions. At high working speeds, the redundant constraints influence especially on the dynamic phenomena.

An algorithm for the calculus of the reactions in kinematic couplings, taking into account the redundant constraints will be established. All theoretical considerations will be particularized on the packet forming mechanism from a butter-packing machine (NAGEMA – type PA1).

2. THEORETICAL ONSIDERATIONS

Methods from the materials resistance, based on the element deformation must be used, to raise the indetermination. Such methods are the effort method or the displacement method [1]. These methods have a great degree of generality, but their application requires some particularizations for every category of mechanisms. Like in other methods, [2, 3, 8], the mechanisms are considered to be spatial, the degree of indetermination being equal to the number of redundant constraints. To determinate all the reactions, in addition to the equations of dynamic equilibrium, supplementary equations are needed, their number being equal to the number of redundant constraints.

In sequel, the specific conditions and the algorithm to raise the indetermination of the packing mechanisms through the effort method will be established.

The raise of the indetermination in the packing mechanisms through the effort method must take into account some particularities. The algorithm for the calculus of the reactions from the couplings in the packing mechanisms with redundant constraints comprises the following steps:

a) The establishing of the mechanism loading scheme: the mechanism is considered to be spatial. All external and internal loads, which act on the mechanism, will be established, including the local and the on contour redundant constraints. A loading scheme of the mechanism with all forces and moments will be established.

b) The base system choosing: through base system is meant the mechanism scheme in which the number of reactions is equal to the number of dynamic equilibrium equations. This system is obtained by removing, from the mechanism loading scheme, a number of reactions equal to the number of redundant constraints. So, for the same mechanism, many base systems can be obtained, depending on the removed reactions, complying with following conditions: 1) the base system must be static determined; 2) the base system must be geometrically undeformable for a given position of the driving element.

c) The reactions determination in the base system: the calculus of reactions in the base system will be made neglecting the friction in the kinematic pairs. There will be considered that the mechanism contains *n* mobile elements, and the element *v* is subjected to: F_v = the resultant of all internal and external forces that act on the element *v*; M_v = the resultant of all internal and external moments that act on element *v*. Relating to the coordinate system *Oxyz,* neglecting the friction in the kinematic pairs, to element *v* are corresponding six equilibrium equations for forces and moments.

For the entire mechanism will result 6n equilibrium equations that can be written as follows:

$$A \cdot R = B \qquad (1)$$

where: A – is the coefficients matrix; R – is the unknowns matrix (the matrix of reactions $F_{(0)uv}$); B – the matrix of external and internal forces and moments that act on the mechanism; $u = \overline{1,n}$, $v = \overline{1,n}$.

The solution of equation (1) is:

$$R = A^{-1} \cdot B \qquad (2)$$

where A^{-1} – is the inverse of matrix A.

d) The determination of the reactions due to unit efforts that act on the direction of the redundant constraints: in this stage, the base system is considered to be loaded, successively, only with one unit effort applied on the direction of one redundant constraint. The unit efforts are denoted with X_1, X_2 ….. X_p, where *p* is the number of redundant constraints. For each loading of the base system with one of the *p* unit efforts, the system of dynamic equilibrium equations will be written (6n equations with 6n unknowns). The reactions $F_{(i)uv}$, due to the unit effort i ($i = \overline{1,p}$) are obtained by solving the system of equations written as in relation (2).

e) The determination of the redundant constraints: the principle that lies at the base of raising the indetermination of the packing mechanisms through the effort method is the following: the displacements sum on each direction of the redundant constraints, due to external loads and to redundant constraints, has to be equal to the real displacement of the mechanism in that direction. We will consider that in the packing mechanisms, the elastic deformations in kinematic pairs are null.

