Inverse problem for a two-dimensional strongly degenerate heat equation

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We consider an inverse problem for a two-dimensional degenerate heat equation in a rectangular domain. Direct problems of this type are mathematical models of various processes such as seawater desalination, movement of liquid in porous medium, financial market behavior.
Inverse problems arise when certain parameters of these processes are unknown. Var- ious types of inverse problems for non-degenerate equations are well investigated and some results may be found in many monograph. In the domain $Q_T := (x, y, t) : 0 < x < h, 0 < y < l, 0 < t < T$ we consider a two- dimensional heat equation with unknown leading coefficient depending on the time variable. We suppose that the equation degenerates at the initial moment as a power with a given exponent $\beta \geq 1$. We choose the case of mixed Dirichlet-Neumann boundary conditions. The additional condition (so-called overdetermination condition) is taken accordingly to the physical sense and it presents the value of the heat flux on the part of the boundary of domain. So, the problem consists of finding a pair of functions $(a(t), u(x, y, t)), a(t) > 0, t \in [0, T]$ that satisfy the degenerate heat equation

$$u_t = t^\beta a(t) \triangle u + f(x, y, t), (x, y, t) \in Q_T,$$

initial condition

$$u(x, y, 0) = \phi(x, y, 0), (x, y) \in \widetilde{D} := [0, h] \times [0, l],$$

boundary conditions

$$u(0, y, t) = \mu_1(y, t), u(h, y, t) = \mu_2(y, t), (y, t) \in [0, l] \times [0, T],$$

$$u_y(x, 0, t) = \nu_1(x, t), u_y(x, l, t) = \nu_2(x, t), (x, t) \in [0, h] \times [0, T]$$

and overdetermination condition

$$a(t)u_x(0, y_0, t) = \chi(t), t \in (0, T],$$

where $y_0 \in (0, l)$ is some arbitrary fixed point. Our goal is to determine the conditions of existence and uniqueness of classical solution of the problem.