Example of ternary non-commutative Moufang loop

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It is demonstrated that there exist ternary Moufang loops that are different from ternary groups. Let $(K, +, \cdot, 1)$ be an associative ring (not necessary commutative) which has characteristic 3, i.e., $x + x + x = 0$ for all $x \in K$.
By $K'(\cdot)$ we denote an abelian subgroup of the group $K^*(\cdot)$. Here $K^* = K \setminus \{0\}$. The map $x \to s \cdot x$ for all $x \in K$ is a permutation for any $s \in K'$. Moreover, we require that $s^2 = 1$ for all $s \in K'$.
In particular case $K = Z_3$ is a ring of residues modulo 3.
On the set $Q = K' \times K = \{< s, k > | \ s \in K', k \in K\}$ we define the following ternary operation

$$A(< s_1, x_1 >, < s_2, x_2 >, < s_3, x_3 >) =< s_1 s_2 s_3, s_2 x_1 + s_3 x_2 + s_1 x_3 > \quad (1)$$

for all $s_1, s_2, s_3 \in K', x_1, x_2, x_3 \in K$.
Algebra $Q(A)$ with operation defined on the set $Q = K' \times K$ by the formula (1) is a ternary non-commutative Moufang loop that is not a ternary group.
Example of ternary commutative Moufang loop that is not a ternary group is also constructed.

Bibliography

[1] V.D. Belousov. *n-Ary Quasigroups*, Stiintsa, Kishinev, (1971).