Based on this principle, for a given mechanism it results the following elastic equilibrium equations system:

$$C \cdot E = D \qquad (3)$$

where: $C = [\delta_{ij}]$; $E = [x_i]^T$; $D = [\delta_{i0}]^T$; C = the matrix of coefficients δ_{ij}; E = the matrix of redundant constraints; D = the matrix of coefficients δ_{i0}; δ_{ij} = the displacement on the direction of the redundant constraint X_i produced by the redundant constraint X_j=1 applied to the base system; δ_{ii} = the displacement on the direction of the redundant constraint X_i produced by the redundant constraint X_i=1 applied to the base system; δ_{i0} = the displacement on the direction of the redundant constraint X_i in the base system, produced by the external loads. The coefficients δ_{ij} are independent on the external loads, they depending only on the geometry of the base system (i=1...p, j=1...p).

The solution of equation (3) is:

$$E = C^{-1} \cdot D \qquad (4)$$

where: C^{-1} = the inverse of matrix C.

The coefficients δ_{ij} and δ_{i0} are calculated on the basis of the general expression of the displacements established with the Mohr-Maxwel method:

$$\delta = \sum_b \int \frac{M \cdot m}{EI} dx \qquad (5)$$

where: M – the bending moment in the current section x; m – the bending moment in the current section x produced by a force equal to unity (P=1) applied in the section where the displacement is calculated; E – the elasticity modulus; I – the axial moment of inertia of the current section x.

From all stresses, for the calculus of the redundant constraints in the packing mechanism, there will be taken into account only that due to the bending loads, the others could being neglected. By particularizing the relation (5), the coefficients of the redundant constraints from relation (3) are obtained:

$$\delta_{ii} = \sum_b \int \frac{m_i^2}{EI} dx \,;\, \delta_{ij} = \sum_b \int \frac{m_i \cdot m_j}{EI} dx \,; \qquad (6)$$
$$\delta_{i0} = \sum_b \int \frac{M_0 \cdot m_i}{EI} dx$$

where: m_i, m_j – the bending moment in current section x produced by a unity load applied on the direction of the redundant constraint i respectively j; M_0 – the bending moment in current section x due to the external forces and moments.

f) The determination of the total reactions in kinematic pairs, with including the effect of the redundant constraints: the total value of a reaction from the base system is calculated on the basis of the effects superimposing principle. On the reaction determined by the external loads is superimposed the effect of the redundant constraints:

$$F_{uv} = F_{(0)uv} + F_{(1)uv} \cdot X_1 + ... + \qquad (7)$$
$$+F_{(i)uv} \cdot X_i + ... + F_{(p)uv} \cdot X_p$$

where: F_{uv} – one of the components of the reaction of the element *u* on the element *v*.

3. NUMERICAL RESULTS

The above-presented algorithm was applied at the forming mechanism from the machine NAGEMA (type PA1) for packing butter in packets. The characteristics of the mechanism are the following: cam driven, structure formed by two independent contours, it contains local and on contour redundant constraints, the rotation speed of the camshaft is 80÷120 rot/min, negligible external forces.

The mechanism is studied in several structural variants that differ through the number of the redundant constraints, with the following loading schemes with external and internal forces:

- the variant a, with 5 redundant constraints on contour and 4 local redundant constraints, in figure 1;

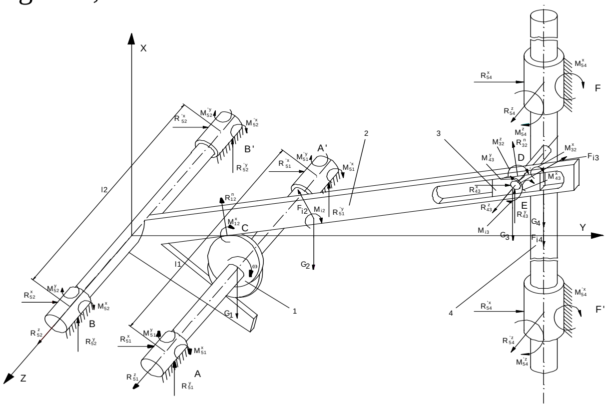

Figure 1. The internal and external forces which act on the forming mechanism from the machine NAGEMA for packing butter in packets.

-the variant b, with 2 redundant constraints on contour and 4 local redundant constraints. The specific elements are presented in figure 2, the other loadings being the same as in figure 1;

-the variant c, with 3 redundant constraints on contour and 4 local redundant constraints. The specific elements are presented in figure 3, the other loadings being the same as in figure 1;

-the variant d, with 2 redundant constraints on contour. The specific elements are presented in figure 4, the other loadings being the same as in figure 1.

