Overview of Nonlinear Partial Differential Equation-based Structural Inpainting Techniques

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Image interpolation, known also as inpainting or completion, represents the process of recovering the missing or highly deteriorated regions of the image, as plausibly as possible, by using the information achieved from the known surrounding areas. The inpainting techniques are divided into three main categories: structure-based, texture-based and combined reconstruction approaches.
We consider only the structural image interpolation domain that comprises variational and partial differential equation (PDE) - based inpainting models. A comprehensive overview of the state of the art nonlinear diffusion-based inpainting techniques is provided here. Our own contributions to this image processing field are also highlighted.
Nonlinear PDE-based interpolation models of various orders are described in this survey. Many of them follow the variational principles, while other PDE inpainting models do not derive from variational schemes, being directly given as evolutionary equations. The state of the art second-order diffusion-based interpolation schemes, given in variational or PDE-based form, are presented first. Thus, we describe the Harmonic Inpainting, the variational inpainting models based on the Mumford-Shah functional and the total variation-based reconstruction algorithms, such as the influential TV Inpainting and other variational schemes derived from it.
Higher-order PDE-based interpolation approaches following variational principles, such as TV^2 Inpainting, Total Generalized Variation (TGV) Inpainting or Euler's Elastica Inpainting, are disscused and compared next. Then, we focus on the non-variational high-order PDE completion models. The third-order PDE-based structural inpainting solutions include the pioneering interpolation model introduced by Bertamio et. al, Navier-Stokes equation-based inpainting and Curvature-driven Diffusion (CDD) Inpainting scheme. The state of the art fourth-order differential models for image interpolation that are described here include Cahn-Hillard Inpainting, TV-H^{-1} Inpainting and LCIS Inpainting. Finite difference-based numerical approximation algorithms and interpolation experiments are provided for the surveyed techniques.
We have also conducted a high amount of research in the diffusion-based image restoration and interpolation domains in the last decade. Some of the most important structure-based inpainting techniques developed by us are also included in this overview. So, we briefly describe some nonlinear parabolic second-order PDE-based inpainting models that can be achieved from variational problems and are derived from our past anisotropic diffusion-based restoration methods by introducing image masks corresponding to the inpainting regions. Variational hybrid interpolation techniques that combine second- and fourth-order nonlinear diffusions are also presented here. Some structural inpainting approaches based on PDE models not following variational principles, which have been proposed by us, are disscused next. They include a nonlinear hyperbolic PDE-based image reconstruction technique and a second-order anisotropic diffusion-based interpolation framwework. Several numerical experiments and method comparison that illustrate the effectiveness of our inpainting methods are also described in this survey.
Keywords. Structural inpainting, Variational scheme, Nonlinear diffusion, Second-order PDE model, High-order PDE, Total variation, Finite difference method, Numerical approximation.