Extreme Points in the Complex of Multy-ary Relations

Sergiu Cataranciuc, Galina Braguţa

State University of Moldova, Chişinău, Republic of Moldova
e-mail: s.cataranciuc@gmail.com, gallinna@yahoo.com

Let $\mathcal{R}^{n+1} = \left(R^1, R^2, ..., R^{n+1}\right)$ be a complex of multy-are relations, defined in the work [1]. We denote by d_k^m the distance function defined on the set R^k [2]. Let $d_k^m - conv(A)$ be the convex hull of a subset A from the metric space (R^k, d_k^m).

Definition 1. The cortege $r = (x_{i_1}, x_{i_2}, ...x_{i_k}) \in R^k$ is called m-extreme point of the set $A \subset R^k$, $1 \leq k < m \leq n+1$ if:

a) $r \in A$;

b) $r \notin d_k^m - conv(A - r)$.

Knowing m-extreme points of a set often simplifies the procedure of convex hull construction and the study of its properties. Let denote by $ext^m(A)$ the set of all m-extreme points of the set A.
Lemma 1. If A is a subset from m-ary relation R^k and $r \in ext^m(A)$ then $r \in ext^m(d_k^m - conv(A))$.
From Lemma 1 results that $ext^m(A) \subset ext^m(d_k^m - conv(A))$.
Let be $r \in A \subset R^k$. We denote by $\Gamma_A^m(r) = \{z \in A : z \cup r \in R^m\}$ the set of all elements from A that are joined with r through a m-dimensional chain of length one. Such a set may be named m-dimensional neighborhood of the element r in A.
A complex of multi-ary relations $\mathcal{R}^{n+1} = \left(R^1, R^2, ..., R^{n+1}\right)$, defined on a set of elements X is complete, if $R^1 = X$ and $R^s = X^s$, $2 \leq s \leq n+1$. The complex $\mathcal{R}^{n+1}$ is named m-complete, if $R^m = X^m$, $2 \leq m \leq n+1$.
Lemma 2. If the complex $\mathcal{R}^{n+1} = \left(R^1, R^2, ..., R^{n+1}\right)$ is m-complete, then it is and t-complete, for any $2 \leq t \leq m$.
Definition 2. The cortege $r \in A \subset R^k$ is named m-simplicial cortege in A, if the set $\Gamma_A^m(r)$ generates a m-complete subcomplex.
Theorem 1. If the cortege $r \in A \subset R^k$ is m-simplicial in A, then it is m-simplicial and in the set $d_k^m - conv(A)$.
It follows conditions in which an arbitrary cortege r from the set $A \subset R^k$ is m-extreme point in A.
Theorem 2. The cortege $r \in A \subset R^k$ is m-extreme point in A, if and only if r is m-simplicial cortege in A.
Theorem 3. If $d_k^m - conv(A)$ is the convex hull of a set $A \subset R^k$, then any m-extreme point from $d_k^m - conv(A)$ is m-extreme point in A.

Bibliography

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