The loading schemes were made with the following hypothesis: 1) the rollers of the follower 2 that are in contact with the cams with conjugated profiles 1 were neglected; 2) only the working phase, in which the contact cam-follower is made between the superior arm of the follower 2 and the corresponding profile of cam 1, was considered.

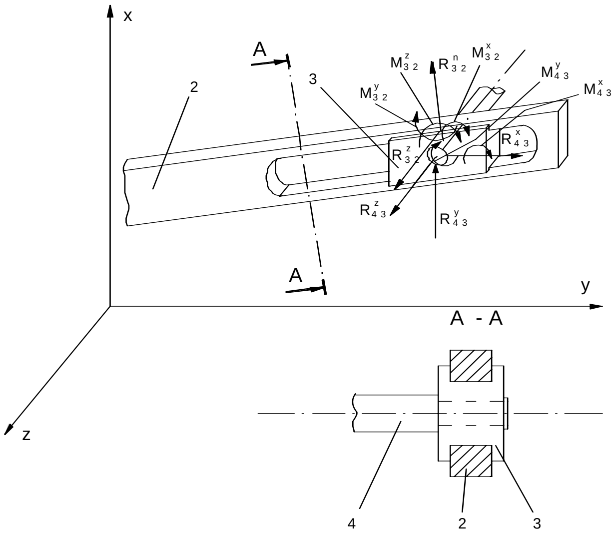

Figure 2. The constructive form of the slider for the variant ***b.***

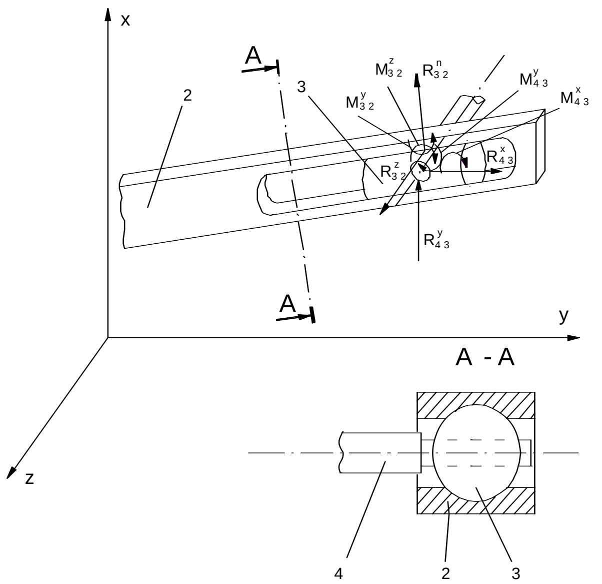

Figure 3. The constructive form of the slider for the variant ***c.***

As an example of results, the figures 5 and 6 are presented.

These figures contain the variation diagrams of the reaction moments about z axis, in pairs F and F' of the mechanism, with or without the effect of the redundant constraints, depending on the position angle of cam. In the figures 5 and 6, the variants that does not appear, from the variants 0, a, b, c, d, have value zero.

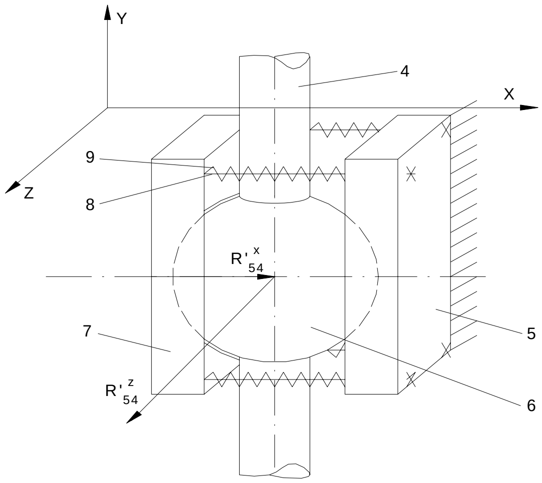

Figure 4. The constructive form of the kinematic pair F' for the variant ***d.***

