Set-Valued Almost Periodic Functions and Perfect Mappings

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Fix a natural number $n \geq 1$. Denote by d the Euclidean distance on the n-dimensional Euclidian space $\mathbb{R}^n$ and by $Com(\mathbb{R}^n\ )$ the space of all non-empty compact subsets of $\mathbb{R}^n$ with the Pompeiu-Hausdorff distance $d_P(A, B)$. The space $\mathbb{R} = \mathbb{R}^1$ is the space of reals and $\mathbb{C} = \mathbb{R}^2$ is the space of complex numbers. The space $(Com(\mathbb{R}^n), d_P)$ is a complete metric space.
Fix a topological space G. By $T(G)$ denote the family of all single-valued continuous mappings of G into G. Relatively to the operation of composition, the set $T(G)$ is a monoid (a semigroup with unity).

A single-valued $\varphi : G \longrightarrow Com(\mathbb{R}^n)$ is called a set-valued function on G. For any two set-valued functions $\varphi, \psi : G \longrightarrow B(\mathbb{R})$ and $t \in \mathbb{R}$ are determined the distance $\rho(\varphi, \psi) = sup\{d_P(\varphi(x), \psi(x)) : x \in G\}$ and the set-valued functions $\varphi + \psi$, $\varphi \cdot \psi$, $-\varphi$, $\varphi \cup \psi$, where $(\varphi \cup \psi)(x) = (\varphi(x) \cup \psi(x)$, and $t\varphi$. We put $(\varphi \circ f)(x) = \varphi(f(x))$ for all $f \in T(G)$, $\varphi \in SF(G)$ and $x \in G$. Let $SF(G, \mathbb{R}^n)$ be the space of all set-valued functions on G with the metric ρ. The space $SF(G, \mathbb{R}^n)$ is a complete metric space.

A set-valued function $\psi : G \longrightarrow \mathbb{R}^n$ is called lower (upper) semicontinuous if the set $\psi^{-1}(H)$ $= \{x \in G : \psi(x) \cap H \neq \emptyset\}$ is an open (a closed) subset of G for any open (closed) subset H of the space R^n. Denote by $LSC(G, \mathbb{R}^n)$ the family of all lower semicontinuous functions and by $USC(G, \mathbb{R}^n)$ the family of all upper semicontinuous functions on the space G.

If $\varphi \in SF(G)$ and $f \in T(G)$, then $\varphi_f = \varphi \circ f$ and $\varphi_f(x) = \varphi(f(x))$ for any $x \in G)$. Evidently, $\varphi_f \in SF(G)$.

Fix a submonoid P of the monoid $T(G)$. We say that P is a monoid of continuous translations of G. The set P is called a transitive set of translations of G if for any two points $x, y \in G$ there exists $f \in P$ such that $f(x) = y$. In particular, $1_G \in P$, where $1G$ is the identical translation on the space G.

For any function $\varphi \in SF(G, \mathbb{R}^n)$ we put $P(\varphi) = \{\varphi_f : f \in P\}$.

Definition 1. *A function $\varphi \in SF(G, \mathbb{R}^n)$ is called a P-periodic function on a space G if the closure $\bar{P}(\varphi)$ of the set $P(\varphi)$ in the space $SF(G, \mathbb{R}^n)$ is a compact set.*

Denote by P-$ap_s(G, \mathbb{R}^n)$ the subspace of all P-periodic set-valued functions on the space G.

Let $\mu : A \longrightarrow B$ be a perfect mapping of a space A onto a space B, $P_A \subset T(A)$, $P_B \subset T(B)$ and $h : P_A \longrightarrow P_B$ is a single valued mapping such that $\mu(f(x)) = h(f)(\mu(x)$ for any $x \in A$ and $f \in P_A$. Consider the mapping $\Phi_{(\mu,h)} : SF(A, \mathbb{R}^n) \to SF(B, \mathbb{R}^n)$ and $\Psi_{(\mu,h)} : SF(B, \mathbb{R}^n) \to SF(A, \mathbb{R}^n)$, where $\Phi_{(\mu,h)}(\varphi) = \mu \circ \varphi$ for each $\varphi \in SF(B, \mathbb{R}^n)$ and $\Psi(\psi)_f(y) = \mu(\psi_g(\mu^{-1}(y))$, where $h(g) = f$. We have $\Psi(\psi)_f(y) = \cup\{\mu(\psi_g(\mu^{-1}(y)) : h(g) = f\}$.

Theorem 1. *The mappings $\Phi_{(\mu,h)}$ and $\Psi_{(\mu,h)}$ are continuous.*

Theorem 2. *$\Phi_{(\mu,h)}(P_B$-$ap_s(B, \mathbb{R}^n) \subset P_A$-$ap_s(A, \mathbb{R}^n)$ and*

$Psi_{(\mu,h)}(P_A$-$ap_s(A, \mathbb{R}^n) \subset P_B$-$ap_s(B, \mathbb{R}^n)$.