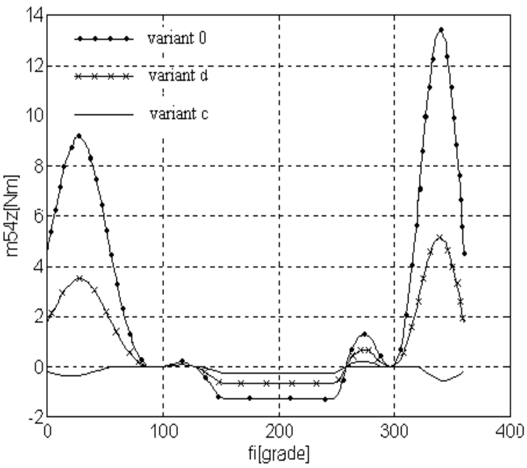

Figure 5. The variation of the reaction force m_{54z} depending on the cam position angle φ, for the variants ***0, d, c.***

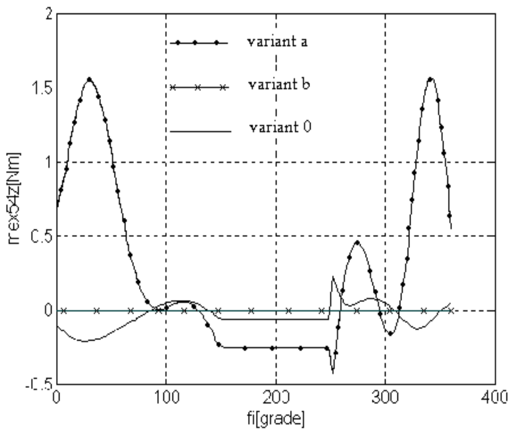

Figure 6. The variation of the reaction force m_{ex54z} depending on the cam position angle φ, for the variants ***0, a, b.***

4. CONCLUSIONS

- The calculus of the reaction forces, taking into account the redundant constraints, is made by adding, to the classical equilibrium equations, new equations which reflect the deformation of the kinematic elements.
- The redundant constraints have relatively small values, but through their position and great number they can have important negative effects on mechanism working.
- In all mechanism couplings, excepting coupling E, it can be observed the increasing of the reactions values, as an effect of the redundant constraints.
- The nonzero reactions have peak values at the piston stroke endpoints; this indicates the presence of shocks during the work of the mechanism.
- The variant d is the most advantageous, the reactions in this case having the most close values to those corresponding to the base system.
- The study in the present paper shows that the redundant constraints can influence considerably the reaction forces in complex mechanisms.

Bibliography

1. ***Buzdugan Gh.*** *Rezistența materialelor, Editura Academiei, Bucureşti, 1986.*
2. ***Dudiță F., Diaconescu D.*** *Optimizarea structurală a mecanismelor, Editura Tehnică, Bucureşti, 1987.*
3. ***Fayet M.*** *Mecanismes multi-boucles-III. Hyperstatisme au sens de la dynamique et au sens de la cinematique-dualite, Mechanism and Machine Theory,* ***30****, 2 (1995).*
4. ***Frolov K. V.*** *Teoriya mehanizmov i mashin, Vîshaya Shkola, Moskva, 1987.*
5. ***Merticaru E., Merticaru V.jr.*** *Elemente de dinamica mecanismelor cu came. Editura Vie, Iaşi, 1999.*
6. ***Merticaru V., Ciauşu V., Merticaru E.*** *Ambalaje şi mecanisme de ambalat, Editura O.I.D.-I.C.M., Bucureşti, 1997.*
7. ***Merticaru V., Ciauşu V.*** *Possibilities for Diminution of Excess Joints Number in Mechanisms, Rev. Roum. Sci. Techn. – Mec. Appl.,* ***39****, 5 (1994).*
8. ***Reshetov L. N.*** *Samoustanavlivayushhijsya mehanizm, Mashinostroenie, Moskva, 1985.*

Recomandat spre publicare: 19.09.2012